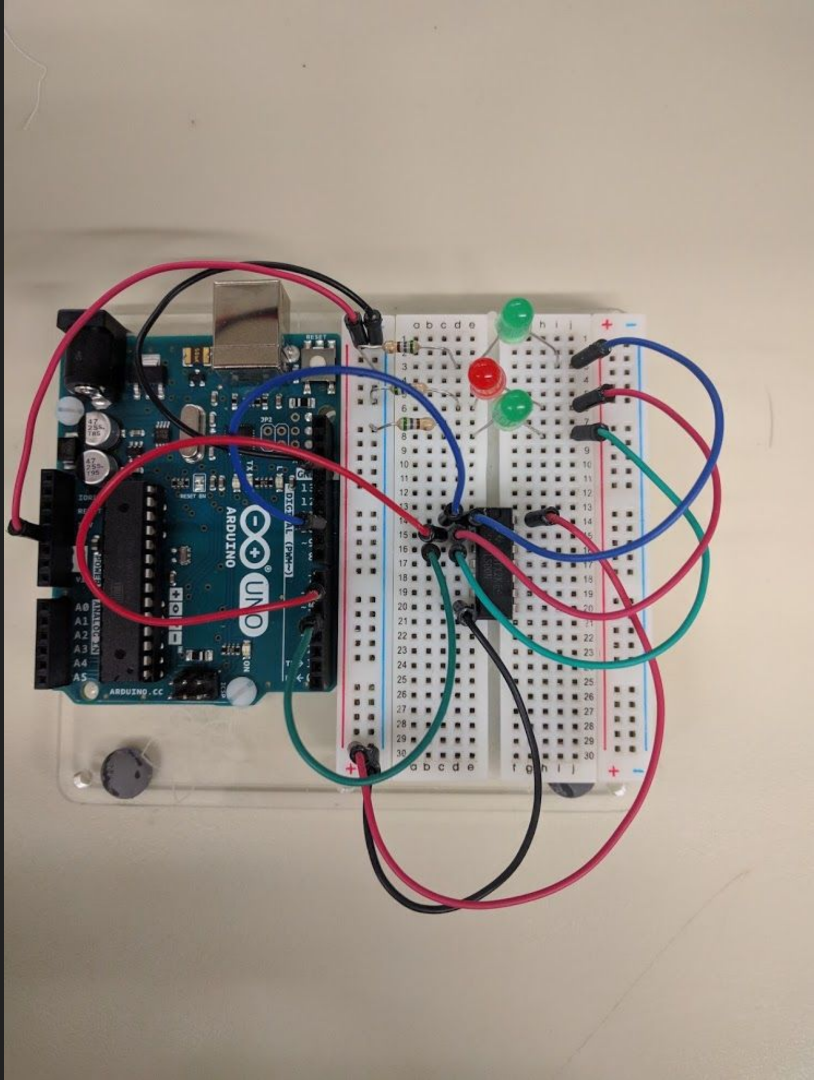
Recitation 6

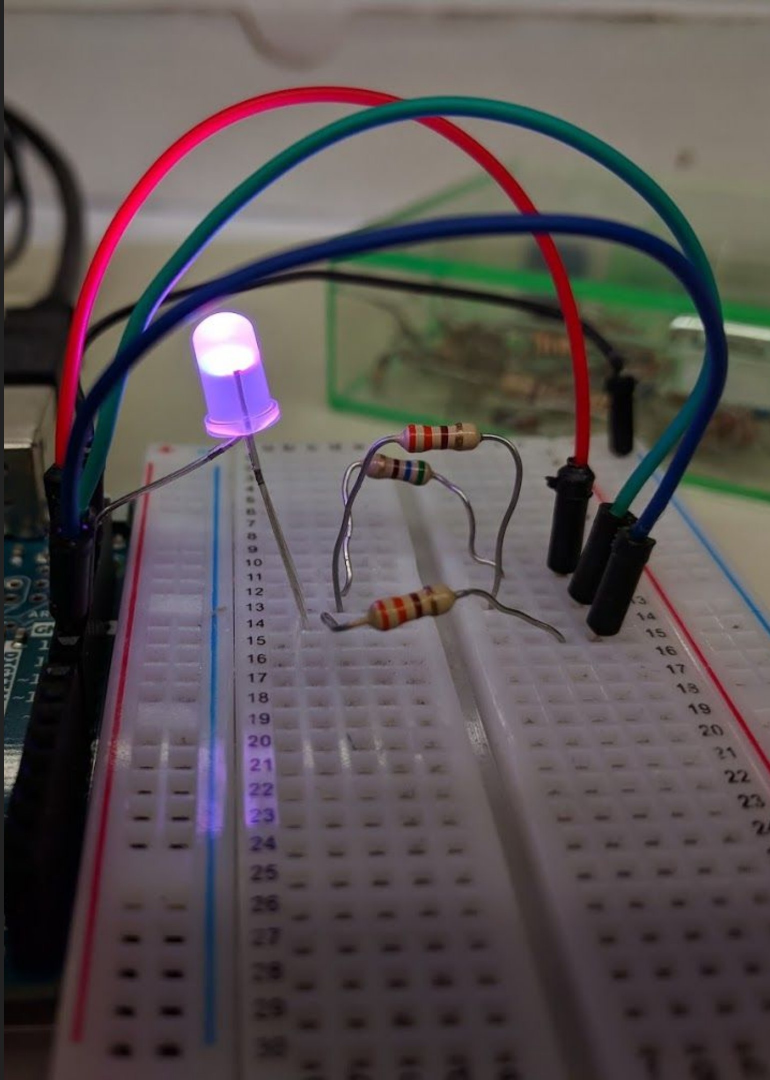
Circuits are my favorite



Review: What You’ve Done (or are about to do)

Circuits

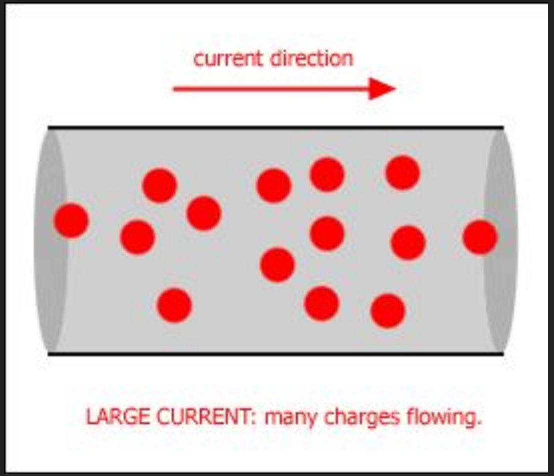
- Components linked together with conductors (wires)
- Electric fields push electrons around to create a current
- We create these electric fields with a potential difference (usually from a battery)

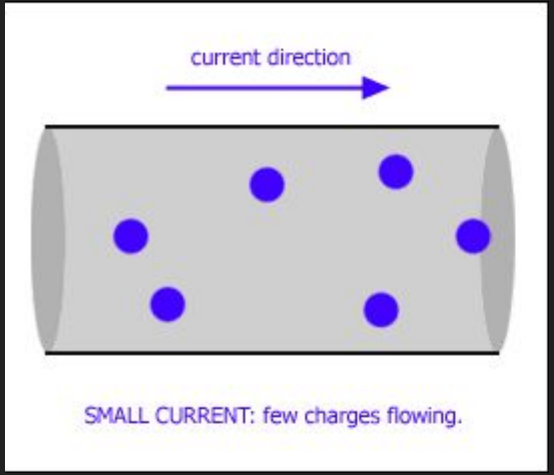


Current

- An amount of charge moving through a cross sectional area of a wire in a given amount of time
- Units of Amperes, A (C/s)
- Measured with an ammeter (1 A is usually a lot!)

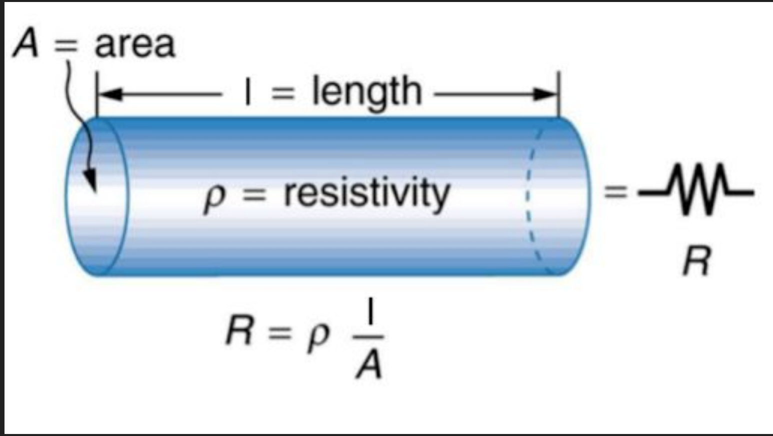
$$I = \frac{\Delta q}{\Delta t}$$





Resistance

- Conductivity σ tells us how easily it is for the free charges to flow in the conductor (microscopic material property)
- Resistivity ρ tells us how much the material resists the free charges to flow (microscopic material property)
- Resistance how much a macroscopic material resists the flow of current (macroscopic material and geometric property)



Conductivity

$$\sigma = \frac{q^2 n \Delta \bar{t}}{m_q}$$

Resistivity

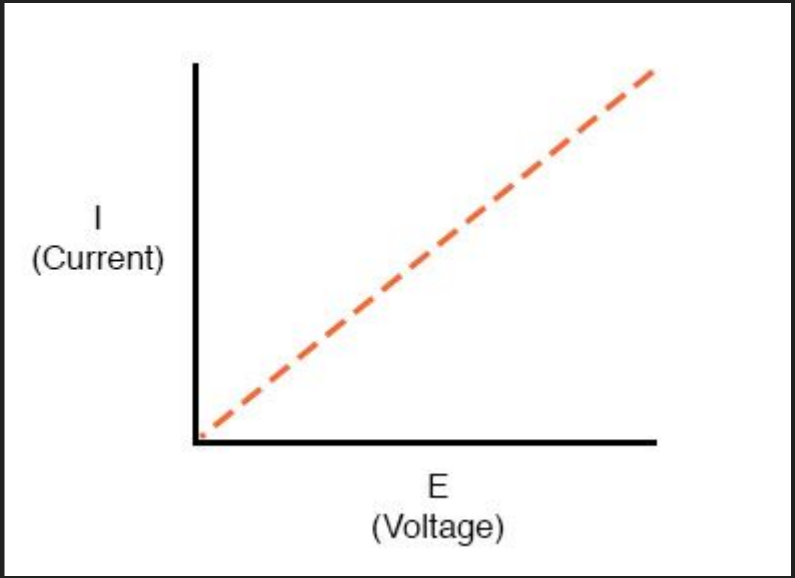
$$\rho = 1/\sigma$$

Resistance

$$R = \frac{\rho L}{A}$$

Relating them all

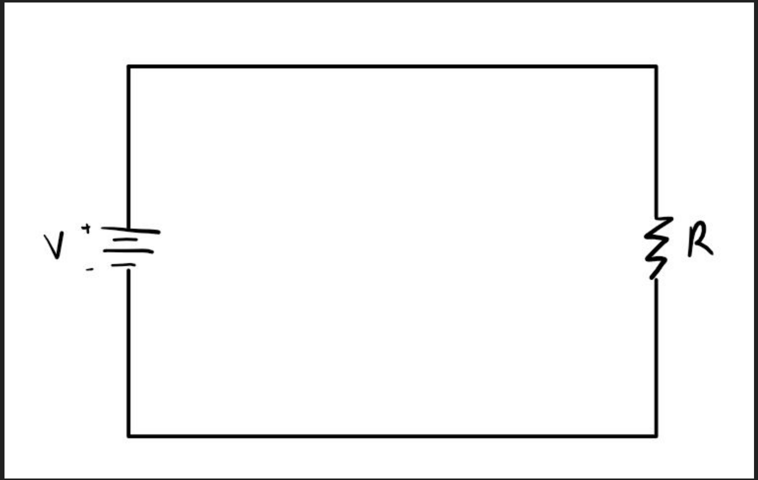
- Ohm's "Law" tells us how current, voltage, and resistance relate for Ohmic devices
- Lightbulbs and resistors are Ohmic devices
- Devices that don't follow Ohm's "Law" are called non-Ohmic



$$V = IR$$

Circuit elements

- Resistors
- Voltage sources
- Lightbulbs (just a glowing resistor)



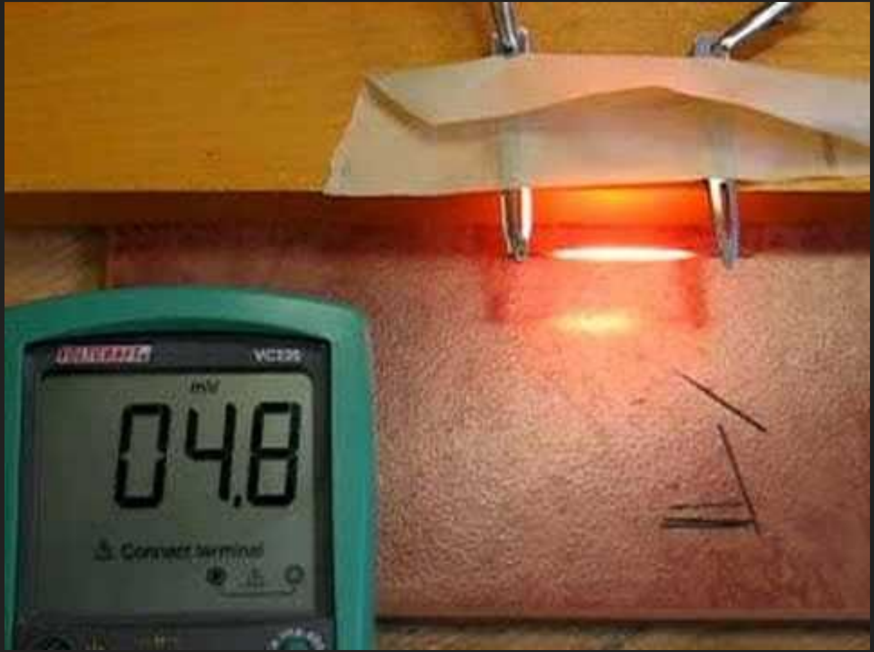
Power

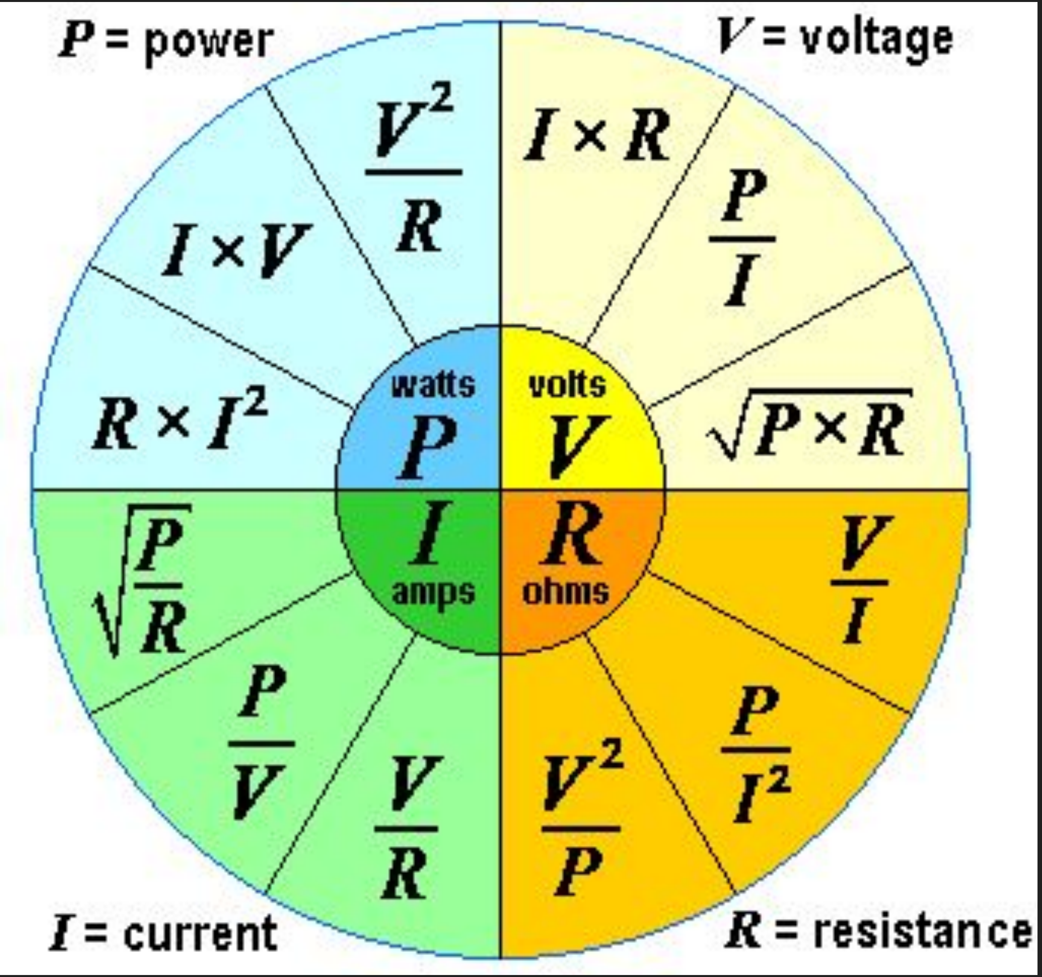
- We can talk about the energy/time a resistor dissipates

$$P = IV$$

$$P = RI^2$$

$$P = V^2/R$$

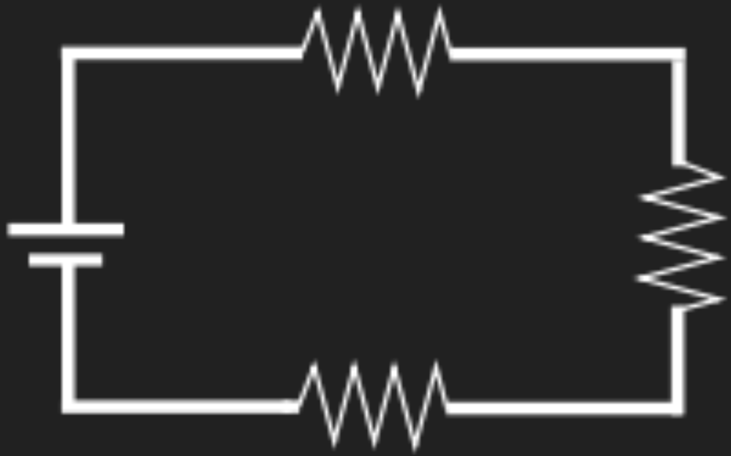




P = power
V = voltage
I = current
R = resistance
watts
P
volts
V
I
amps
R
ohms
$\frac{V^2}{R}$
$I \times V$
$R \times I^2$
$I \times R$
$\frac{P}{I}$
$\sqrt{P \times R}$
$\sqrt{\frac{P}{R}}$
$\frac{P}{V}$
$\frac{V}{R}$
$\frac{V}{I}$
$\frac{P}{I^2}$
$\frac{V^2}{P}$

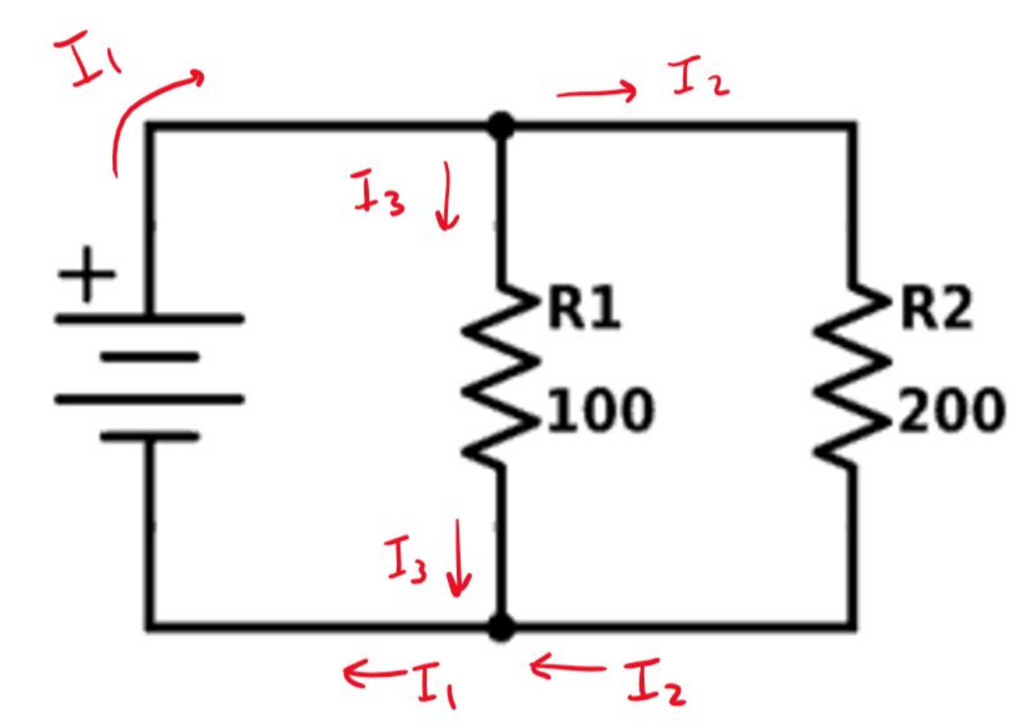
Connecting in Series

- Resistors connected in a single conducting path
- Any given electron will pass through each of the resistors
- If we cut one of the resistors out, no current will flow
- Same current going through each resistor

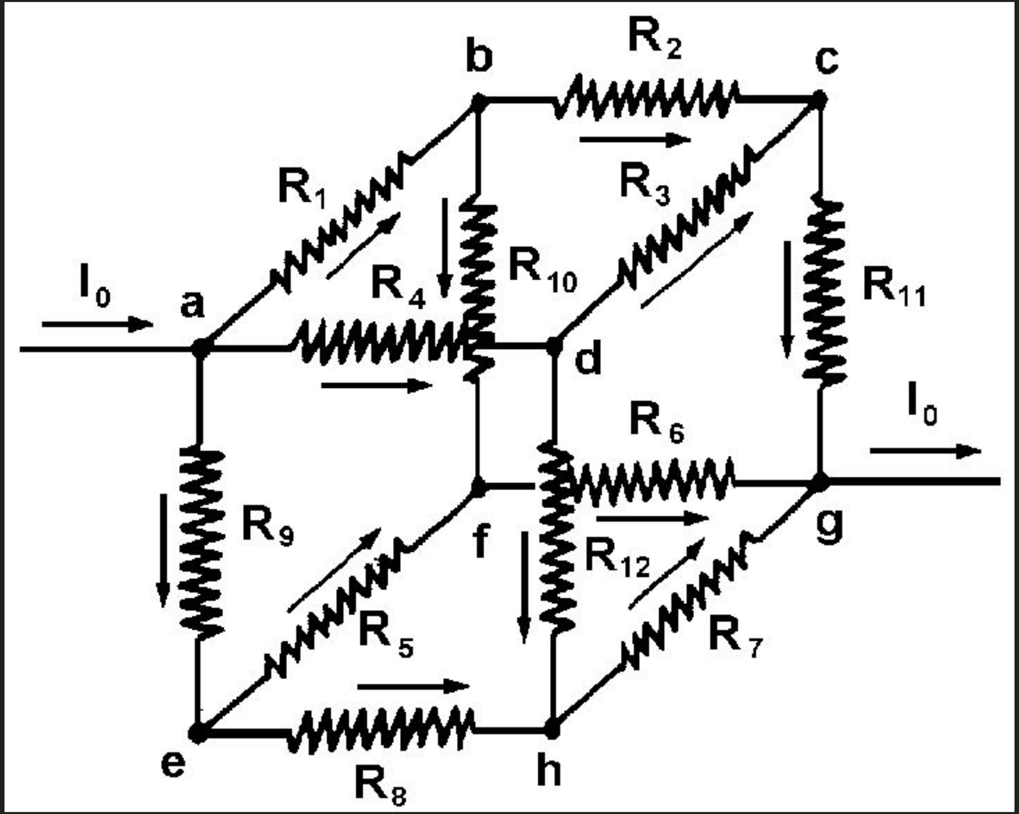


Connecting in Parallel

- Circuit elements are connected by multiple conducting paths
- A single electron travels one of the paths
- Current splits up at the nodes (aka junctions)
- Voltage drops the same amount across a branch



Our circuits can get pretty complicated...

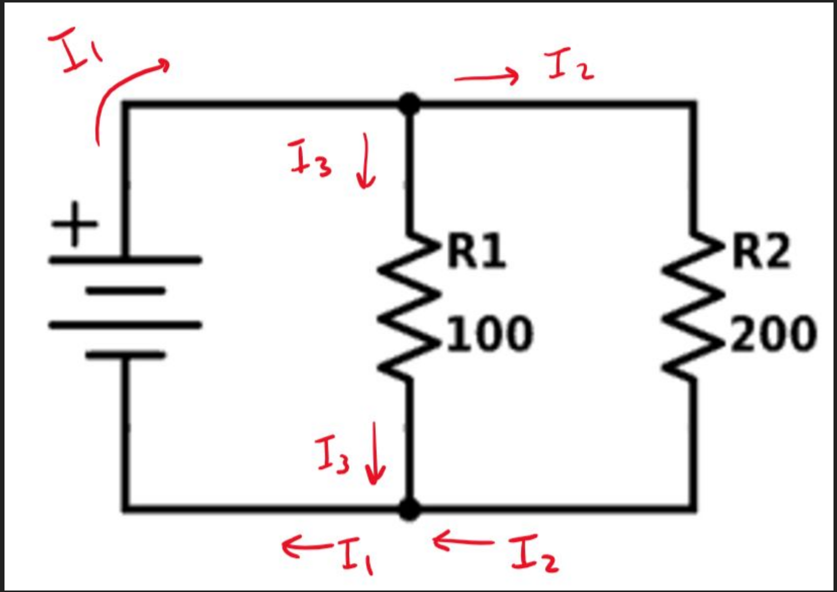


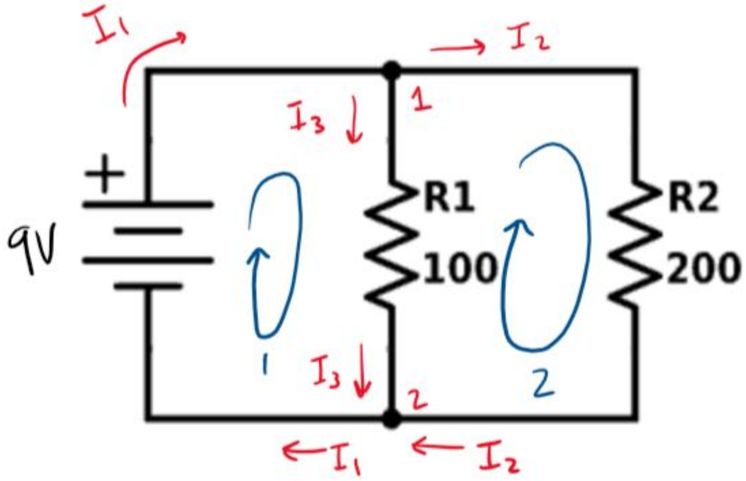
How can we handle this stuff?

- Kirchoff’s loop law
- Junction law
- Equivalent Circuits

Kirchoff’s Loop Law and Junction Rule

- Pick a direction for the currents
- Pick the loops and write down the sum ΔV for each loop
- Pick junctions and find $I_{in} = I_{out}$
- Solve the system of equations





Loop 1

$$\sum V_{Loop} = 0V = +9V - I_3 R_1$$
$$= 9V - I_3 (100\Omega)$$

Loop 2

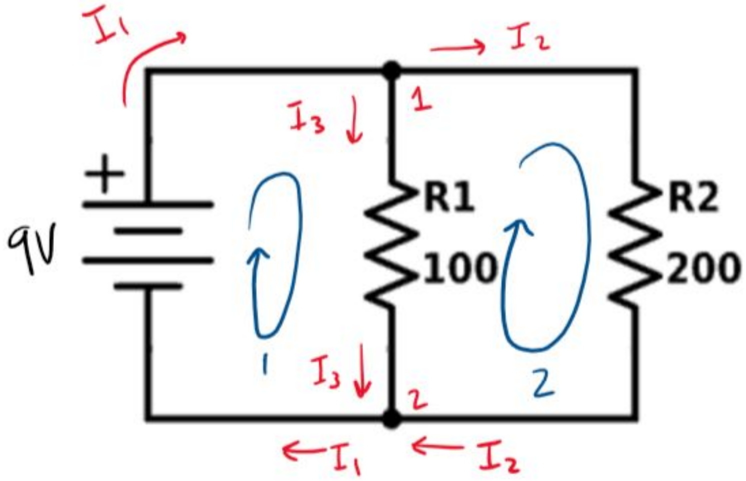
$$\sum V_{loop} = 0V = -I_2 R_2 + I_3 R_1$$
$$= -I_2 (200\Omega) + I_3 (100\Omega)$$

Junction 1

$$I_1 = I_2 + I_3$$

Junction 2

$$I_3 + I_2 = I_1$$



$$9V = I_3(100\Omega) \rightarrow I_3 = \frac{9V}{100\Omega}$$

$$I_2(200\Omega) = I_3(100\Omega) \rightarrow I_3 = \frac{I_2(200\Omega)}{100\Omega}$$

System of Equation

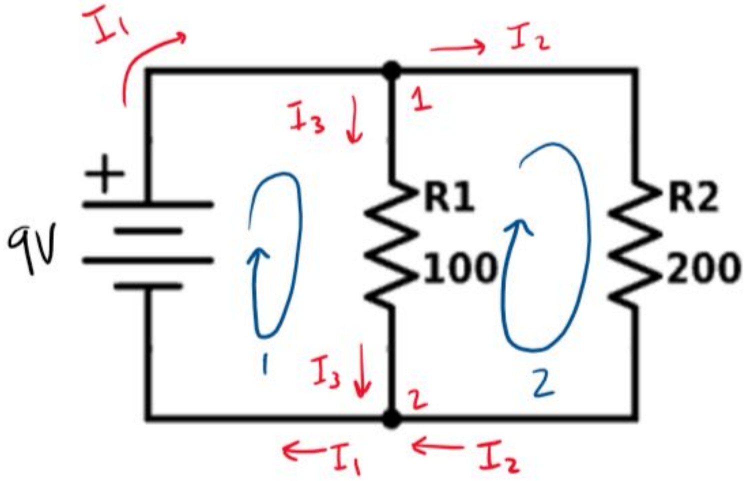
$$0 = 9V - I_3(100\Omega)$$

$$0 = -I_2(200\Omega) + I_3(100\Omega)$$

$$I_1 = I_2 + I_3$$

$$\frac{9V}{100\Omega} = \frac{I_2(200\Omega)}{100\Omega}$$

$$I_2 = \frac{9V(100\Omega)}{(100\Omega)(200\Omega)} = 0.045\,A$$



$$I_3 = \frac{I_2 (200\,\Omega)}{100\,\Omega}$$

$$= \frac{(0.045\,A)(200\,\Omega)}{100\,\Omega}$$

$$= 0.09\,A$$

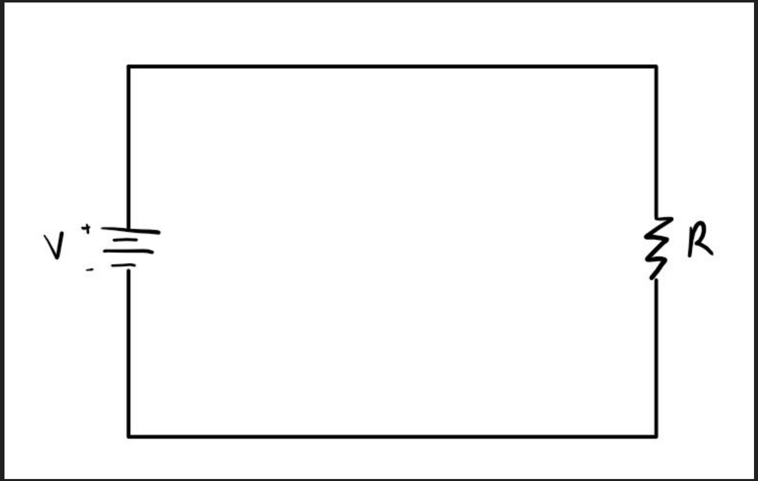
$$I_1 = I_2 + I_3$$

$$= 0.045\,A + 0.09\,A$$

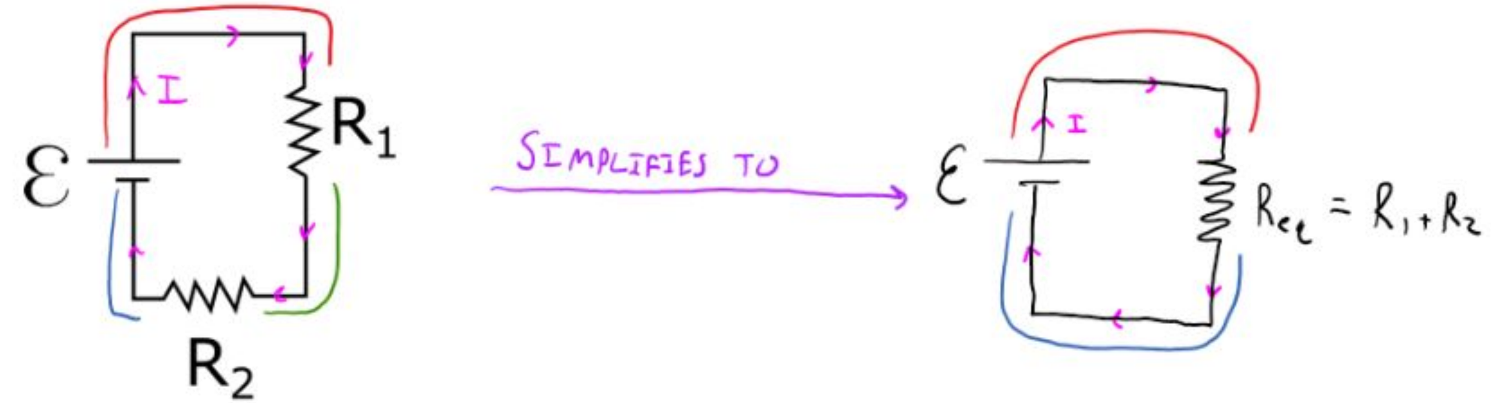
$$I_1 = 0.135\,A$$

Equivalent Circuits

- A simplified model of a circuit
- Turn all the voltage sources into one voltage source
- Turn all the resistors into one resistance source



Resistors in Series



$$\Delta V_{BAT} + \Delta V_1 + \Delta V_2 = 0$$

$\Delta V = IR$

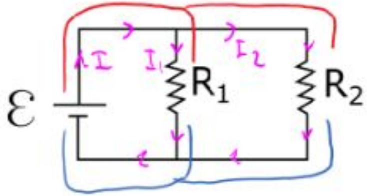
$$\mathcal{E} - IR_1 - IR_2 = 0$$

$$\mathcal{E} - I\underbrace{(R_1 + R_2)}_{R_{eq}} = 0$$

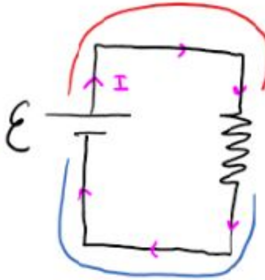
IN GENERAL

$$R_{eq} = R_1 + R_2 + R_3 + \ldots$$

Resistors in Parallel



SIMPLIFIES TO →



$$R_{eq} = \frac{1}{\left(\frac{1}{R_1} + \frac{1}{R_2}\right)}$$

$$I = I_1 + I_2$$

$\Delta V = IR$

$$I = \frac{\Delta V_1}{R_1} + \frac{\Delta V_2}{R_2}$$

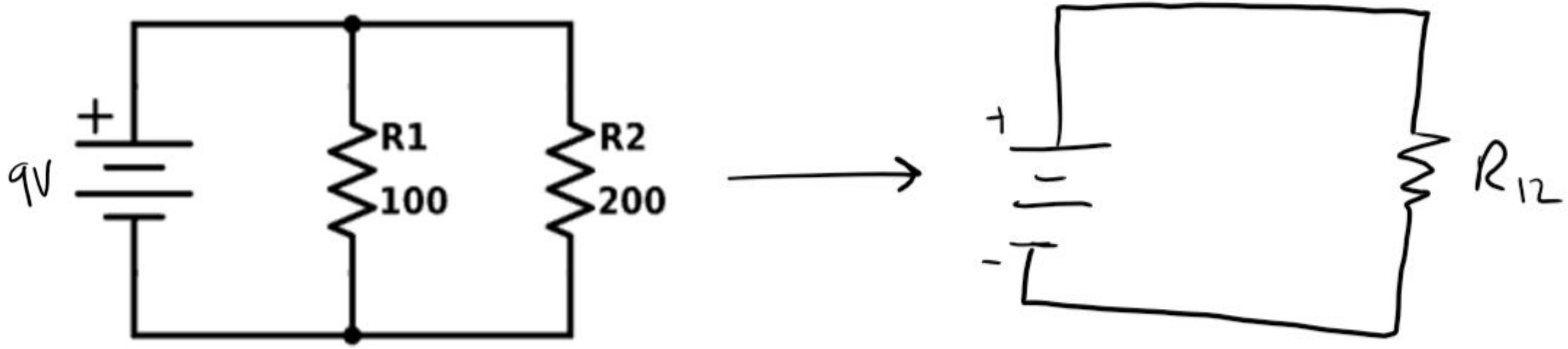
$$I = \frac{\Delta V_{BATT}}{R_1} + \frac{\Delta V_{BATT}}{R_2}$$

$$I = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)\Delta V_{BATT}$$

$$\mathcal{E} = \underbrace{\left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1}}_{R_{eq}} I$$

IN GENERAL

$$R_{eq} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots\right)^{-1}$$



$$\frac{1}{R_{12}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\frac{1}{R_{12}} = \frac{1}{100\Omega} + \frac{1}{200\Omega}$$

$$\frac{1}{R_{12}} = \frac{2+1}{200\Omega}$$

$$R_{12} = \frac{200\Omega}{3} = 66.67\,\Omega$$

$$\Delta V = I R$$

$$9V = I_1 R_{12}$$

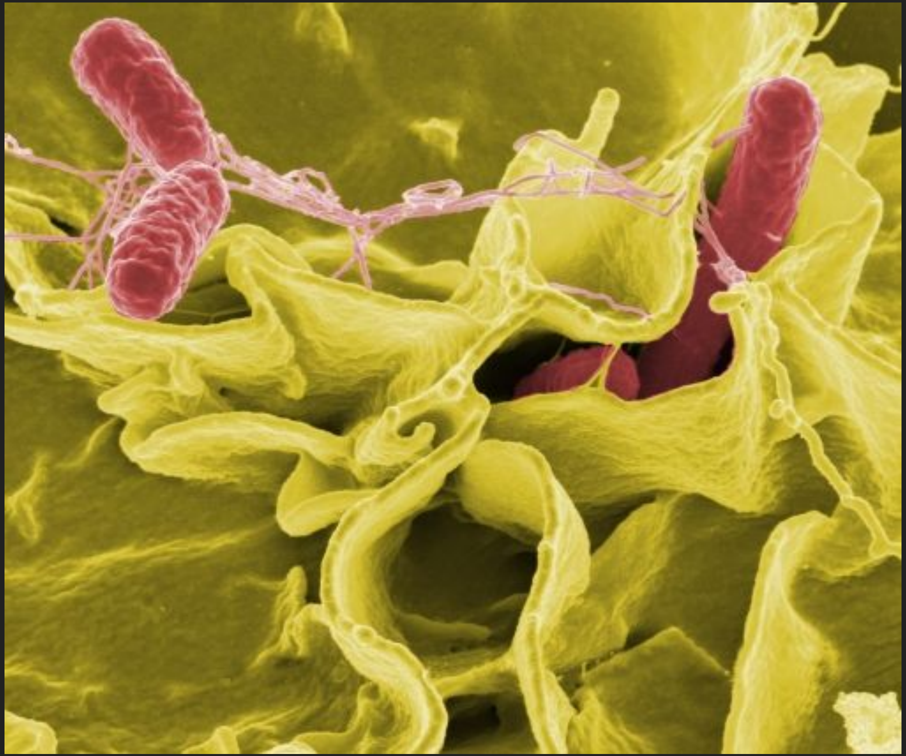
$$I_1 = \frac{9V}{66.67\Omega}$$

$$I_1 = 0.135\ A$$

Why it’s cool

Biological Circuits at MIT

- Can build circuits from biological elements like proteins to detect environmental changes
- These biological circuits all talk to each other, which makes things hard
- Researchers developing new ways to isolate the circuits and make them all work simultaneously



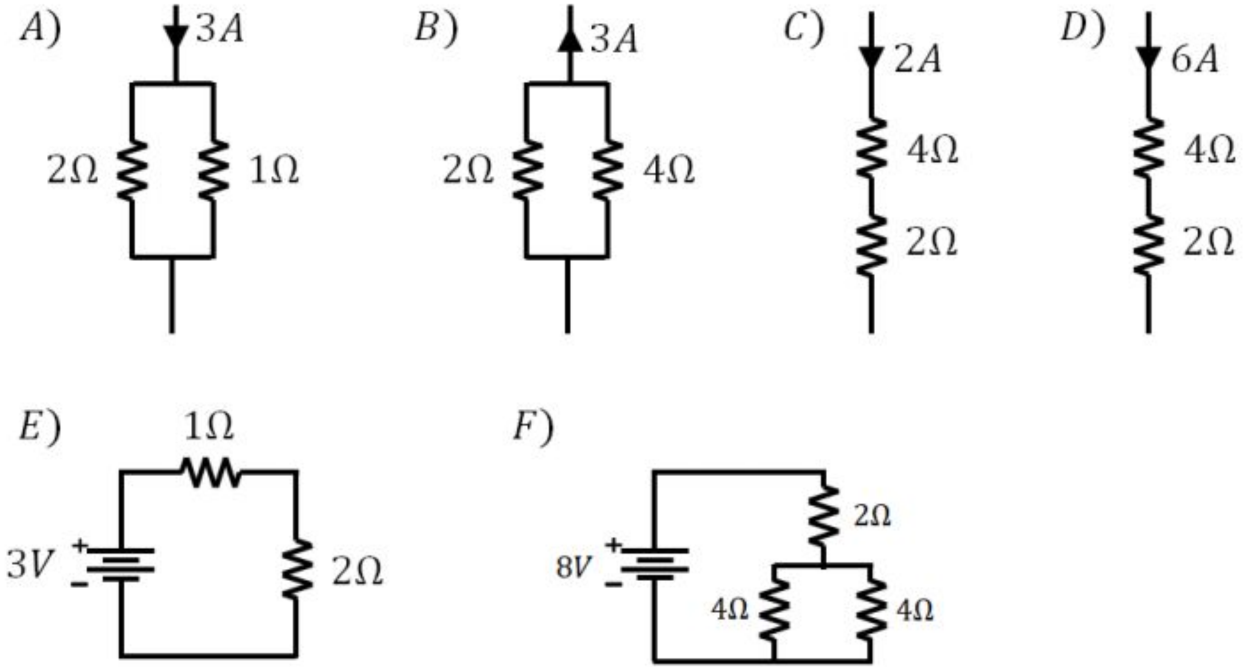
http://news.mit.edu/2012/complex-biological-circuit-1007

Putting It to Practice

S18 Final Exam

3. Which of the following circuits will cause the 2 Ω resistor to dissipate 8 W.

- ☐ a)
- ☐ b)
- ☐ c)
- ☐ d)
- ☐ e)
- ☐ f)



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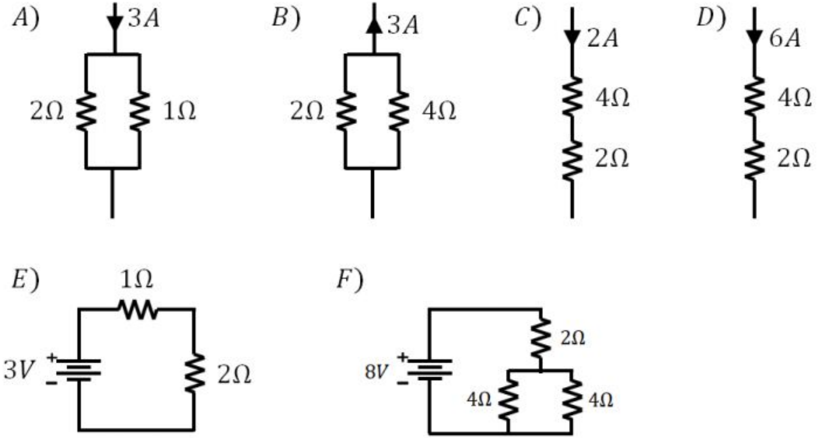

$$P_2 = RI^2$$
$$= (2\Omega)(1A)^2$$
$$= 2W$$

$$3A = I_2 + I_3$$
$$-I_2(1\Omega) + I_3(2\Omega) = 0$$
$$I_2 = \frac{I_3(2\Omega)}{(1\Omega)}$$

$$I_2 = 2I_3$$
$$3A = I_3 + 2I_3$$
$$3A = 3I_3$$
$$I_3 = 1A, \quad I_2 = 2A$$

S18 Final Exam

3. Which of the following circuits will cause the 2 Ω resistor to dissipate 8 W.

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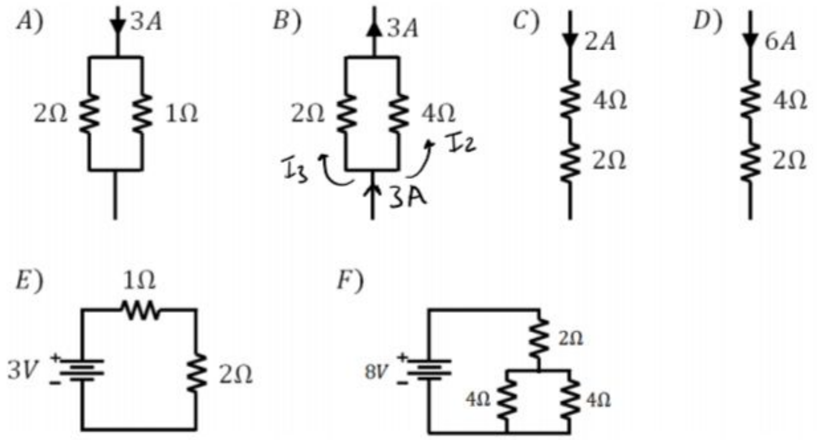


$$P = (2\Omega)(2A)^2$$
$$= 8W$$

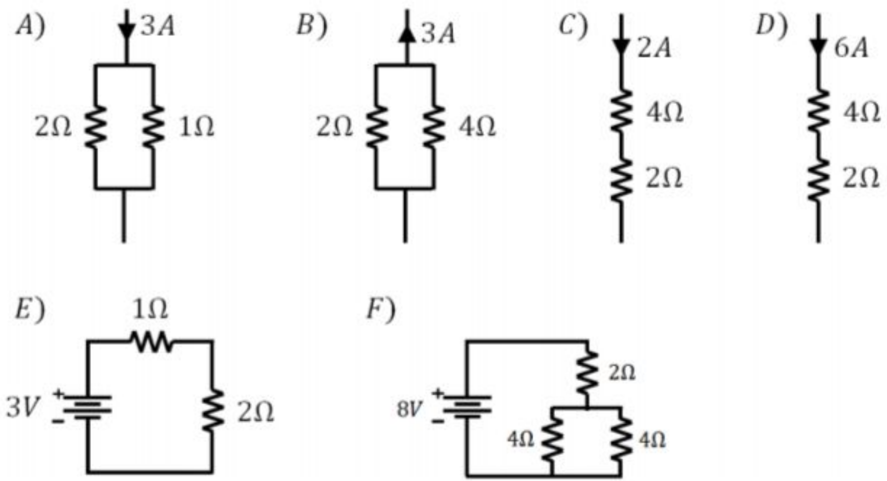
$$3A = I_2 + I_3$$
$$-I_3(2\Omega) + I_2(4\Omega) = 0$$
$$I_2 = \frac{I_3(2\Omega)}{4\Omega} = \frac{1}{2} I_3$$

$$3A = I_3 + \frac{1}{2} I_3$$
$$3A = \frac{3}{2} I_3$$
$$I_3 = 2A$$

S18 Final Exam

3. Which of the following circuits will cause the 2 Ω resistor to dissipate 8 W.

- ☒ a)
- ■ b)
- ☐ c)
- ☐ d)
- ☐ e)
- ☐ f)

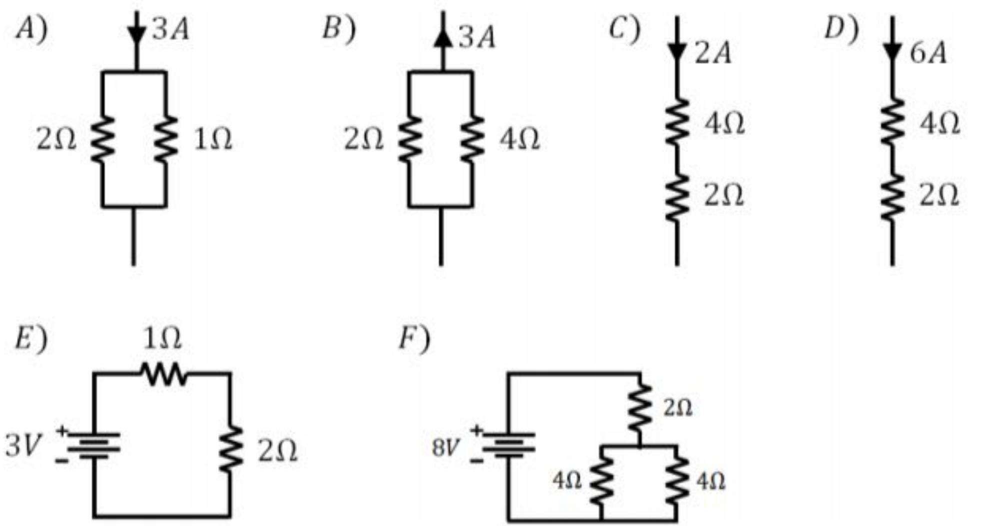


$$P = (2\Omega)(2A)^2 = 8W$$

S18 Final Exam

3. Which of the following circuits will cause the 2 Ω resistor to dissipate 8 W.

- ☒ a)
- ■ b)
- ■ c)
- ☐ d)
- ☐ e)
- ☐ f)

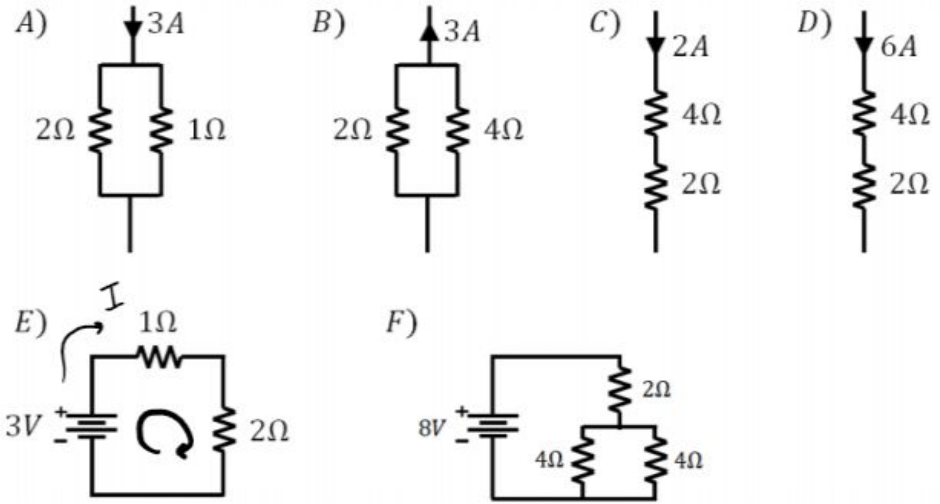


$P = (2\Omega)(6A)^2 = 72W$

S18 Final Exam

3. Which of the following circuits will cause the 2 Ω resistor to dissipate 8 W.

- [x] a)
- [ ] b)
- [ ] c)
- [x] d)
- [ ] e)
- [ ] f)



$0 = 3V - I(1\Omega) - I(2\Omega)$

$-3V = -I(1\Omega + 2\Omega)$

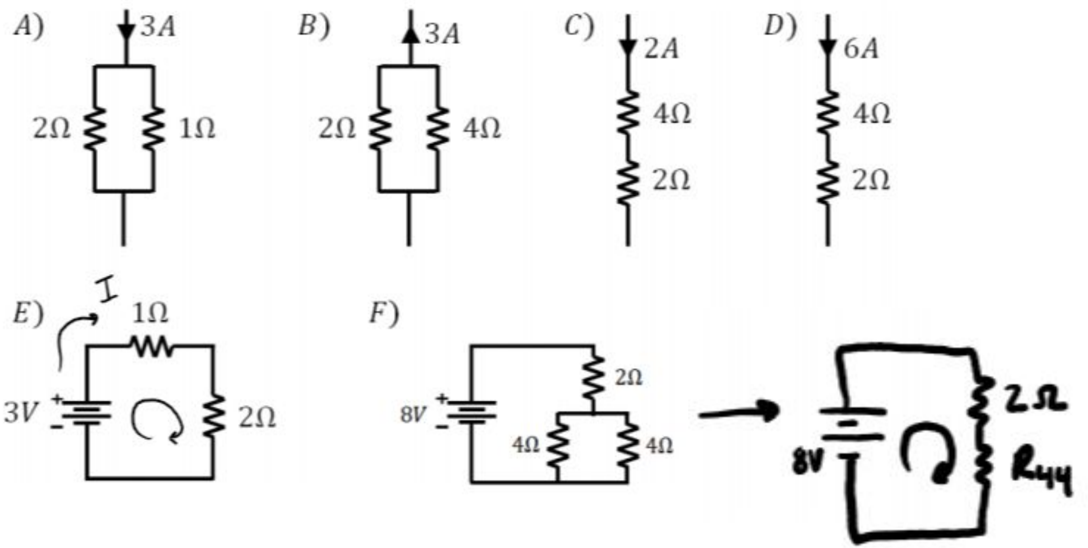
$\frac{3V}{3\Omega} = I = 1A$

$P = (2\Omega)(1A)^2 = 2W$

S18 Final Exam

3. Which of the following circuits will cause the 2 Ω resistor to dissipate 8 W.

- ☒ a)
- ■ b)
- ■ c)
- ☒ d)
- ☒ e)
- ☐ f)



$$\frac{1}{R_{44}} = \frac{1}{4\Omega} + \frac{1}{4\Omega}$$

$$\frac{1}{R_{44}} = \frac{2}{4\Omega}$$

$$R_{44} = 2\Omega$$

$$0 = 8V - I(2\Omega) - I(2\Omega)$$

$$8V = I(2\Omega + 2\Omega)$$

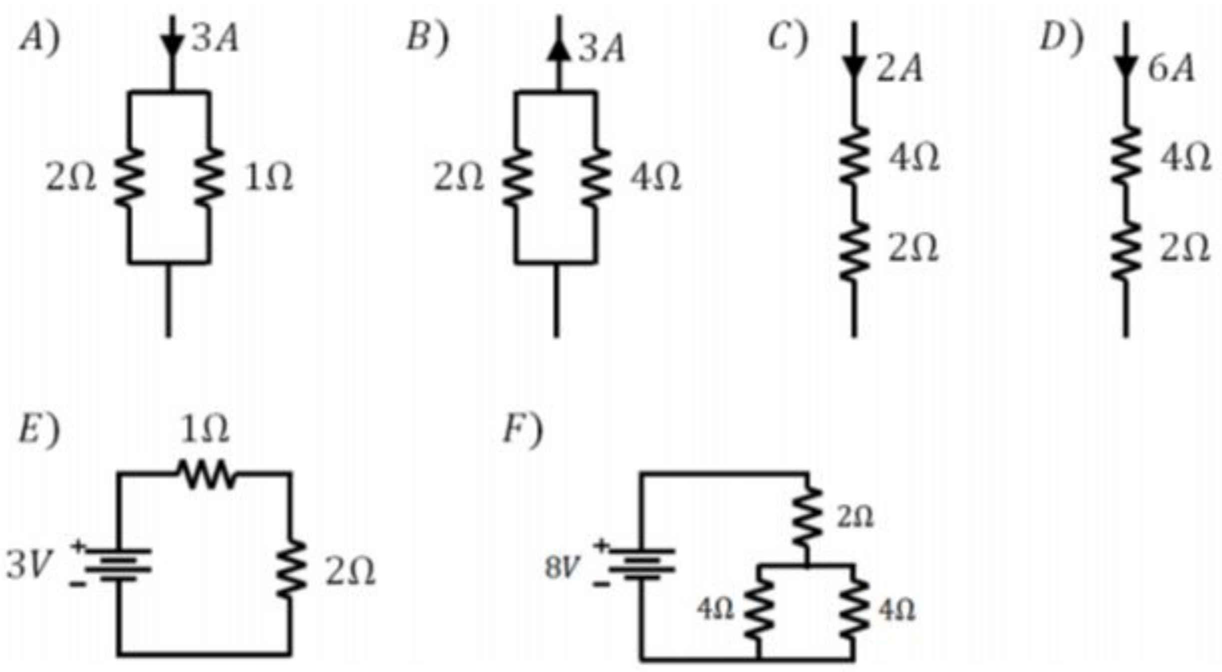
$$I = 2A$$

$$P = (2\Omega)(2A)^2 = 8W$$

S18 Final Exam

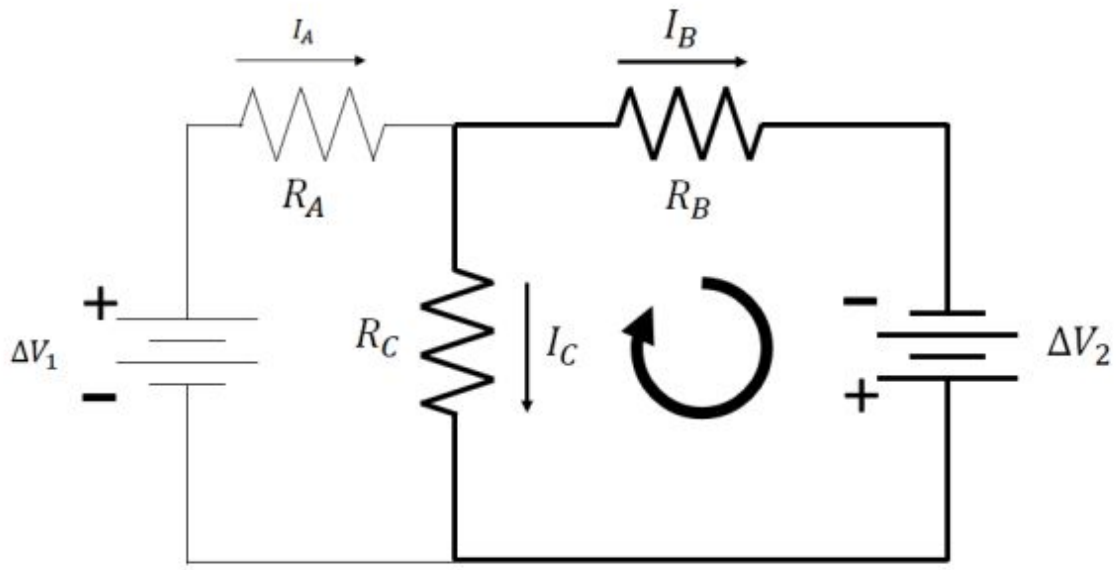
3. Which of the following circuits will cause the 2 Ω resistor to dissipate 8 W.

- [ ] a)
- [x] b)
- [x] c)
- [ ] d)
- [ ] e)
- [x] f)



S18 Final Exam

6. (3 points) The following circuit has the specified voltages, currents, and resistances. Use Kirchhoff's loop rule to write an equation that describes the loop indicated by the bold circle and darker wires.



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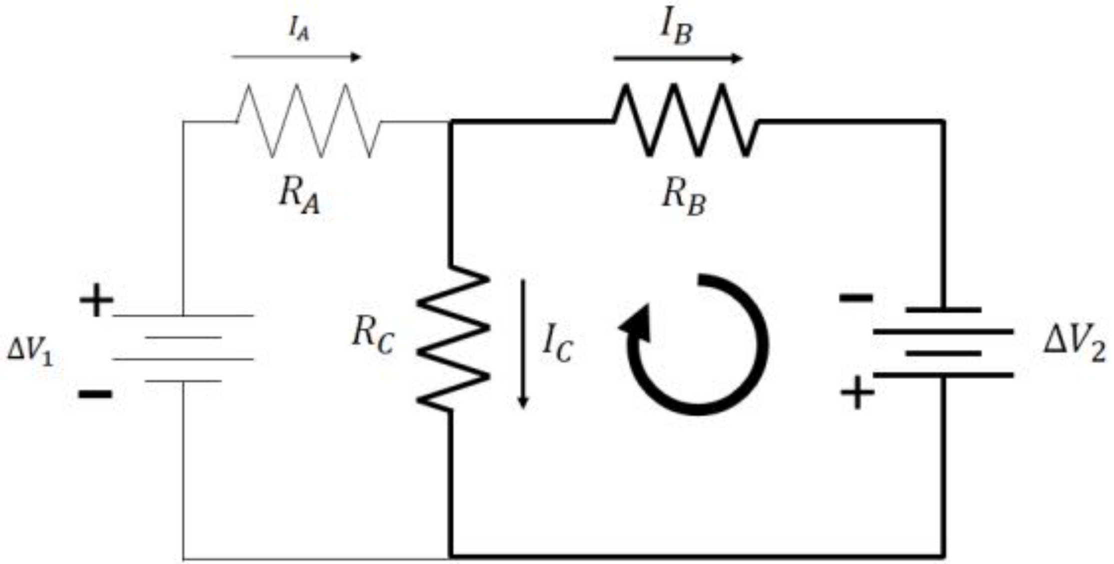

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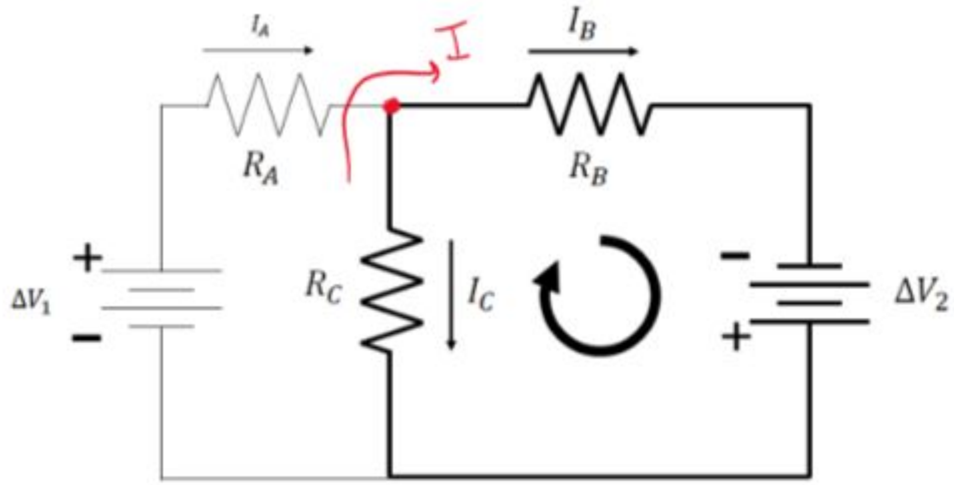


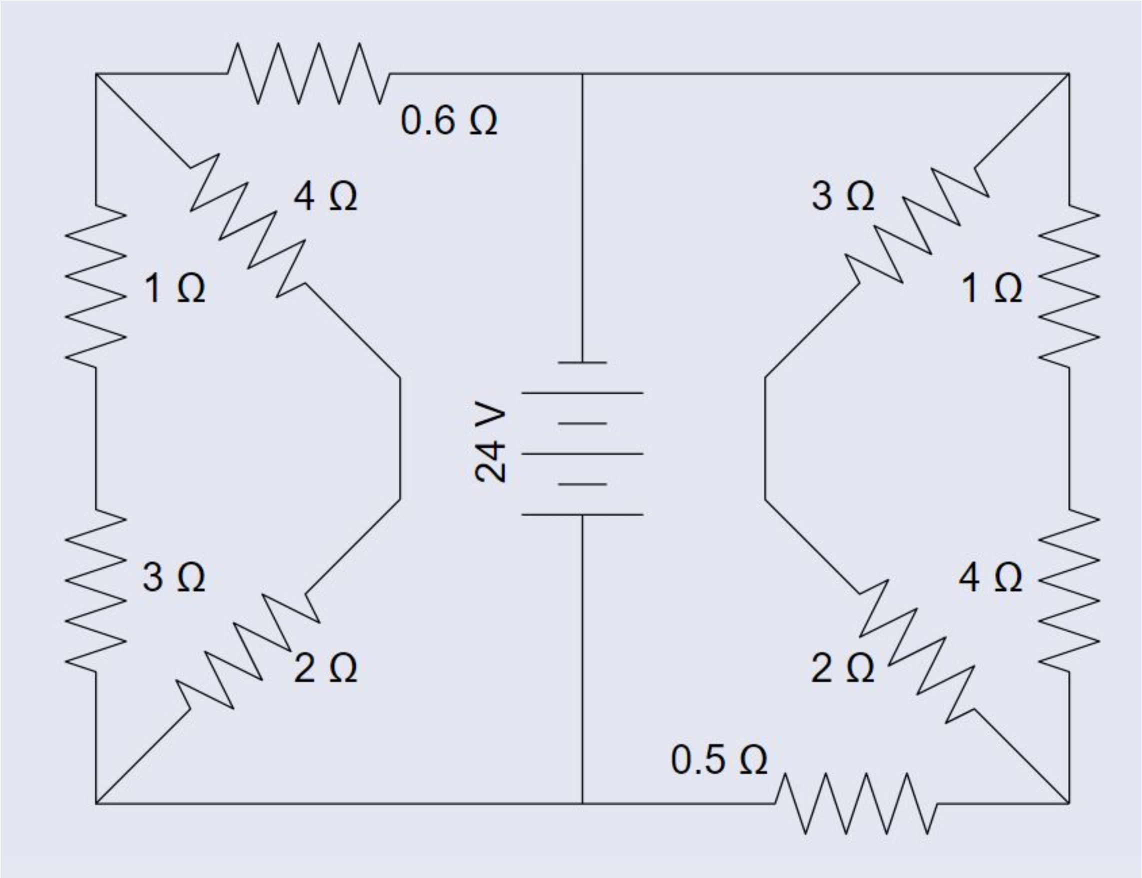
$\sum V_{Loop} = 0$

$\sum V_{Loop} = 0 = -I_B R_B$

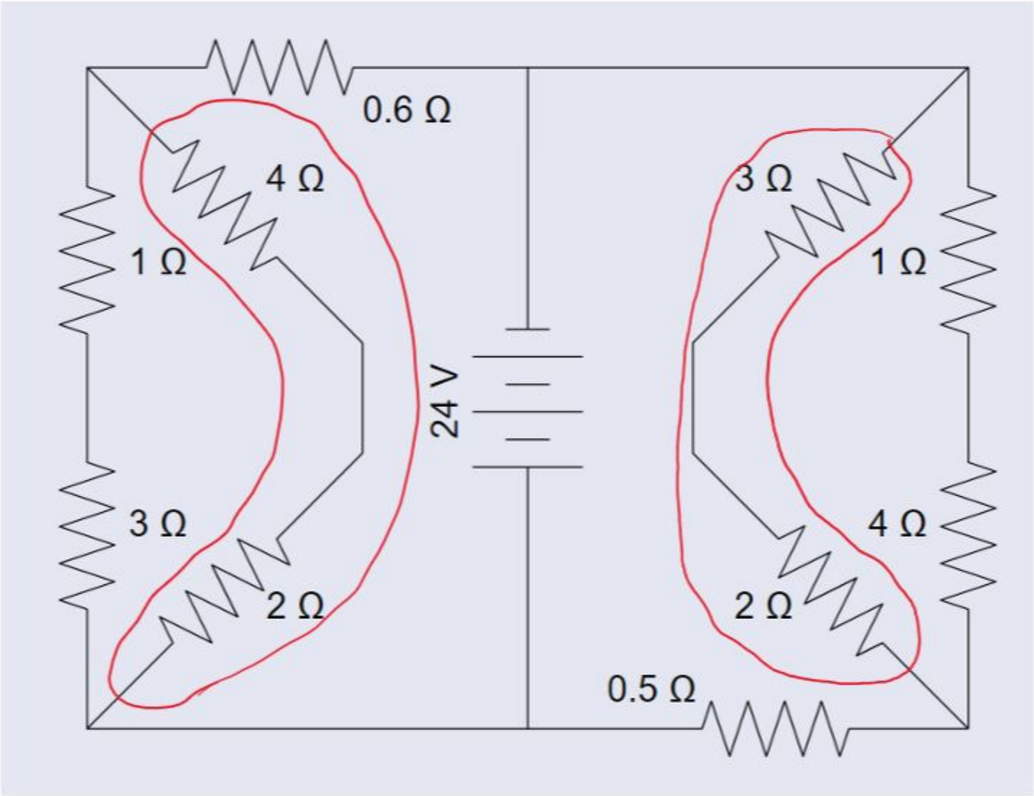
$\sum V_{Loop} = 0 = -I_B R_B + \Delta V_2$

$\sum V_{Loop} = 0 = -I_B R_B + \Delta V_2 + I_c R_c$

Find the equivalent resistance of the circuit



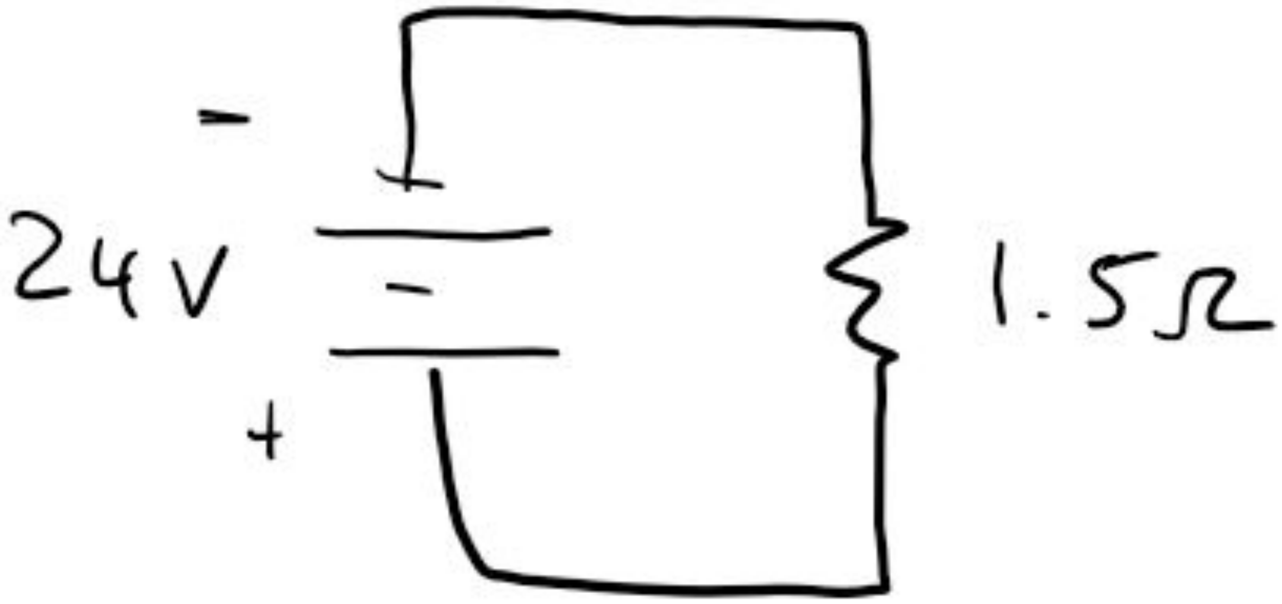
Find the equivalent resistance of the circuit



$4\,\Omega + 2\,\Omega = 6\,\Omega$

$3\,\Omega + 2\,\Omega = 5\,\Omega$

Find the equivalent resistance of the circuit



Find the equivalent resistance of the circuit