PH202 Recitation 1 Rotational Kinematics: Solutions

Arc Length

Arc Length

Change in Angular Position
(Measured in radians)

Radius

$$S = \Delta\theta \, r$$

In words: The **arc Length** (distance) is equal to the **change in angular position** multiplied by the **radius**.

Change in Angular Position

Change in Time

Change in Time squared

$$\Delta\theta = \omega_i \Delta t + \frac{1}{2}\alpha \Delta t^2$$

Initial Angular Velocity

Angular Acceleration

In words: The change in angular position is equal to the product of the initial angular velocity and the change in time plus one half the product of the angular acceleration and the change in time squared.

Rotational Kinematic Equation II

Final Angular Velocity | Initial Angular Velocity | Angular Acceleration | Change in Time

$$\omega_f = \omega_i + \alpha\, \Delta t$$

In words: The **final angular velocity** is equal to the **initial angular velocity** plus the product of the **angular acceleration** and **change in time**

Rotational Kinematic Equation III

Final Angular Velocity | Initial Angular Velocity | Angular Acceleration | Change in Angular Position

$$\omega_f^2 = \omega_i^2 + 2\alpha\Delta\theta$$

In words: The square of the **final angular velocity** is equal to the square of the **initial angular velocity** plus twice the product of the **angular acceleration** and the **change in angular position**.

Discussion Question 1:

Earth rotates once per day.
Stand where for smallest tangential speed and largest tangential speed?

Well, we know tangential speed is given by $V_t = r\omega$
ω = angular velocity = same for any point on earth.
So small $r \Rightarrow V_t$ small $\Rightarrow$ stand on axis (poles)
larger $r \Rightarrow V_t$ large $\Rightarrow$ equator

Discussion Question 2:

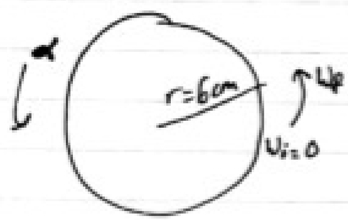
2 points on wheel that is rotating with a decreasing angular velocity. A is on the rim, B halfway between rim and axis.

A) false, $a_r = \omega^2 r$ and r is different
B) false, $a_t = \alpha r$ and r is different
C) false, ω is the same for all r
D) false, $\Delta\theta$ is same for all r

So all answers are false.

PH202 Recitation 1

Problem 1: 6cm radius CD starts at rest and then speeds up to 3600 $\frac{rev}{min}$ in 4s. What is angular acceleration?



k	uk
$\Delta t = 4s$	$\Delta\theta$
$\omega_i = 0$	α
$\omega_f = 3600\,\frac{rev}{min}$	

We know everything we need to solve for α directly.

$$\omega_f = \overset{\nearrow 0}{\cancel{\omega_i}} + \alpha\Delta t \qquad \omega_f = \frac{3600\,rev}{min}\cdot\frac{2\pi\,rad}{1\,rev}\cdot\frac{1\,min}{60\,s} = 120\pi\,\frac{rad}{s}$$

$$\Rightarrow \quad \alpha = \frac{\omega_f}{\Delta t}$$

$$\Rightarrow \quad \alpha = \frac{120\pi\,\frac{rad}{s}}{4s} = 30\pi\,\frac{rad}{s^2}$$

How far did a point on the edge travel during that 4s?

$$S = r\Delta\theta$$

$$\Delta\theta = \overset{0}{\cancel{\omega_i\Delta t}} + \frac{1}{2}\alpha\Delta t^2$$

$$\Delta\theta = \frac{1}{2}\alpha\Delta t^2$$

$$= \frac{1}{2}\left(30\pi\,\frac{rad}{s^2}\right)(4s)^2$$

$$= 240\pi\,rad$$

$$\Rightarrow \quad S = .06m\cdot 240\pi\,rad = 14.4\pi\,m = 45.2\,m$$

If the CD takes 6s to come to rest what is the angular acceleration?

$$\omega_f = \omega_i + \alpha\Delta t$$

$$\Rightarrow \quad \alpha = \frac{\omega_f - \omega_i}{\Delta t} = \frac{0 - 120\pi\,\frac{rad}{s}}{6s} = -20\pi\,\frac{rad}{s^2}$$

Problem 2: At $t=0$ a wheel rotating about a fixed axis has $\omega_i = 2\,\frac{rad}{s}$ and is constantly accelerating.
2 seconds later we have completed 5 revolutions. What is the angular acceleration and angular velocity of the wheel after those 2 seconds?

k	uk
$\omega_i = 2\,\frac{rad}{s}$	ω_f
$\Delta t = 2s$	α
$\Delta\theta = 5\,rev = 10\pi\,rad$	

We can solve for α directly.

$$\Delta\theta = \omega_i \Delta t + \tfrac{1}{2}\alpha \Delta t^2$$

$$\Rightarrow \quad \Delta\theta - \omega_i \Delta t = \tfrac{1}{2}\alpha\Delta t^2$$

$$\Rightarrow \quad \frac{2(\Delta\theta - \omega_i\Delta t)}{\Delta t^2} = \alpha$$

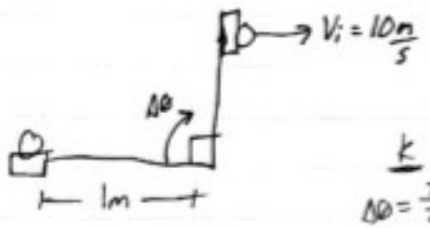
$$\Rightarrow \quad \alpha = \frac{2\left(10\pi\,rad - 2\frac{rad}{s}\cdot 2s\right)}{(2s)^2} = 13.7\,\frac{rad}{s^2} = 5\pi - 2\,\frac{rad}{s^2}$$

We can solve for ω_f directly now.

$$\omega_f = \omega_i + \alpha\Delta t$$

$$\Rightarrow \quad \omega_f = 2\,\frac{rad}{s} + 13.7\,\frac{rad}{s^2}\cdot 2s = 29.4\,\frac{rad}{s}$$

Problem 3: A lacrosse ball is thrown by rotating the stick through 90° and then releasing the ball when vertical. If the stick starts at rest, is 1 m long, and the ball has linear velocity of 10 m/s at launch, what is the angular acceleration of the stick.



k	uk
$\Delta\theta = \frac{\pi}{2}$ rad	ω_f
$\omega_i = 0$	Δt
$V_f = 10 m/s$	α
$L = 1m$	

We know $\Delta\theta = 90° = \frac{\pi}{2}$ rad

V_f = tangential velocity $\Rightarrow V = \omega r$

$\Rightarrow \omega_f = \frac{V_f}{r} = \frac{10 m/s}{1m} = 10 \frac{rad}{s}$

So then we can solve for α directly.

$$\omega_f^2 = \omega_i^2 + 2\alpha\Delta\theta$$

$$\Rightarrow \omega_f^2 - \omega_i^2 = 2\alpha\Delta\theta$$

$$\Rightarrow \alpha = \frac{\omega_f^2 - \omega_i^2}{2\Delta\theta} = \frac{100\frac{rad^2}{s^2} - 0}{2(\frac{\pi}{2} rad)} = \frac{100}{\pi}\frac{rad}{s^2} = 31.8\frac{rad}{s^2}$$