Recitation 1 Answer

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1 Problem Statement

A 475 nm wavelength laser produces a single-slit interference pattern using a slit of spacing a. What is the range of values a can have that will produce exactly 15 bright spots on a screen 10.0 m away?

2 Solution

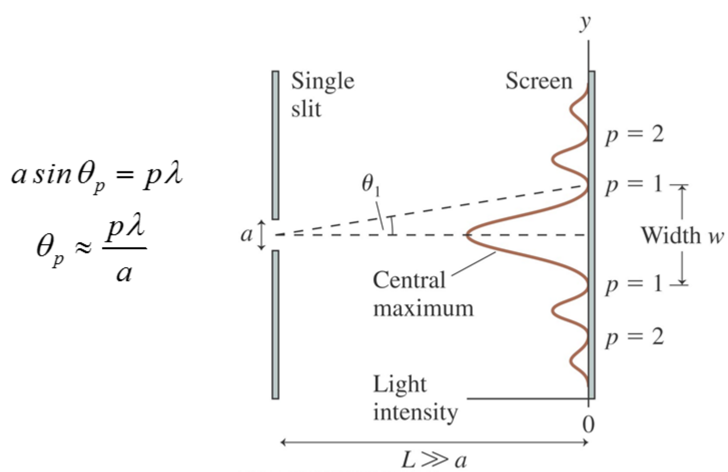

Figure 1: the interference pattern that a single-slit produces

First we must recognize that the way we counted for answering this problem with the diffraction grating has changed. The equation for single-slit interference ($p\lambda = asin\theta_p$) tells us where the dark fringes are. We still want 15 bright fringes, so our way of figuring out the order of p will be different from figuring out our order of m in the diffraction grating case. Consider the image in Figure 1. There are 5 bright fringes and 6 dark fringes. By looking at this figure, we can come up with an equation that will tell us what order of p we need to consider to get 15 dark fringes. The number of dark fringes is just

$$\# \text{ dark fringes} = 2p \tag{1}$$

From Figure 1, we see there is one less bright spot than there is dark spots. If you split this interference pattern into half, you see that at the center is half of a bright spot. We can say that

$$\# \text{ bright fringes} = 2p - 1 \tag{2}$$

Since we want exactly 15 bright spots, we need p=8 when $\theta = 90^o$. So then using the model for single slit interference,

$$p\lambda = asin\theta \tag{3}$$
$$8\lambda = asin90^o \tag{4}$$
$$8\lambda = a \tag{5}$$
$$8(475nm) = a \tag{6}$$
$$3.8\mu m = a \tag{7}$$

The slit spacing needs to be at least $3.8\mu m$ in order to get 15 bright spots for the single slit. Just as before though, we can increase the width of this slit and still get 15 bright spots. If we increase the slit width by too much though, we would see 17 bright spots instead. So we need to figure out what our maximum spacing is to get 15 bright fringes by figuring out the spacing of a where we get 17 fringes. This will give us our range for a! If we increase the order of p to 9, we get 18 dark fringes and 17 bright fringes. So then

$$p\lambda = asin\theta \tag{8}$$
$$9\lambda = asin90^o \tag{9}$$
$$9\lambda = a \tag{10}$$
$$9(475nm) = a \tag{11}$$
$$4.3\mu m = a \tag{12}$$

So then a can be $3.8\mu m \leq a < 4.3\mu m$. If a is slightly less than $3.8\mu m$, then we would only see up to p=7. Here's a quick proof of that

$$p\lambda = 3.79\mu m sin\theta \tag{13}$$
$$p\lambda = 3.79\mu m sin90^o \tag{14}$$
$$p\lambda = 3.79\mu m \tag{15}$$
$$p(475nm) = 3.79\mu m \tag{16}$$
$$p = 7.98 \tag{17}$$

We get a value of $p = 7.89$ for a value just outside of the range of slit-spacing a that gives us 15 bright fringes. We know that p can only be an integer value, so we round p down to 7 because this means that the order p=7 fringes are visible, but the order p=8 fringes are not visible. If up to order p=7 is visible, we see only 13 bright fringes which is not what we want! You can do similarly for values of $4.3\mu m$ and above to see that this will also result in the wrong number of fringes.