

# Recitation W6

Jihan Kim

# Note

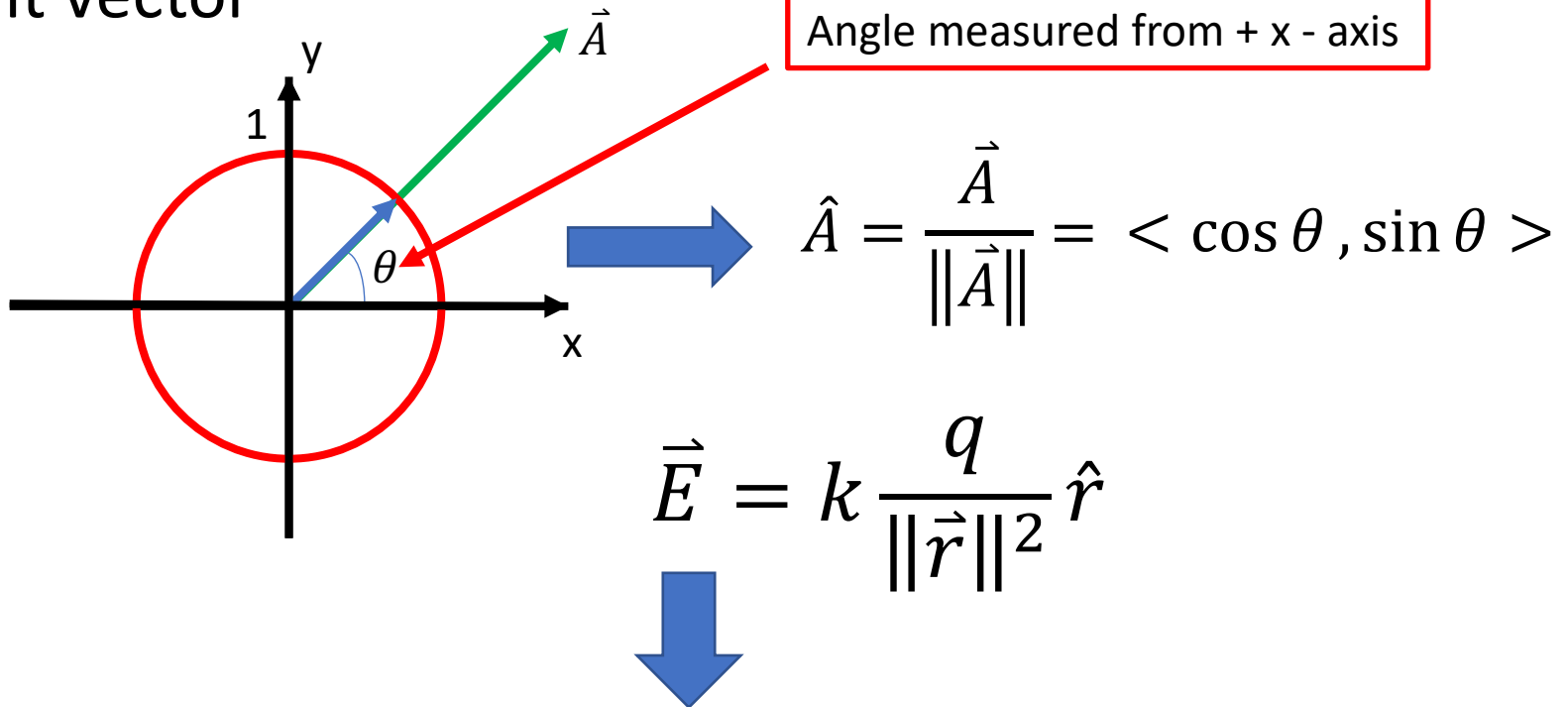
- Mid-term II : May 15<sup>th</sup>
- Tuesday: Mid-term prep session
- Thursday: Mid-term review
- Wednesday: office hour 1-2 p.m.

# Topics

- Electric field 2D
- Electric potential & energy

# Electric field in 2D

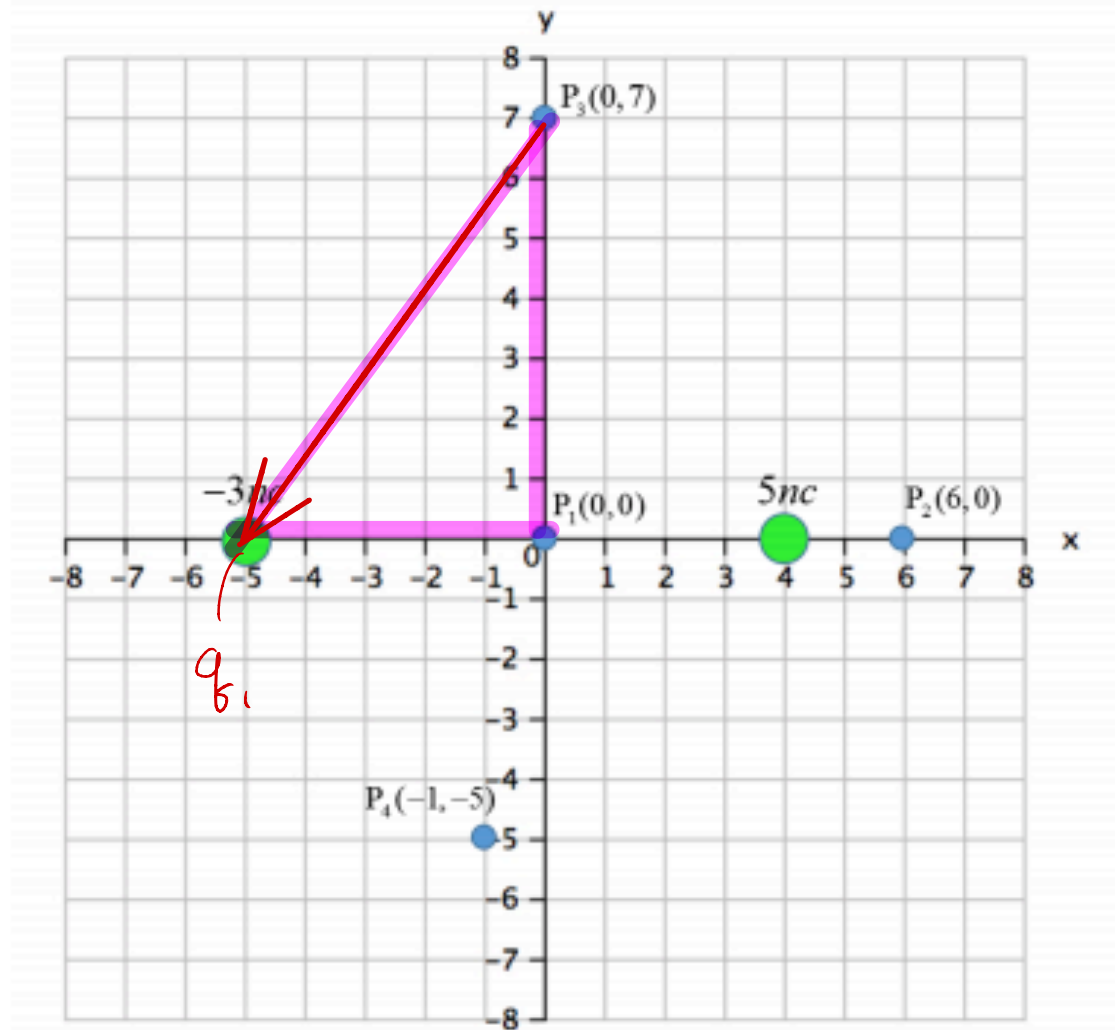
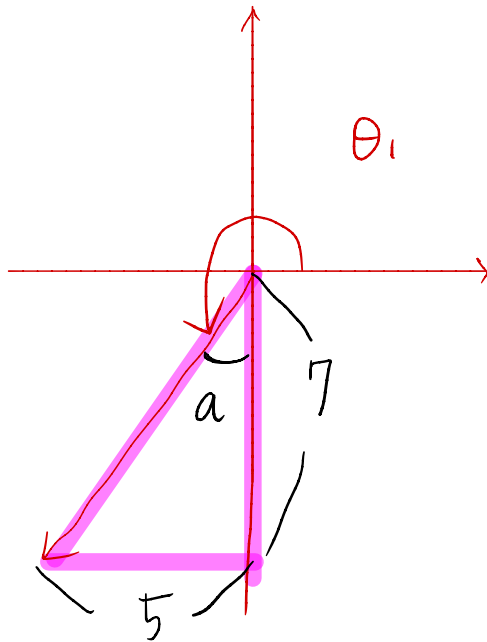
- Unit vector



$$\vec{E} = \left| \frac{kq}{r^2} \right| \langle \cos \theta, \sin \theta \rangle$$

4. (Continued...)

(e) In the diagram to the right there are two point charges. Determine the electric field (vector) at each of the four points indicated on the diagram.



$$\theta_1 = 270^\circ - a$$

$$a = \tan^{-1}\left(\frac{5}{7}\right) = 35.5^\circ$$

$$\Rightarrow \theta_1 = 270^\circ - 35.5^\circ = 234.5^\circ$$

$$\vec{E}_1 = \left| \frac{k q_1}{r_1^2} \right| \langle \cos \theta_1, \sin \theta_1 \rangle$$

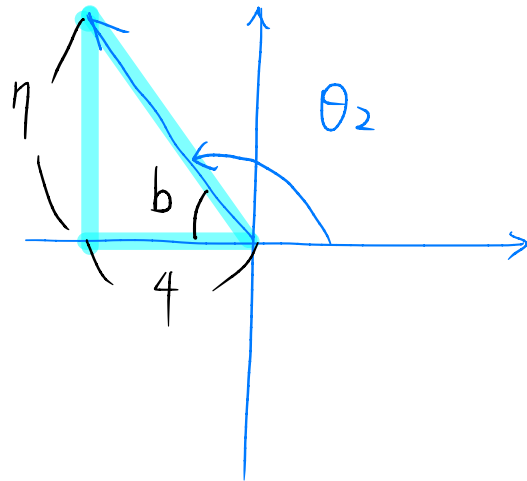
$$= \left| \frac{9 \times 10^9 \text{ N} \frac{\text{m}^2}{\text{C}^2} \cdot (-3 \times 10^{-9} \text{ C})}{5^2 + 7^2 \text{ m}^2} \right| \langle \cos 234.5^\circ, \sin 234.5^\circ \rangle$$

$$= 0.36 \langle -0.58, -0.81 \rangle \frac{\text{N}}{\text{C}}$$

$$= \langle -0.21, -0.29 \rangle \frac{\text{N}}{\text{C}}$$

4. (Continued...)

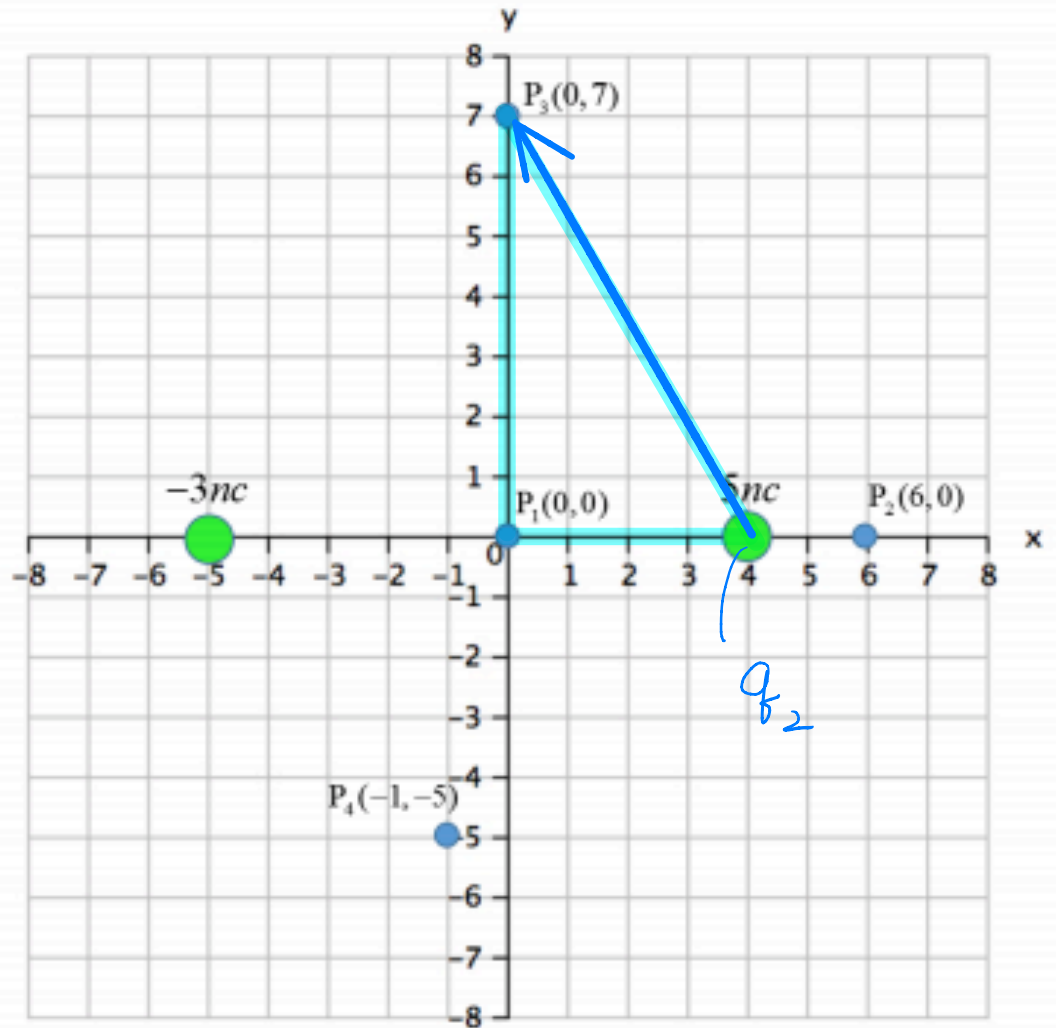
(e) In the diagram to the right there are two point charges. Determine the electric field (vector) at each of the four points indicated on the diagram.



$$\theta_2 = 180^\circ - b$$

$$b = \tan^{-1}\left(\frac{7}{4}\right) = 60.3^\circ$$

$$\Rightarrow \underline{\theta_2 = 180^\circ - 60.3^\circ = 119.7^\circ}$$



$$\vec{E}_2 = \left| \frac{kq_2}{r_2^2} \right| \langle \cos \theta_2, \sin \theta_2 \rangle$$

$$= \left| \frac{9 \times 10^9 \text{ N} \frac{\text{m}^2}{\text{C}^2} \times 5 \times 10^{-9} \text{ C}}{7^2 + 4^2 \text{ m}^2} \right| \langle \cos 119.7^\circ, \sin 119.7^\circ \rangle$$

$$= 0.69 \langle -0.5, 0.89 \rangle \frac{\text{N}}{\text{C}}$$

$$= \langle -0.35, 0.61 \rangle \frac{\text{N}}{\text{C}}$$

$$\Rightarrow \vec{E} = \vec{E}_1 + \vec{E}_2$$

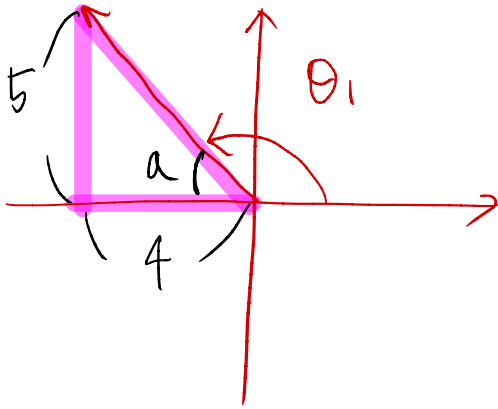
$$= \langle -0.21, -0.29 \rangle + \langle -0.35, 0.61 \rangle \frac{\text{N}}{\text{C}}$$

$$= \langle -0.56, 0.32 \rangle \frac{\text{N}}{\text{C}}$$

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4. (Continued...)

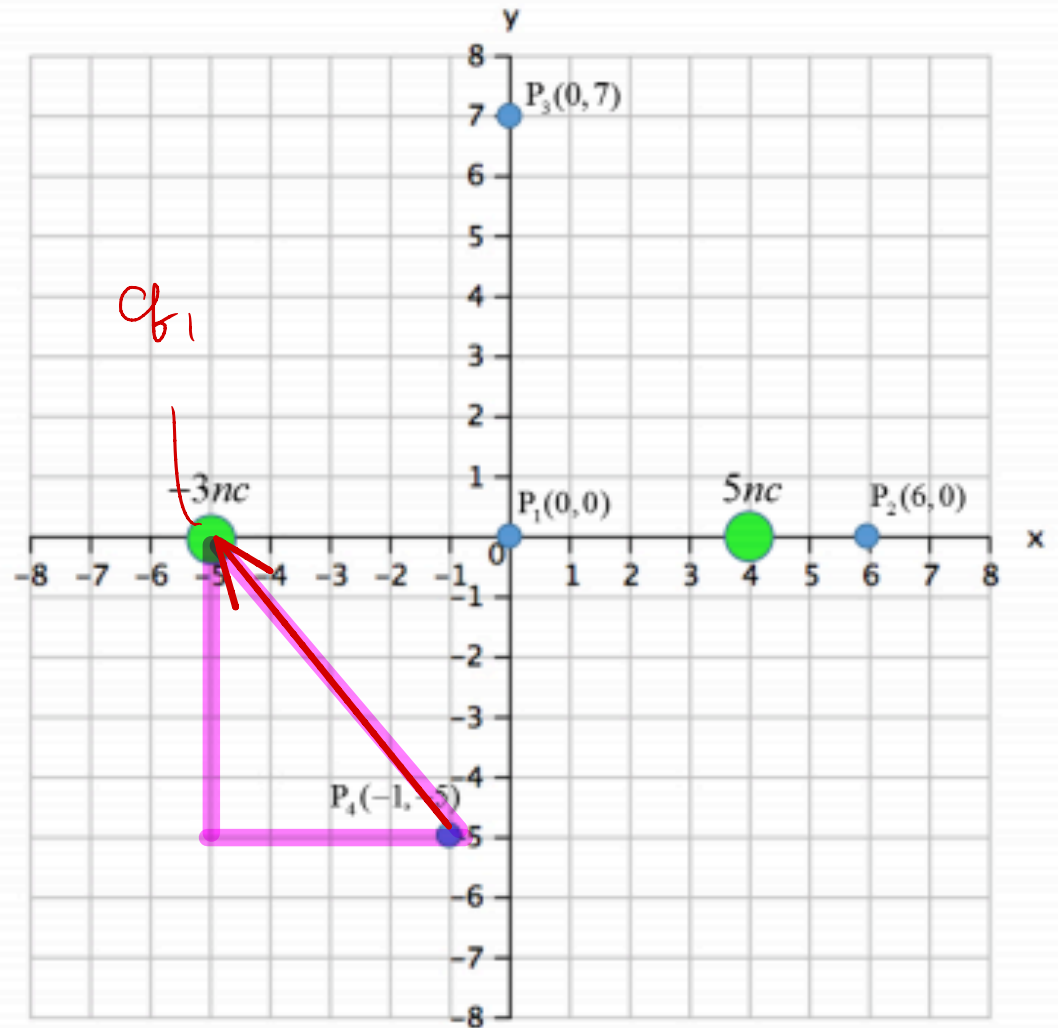
(e) In the diagram to the right there are two point charges. Determine the electric field (vector) at each of the four points indicated on the diagram.



$$\theta_1 = 180^\circ - a$$

$$a = \tan^{-1}\left(\frac{5}{4}\right) = 51.3^\circ$$

$$\theta_1 = 180^\circ - 51.3^\circ = 128.7^\circ$$



$$\vec{E}_1 = \left| \frac{kq_1}{r_1^2} \right| \langle \cos \theta_1, \sin \theta_1 \rangle$$

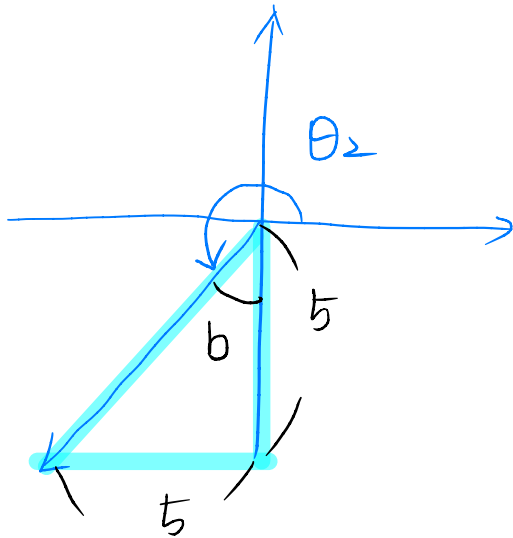
$$= \left| \frac{9 \times 10^9 \text{ N} \frac{\text{m}^2}{\text{C}^2} \cdot (-3 \times 10^{-9} \text{ C})}{5^2 + 4^2} \right| \langle \cos 128.7^\circ, \sin 128.7^\circ \rangle$$

$$= 0.66 \langle -0.63, 0.78 \rangle$$

$$= \langle -0.42, 0.51 \rangle \frac{\text{N}}{\text{C}}$$

4. (Continued...)

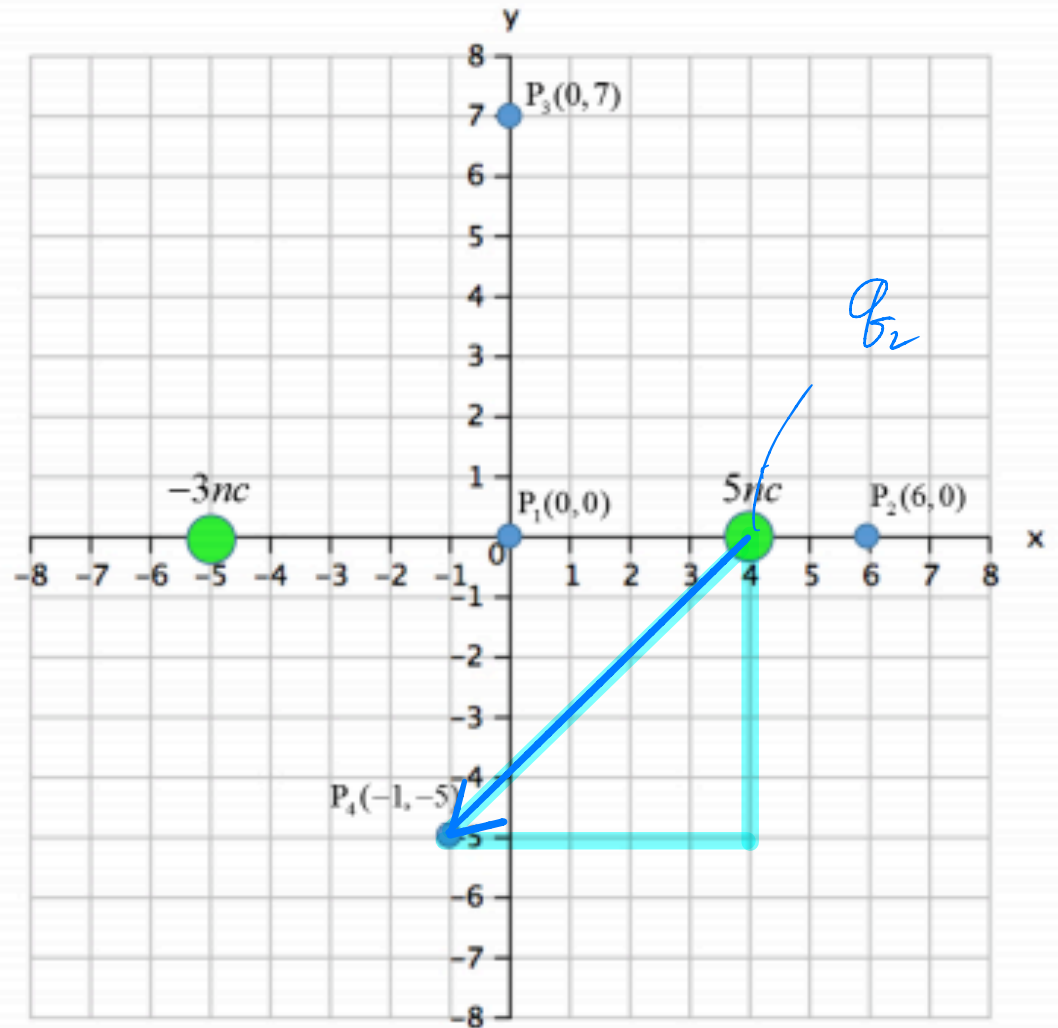
(e) In the diagram to the right there are two point charges. Determine the electric field (vector) at each of the four points indicated on the diagram.



$$\theta_2 = 270^\circ - b$$

$$b = \tan^{-1}\left(\frac{5}{5}\right) = 45^\circ$$

$$\theta_2 = 270^\circ - 45^\circ = 225^\circ$$

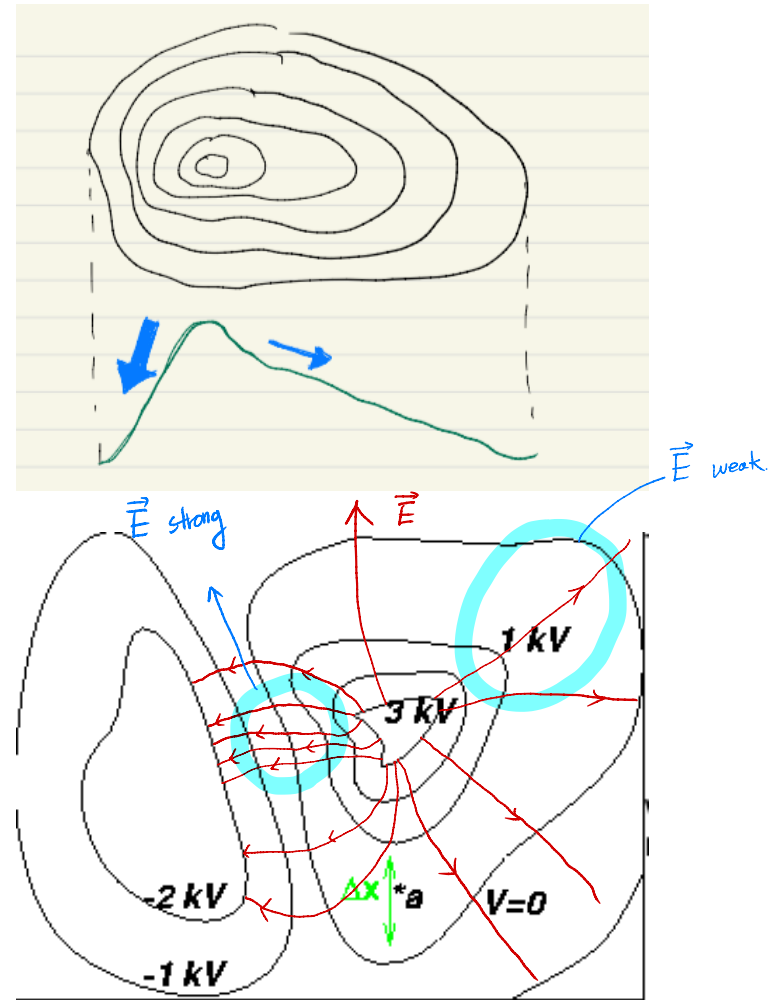
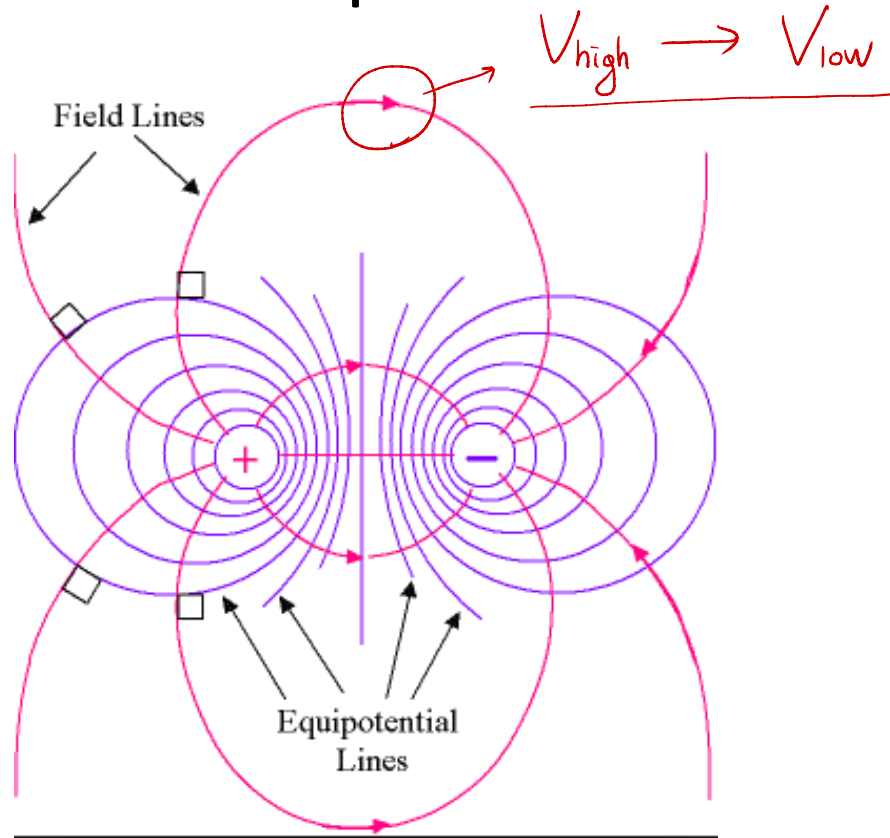


$$\begin{aligned}
\vec{E}_2 &= \left| \frac{kq_2}{r_2^2} \right| \langle \cos \theta_2, \sin \theta_2 \rangle \\
&= \left| \frac{9 \times 10^9 \text{ N} \frac{\text{m}^2}{\text{C}^2} \cdot 5 \times 10^{-9} \text{ C}}{5^2 + 5^2 \text{ m}^2} \right| \langle \cos 225^\circ, \sin 225^\circ \rangle \\
&= 0.9 \langle -0.71, -0.71 \rangle \frac{\text{N}}{\text{C}} \\
&= \langle -0.64, -0.64 \rangle \frac{\text{N}}{\text{C}}
\end{aligned}$$

$$\begin{aligned}
\vec{E} &= \vec{E}_1 + \vec{E}_2 \\
&= \langle -0.42, 0.51 \rangle + \langle -0.64, -0.64 \rangle \frac{\text{N}}{\text{C}} \\
&= \langle -1.06, -0.13 \rangle \frac{\text{N}}{\text{C}}
\end{aligned}$$

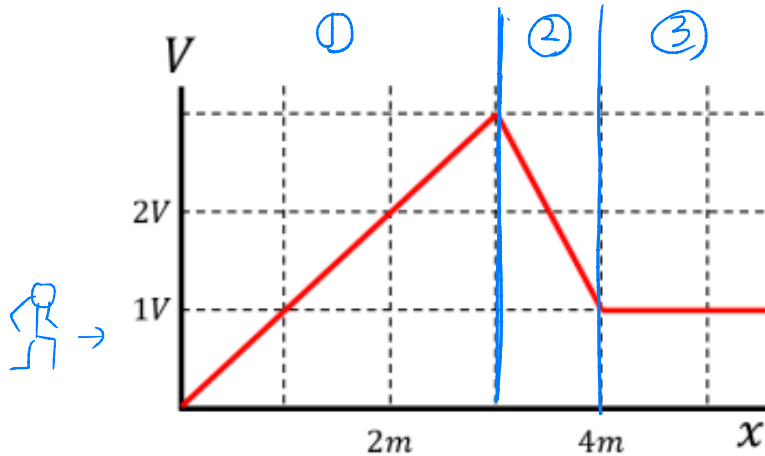

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# Electric potential & energy



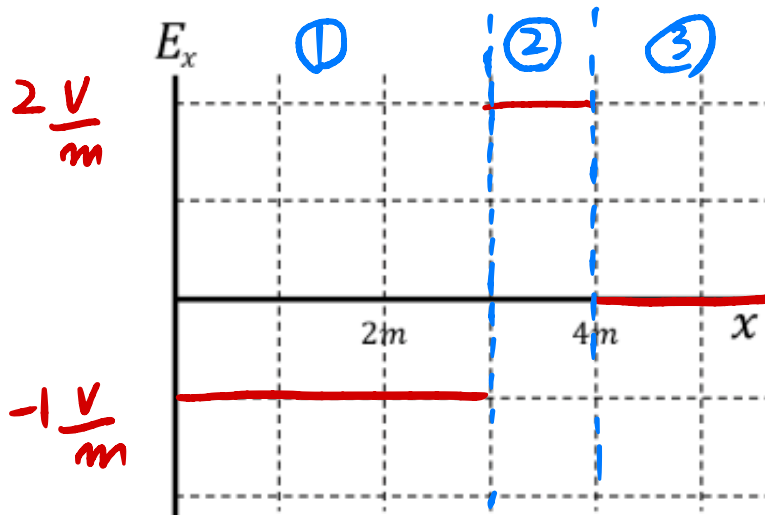
- $\vec{E} = -\frac{\Delta V}{\Delta r} \hat{r}$
- $\vec{E}$  is perpendicular to the equipotential line

6. (5 points) The electric potential along the x-axis is given by the following plot. Create a graph of the x component of the electric field along the same x-axis. (Make sure to add a scale to the electric field axis!)



$$\begin{aligned} \textcircled{1} \quad \Delta V &= +3V \\ \Delta x &= +3m \\ \Rightarrow \vec{E}_{\textcircled{1}} &= - \left( \frac{+3V}{3m} \right) \langle 1, 0 \rangle \\ &= -1 \frac{V}{m} \langle 1, 0 \rangle \end{aligned}$$

$$\begin{aligned} \textcircled{2} \quad \Delta V &= -2V \\ \Delta x &= 1m \\ \Rightarrow \vec{E}_{\textcircled{2}} &= - \left( \frac{-2V}{1m} \right) \langle 1, 0 \rangle \\ &= 2 \frac{V}{m} \langle 1, 0 \rangle \end{aligned}$$



$$\begin{aligned} \textcircled{3} \quad \Delta V &= 0 \\ \Rightarrow \vec{E}_{\textcircled{3}} &= 0 \end{aligned}$$

- $V = \frac{kq}{r}$  : electric potential of a point charge
- $U = qV$  : electric potential energy
- Energy eq

$$\Rightarrow KE_i + U_{gi} + U_{si} + U_{Ei} + W_{ext} = KE_f + U_{gf} + U_{sf} + U_{Ef}$$

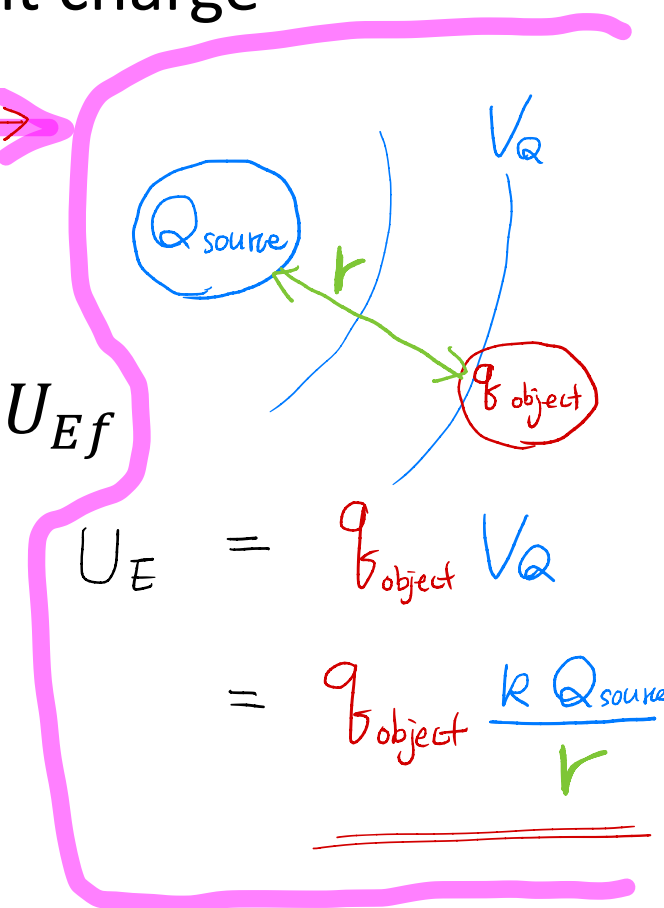
- $KE = \frac{1}{2}mv^2$

- $U_g = mg\Delta h$

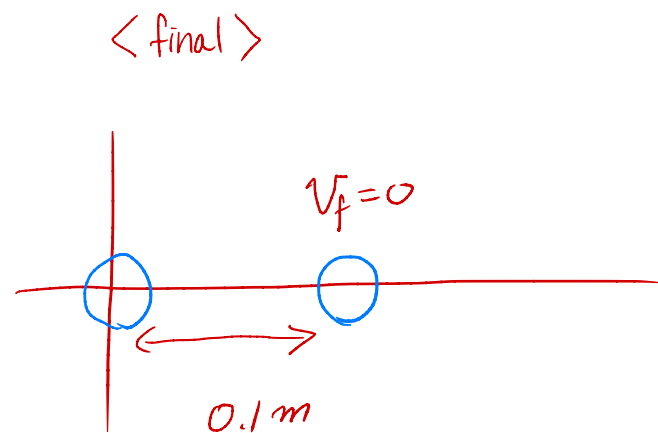
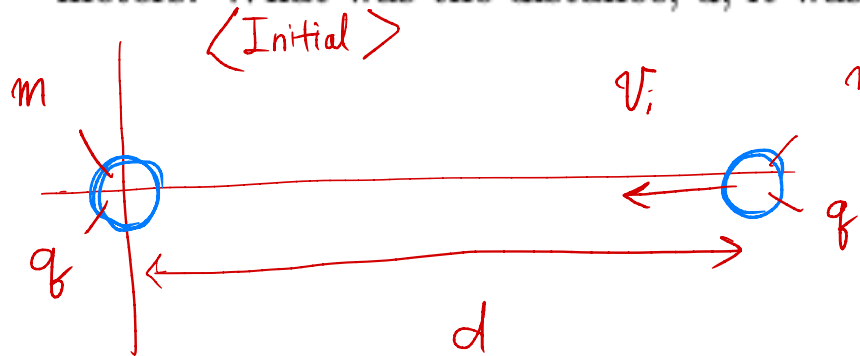
- $U_s = \frac{1}{2}k\Delta x^2$

- $U_E = qV$

- $W_{ext} = F_{ext} d \cos \theta$



7. (8 points) Two identical point charges have mass 1.0 kg and charge +0.0030 C. One is fixed to the origin and cannot move. The other is fired towards the origin from a cannon located a distance  $d$  away. The speed of the moving charge is 900.0 m/s immediately after it is fired from the cannon. The closest it gets to the origin is 0.10 meters. What was the distance,  $d$ , it was fired from?



$$E_i = KE_i + U_{E_i}$$

$$= \frac{1}{2} m v_i^2 + \frac{kq}{d} \cdot q$$

$$E_f = KE_f + U_{E_f}$$

$$= \frac{kq}{(0.1) \text{ m}} \cdot q$$

$$E_i = E_f$$

$$\frac{1}{2} m v_i^2 + \frac{kq^2}{d} = \frac{kq^2}{0.1}$$

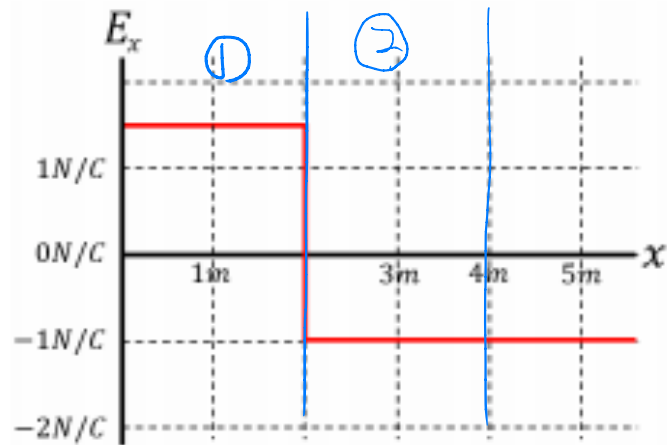
$$\frac{kq^2}{d} = \frac{kq^2}{0.1} - \frac{1}{2} m v_i^2$$

$$d = \frac{kq^2}{\left(\frac{kq^2}{0.1} - \frac{1}{2} m v_i^2\right)} = \frac{9 \times 10^9 \times (3 \times 10^{-3})^2}{\left(\frac{9 \times 10^9 \times (3 \times 10^{-3})^2}{0.1} - \frac{1}{2} (1) (900)^2\right)}$$

$= 0.2$

$$\Rightarrow \underline{\underline{d = 0.2 \text{ m}}}$$

7. (10 points) An electric field points only in the x direction (no y or z components). The x component is shown in the following graph. The potential energy of a 2 C charge is  $U = 10 \text{ J}$  at  $x = 0 \text{ m}$ .



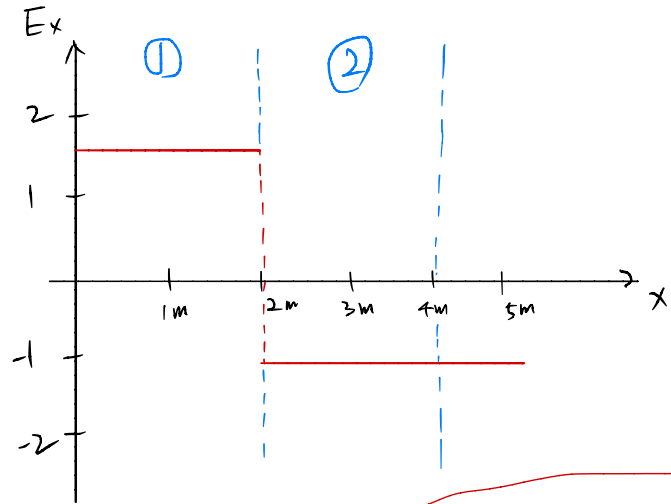
$$U_{E @ 0m} = q V_{@ 0m}$$

$$10 \text{ J} = (2 \text{ C}) V_{@ 0m}$$

$$\Rightarrow \underline{V_{@ 0m} = 5 \text{ V}}$$

a) (8 points) If the charge starts from rest at the origin and experiences no outside forces, what will its kinetic energy be at  $x = 4$  meters?

b) (2 points) What is the potential energy of the charge at  $x = 2$  meters? (hint: this is the easy way to solve part a!)



$$\textcircled{1} E_x = 1.5 \frac{N}{C} = - \frac{\Delta V}{\Delta X}$$

$$\Delta V = V_{\textcircled{2}2m} - V_{\textcircled{2}0m} = V_{\textcircled{2}2m} - 5V$$

$$\Delta X = 2m - 0m = 2m$$

$$\Rightarrow 1.5 = - \left( \frac{V_{\textcircled{2}2m} - V_{\textcircled{2}0m}}{2m} \right)$$

$$3 = -V_{\textcircled{2}2m} + V_{\textcircled{2}0m}$$

$$\underline{V_{\textcircled{2}2m}} = \underline{V_{\textcircled{2}0m}} - 3V = \underline{2V}$$

$\downarrow$   
 5V from

$$\textcircled{2} E_x = -1 \frac{N}{C} = - \frac{\Delta V}{\Delta X}$$

$$\Delta V = V_{\textcircled{2}4m} - V_{\textcircled{2}2m}$$

$$\Delta X = 4m - 2m = 2m$$

$$\Rightarrow -1 = - \frac{(V_{\textcircled{2}4m} - V_{\textcircled{2}2m})}{2m}$$

$$-2 = -V_{\textcircled{2}4m} + V_{\textcircled{2}2m}$$

$$V_{\textcircled{2}4m} = V_{\textcircled{2}2m} + 2V = 4V$$

a) What is kinetic energy at  $x=4\text{m}$

$$E_i = 10\text{J} \quad \text{at} \quad x = 0\text{m}$$

$$E_f = KE + U_E \quad \text{at} \quad x = 4\text{m}$$

$$U_E @ 4\text{m} = qV @ 4\text{m} = (2\text{C})(4\text{V}) = 8\text{J}$$

$$\Rightarrow E_i = E_f$$

$$10\text{J} = KE + 8\text{J}$$

$$\therefore KE @ 4\text{m} = 2\text{J}$$

b) What is  $U_E$  at  $x = 2\text{m}$ .

$$\begin{aligned}U_E @ 2\text{m} &= qV @ 2\text{m} \\ &= (2\text{C})(2\text{V}) = \underline{\underline{4\text{J}}}\end{aligned}$$