PH202 Recitation 4 Thermodynamics 1 Solutions

Discussion Question 1:

What is average kinetic energy of monatomic gas molecule at 20°C?

well, $\overline{KE} = \frac{1}{2}m\bar{v}^2$

we also know for 1 molecule that $KE = \frac{3}{2}Nk_BT = \frac{3}{2}k_BT$

$$\Rightarrow \frac{1}{2}m\bar{v}^2 = \frac{3}{2}k_BT = \overline{KE}$$

so $\overline{KE} = \frac{3}{2}k_BT$

$$= \frac{3}{2}(1.38\times10^{-23}\,J/K)(20°C+273\,K)$$

$$\Rightarrow \overline{KE} = 6.07\times10^{-21}\,J$$

b) rms speed of He at this temperature?

well we know $\overline{KE} = \frac{3}{2}k_BT = \frac{1}{2}m\bar{v}^2$

$$\Rightarrow 3k_BT = m\bar{v}^2$$

$$\Rightarrow \bar{v} = V_{rms} = \sqrt{\frac{3k_BT}{m}}$$

$T = 20°C = 20+273 = 293K$

atomic mass $= 4g/mol$

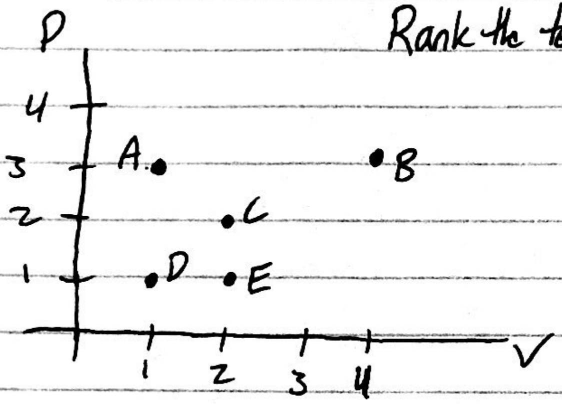
$$\frac{4g}{mol}\cdot\frac{1kg}{1000g}\cdot\frac{1mol}{6.02\times10^{23}\,mol^{-1}} = 6.64\times10^{-27}\,kg$$

so $$V_{rms} = \sqrt{\frac{3\cdot(1.38\times10^{-23}\,J/K)(293K)}{6.64\times10^{-27}\,kg}}$$

$$V_{rms} = 1352\,m/s$$

Discussion Question 2: Ideal gas

Rank the temperatures of the different states



well, ideal gas => ideal gas law applies

$PV = nRT$ or $PV = Nk_BT$

since $nR = Nk_B$ since $n = 1 = \text{constant}$ we know

$PV \propto T$

so rank PV for each point

A: $3P_0(1V_0) = 3P_0V_0$ B: $3P_0(4V_0) = 12P_0V_0$ C: $2P_0(2V_0) = 4P_0V_0$

D: $1P_0(1V_0) = 1P_0V_0$ E: $1P_0(2V_0) = 2P_0V_0$

so rank $T_D < T_E < T_A < T_C < T_B$

Discussion Question 3: ideal gas in sealed container, pressure change if?

A: volume doubled and temperature tripled?

$PV = Nk_BT$

$\Rightarrow P = \frac{Nk_BT}{V}$

$V \to 2V$ and $T \to 3T$

$\Rightarrow P_f \to \frac{Nk_B(3T)}{2V} = \frac{3}{2}P_i$

B: volume halved and temperature tripled?

$V \to \frac{1}{2}V$ and $T \to 3T$

$\Rightarrow P_f \to \frac{Nk_B(3T)}{\frac{1}{2}V} = 6P_i$

Discussion Question 4:

calculate number of molecules in cubic meter of ideal gas at STP?

ideal gas => $PV = Nk_BT$

[redacted]

$$N = \frac{PV}{k_BT}$$

STP: $P = 1\,atm = 101325\,Pa$ $\quad V = 1\,m^3$ $\quad T = 0°C = 273\,K$

$$N = \frac{(101325\,Pa)(1\,m^3)}{(1.38\times10^{-23}\,J/K)(273\,K)} = 2.69\times10^{25} \text{ molecules}$$

How many moles?

$$PV = nRT$$

$$n = \frac{PV}{RT}$$

$$n = \frac{(101325\,Pa)(1\,m^3)}{(8.314\,J/mol\,K)(273\,K)} = 44.6\,mol$$

Check: $44.6\,mol \cdot 6.02\times10^{23}\,\frac{molecules}{mol} = 2.69\times10^{25}$ molecules ✓

Question 1: 2 moles of monatomic gas with $T_i = 500K$
5000J heat added to the gas, gas performs 7.5kJ of work
what is T_f?

well we know $\Delta U = Q + W$
and $\Delta U = \frac{3}{2} N k_B \Delta T = \frac{3}{2} n R \Delta T$

so $\frac{3}{2} n R \Delta T = Q + W$

$$\Rightarrow \Delta T = \frac{2(Q+W)}{3nR}$$

$Q = 5000J$ added $= +5000J$
$W = 7.5kJ$ of work gas does $= -7500J$

$$\Rightarrow \Delta T = \frac{2(5000J - 7500J)}{3(2mol)(8.314 J/mol\,K)} \quad -100K$$

$$\Rightarrow T_f = 400K$$

Question 2: Compute ΔE and ΔT. for 1 mole of ideal monatomic gas

A: 1500J heat added no work

$$\Delta E = Q + W = 1500J \text{ added} = +1500J$$

$$\Delta E = \frac{3}{2} nR\Delta T$$

$$\Rightarrow \frac{2\Delta E}{3nR} = \Delta T = \frac{2(1500J)}{3(1)(8.314 J/mol K)} = +120.3K$$

B: 1500J of work done on gas no heat

$$\Delta E = Q + W = 1500J \text{ work done} = +1500J$$

$$\Delta E = \frac{3}{2} nR\Delta T$$

$$\Rightarrow \Delta T = \frac{2\Delta E}{3nR} = +120.3K$$