PH201 Recitation 2 Average Quantities: Solutions

Position

Position Vector | x-component | y-component

$$\vec{r} = \langle x, y \rangle$$

In words: A position vector is defined by its x and y components.

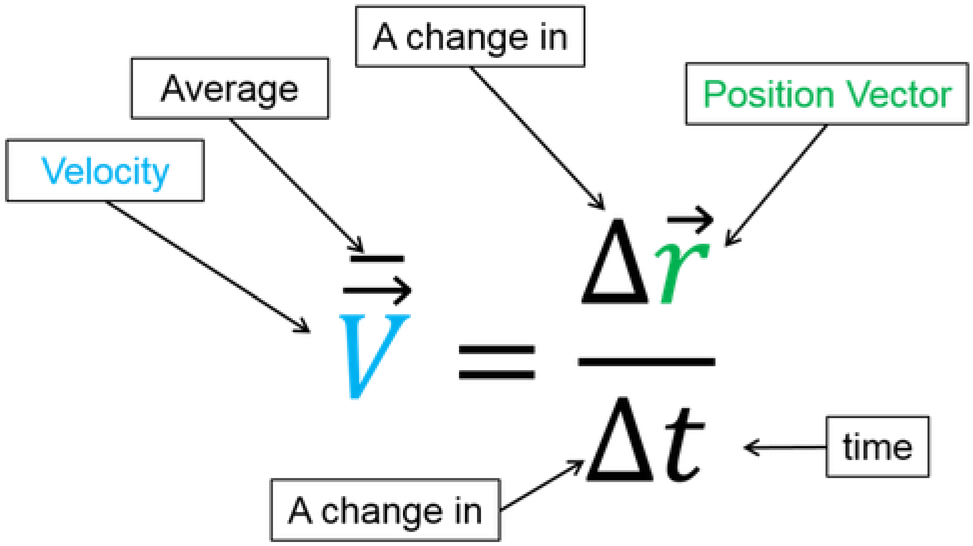
Displacement

A change in | Position Vector | Final position | Initial position

$$\Delta\vec{r} = \vec{r}_f - \vec{r}_i$$

In words: The **change in position** (or displacement) is equal to the **final position** minus the **initial position**.

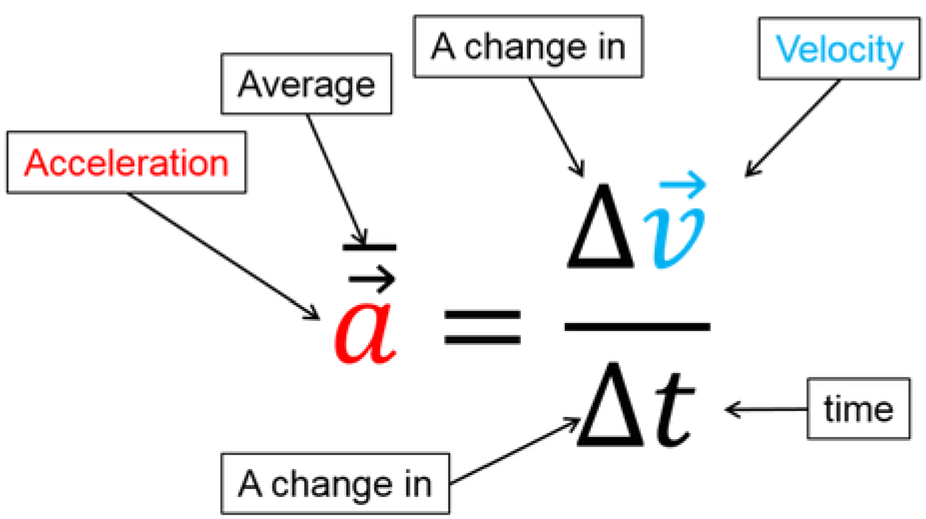
Average Velocity



$$\bar{\vec{V}} = \frac{\Delta \vec{r}}{\Delta t}$$

In words: The **average velocity** is equal to the **change** in **position** divided by the **change** in **time**.

Average Acceleration



$$\bar{\vec{a}} = \frac{\Delta \vec{v}}{\Delta t}$$

In words: The **average acceleration** is equal to the **change** in **velocity** divided by the **change** in **time**.

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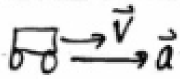
Discussion question 1: car driving down the road
constant acceleration <u>in the direction</u> of the car's motion

picture: assume car's motion is to the right, motion being velocity



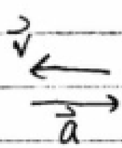
$\vec{a} = \frac{\Delta \vec{v}}{\Delta t} \Rightarrow \Delta \vec{v} \uparrow$ (to the right; since in the same direction) $\Rightarrow$ speed of the car increases $\Rightarrow$ answer A

part 2: constant acceleration opposite direction of the car's motion

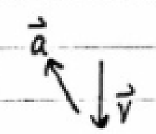
picture: assume car's motion (velocity) is to the right, so acceleration is left

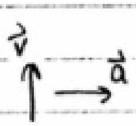
$\vec{a} = \frac{\Delta \vec{v}}{\Delta t} \Rightarrow \Delta \vec{v} \downarrow$ (to the left; since opposite direction) $\Rightarrow$ speed of the car decreases $\Rightarrow$ answer B

Discussion question 2: In which of the cases is the object slowing down and turning to <u>ITS</u> right?

a)  velocity left, acceleration right, so object will slow down and <u>turn around in its</u> perspective

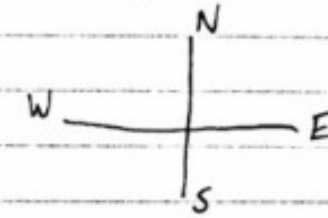
b)  acceleration points in same direction as $\Delta \vec{v}$, so object is turning to <u>its</u> right but increasing velocity (speeding up, not slowing down)

c)  $\vec{a}$ points in same direction as $\Delta \vec{v}$, so object is turning to <u>its</u> right and slowing down ✓

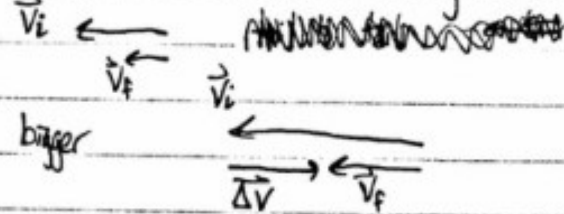
d)  $\vec{a}$ points in same direction as $\Delta \vec{v}$, so object is turning to <u>its</u> right and speeding up

e)  $\vec{a}$ points in same direction as $\Delta \vec{v}$, so object is turning to <u>its</u> left

Discussion question 3: in which situations does the car <u>necessarily</u> have a westward acceleration?

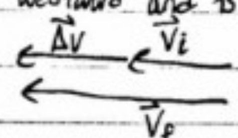


a) car travels westward and is slowing down



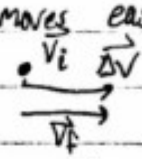
$\vec{a}$ points in same direction as $\vec{\Delta v}$ => $\vec{a}$ to east

b) car travels westward and is speeding up



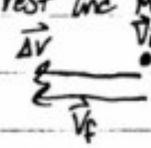
$\vec{a}$ points in same direction as $\vec{\Delta v}$ => $\vec{a}$ to west ✓

c) car starts from rest and moves east



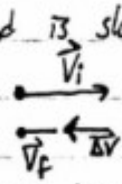
$\vec{a}$ points in same direction as $\vec{\Delta v}$ => $\vec{a}$ to east

d) car starts from rest and moves west



$\vec{a}$ points in same direction as $\vec{\Delta v}$ => $\vec{a}$ to west ✓

e) car travels eastward and is slowing down



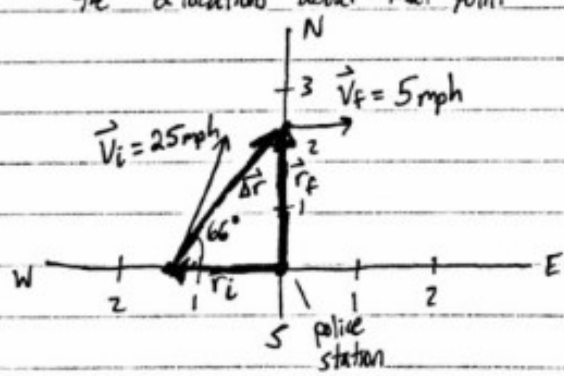
$\vec{a}$ points in same direction as $\vec{\Delta v}$ => $\vec{a}$ to west ✓

Problem of the Day: Gps tells you you're moving 25.0 mph in a direction of 66° N of E at a location 1.33 miles West of the police station. 5 minutes later you are now 2.1 miles north of the police station and are moving east at a speed of 5 mph

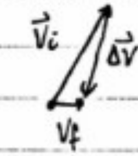
problem orientation: descriptive representation: we are in a location directly west of the origin traveling at a velocity. 5 minutes later we are directly north of the origin traveling with a new velocity.

trying to solve for average velocity, average acceleration, and distance/direction you are away from original reading you took = displacement from initial position

physical representation: choose police station as origin since we know the 2 locations about that point



vector ops for $\vec{a}$ / $\Delta \vec{V}$
tail to tail $\vec{V}_i$ and $\vec{V}_f$



let's convert these quantities into SI units:

$$|\vec{V}_f| = \frac{5\ mi}{hr} \Big| \frac{1609\ m}{1\ mi} \Big| \frac{1\ hr}{3600\ s} = 2.23\ \frac{m}{s} \quad |\vec{V}_f|$$

$$|\vec{V}_i| = \frac{25\ mi}{hr} \Big| \frac{1609\ m}{1\ mi} \Big| \frac{1\ hr}{3600\ s} = 11.17\ \frac{m}{s} \quad [\vec{V}_i]$$

$$|\vec{r}_f| = \frac{2.1\ mi}{} \Big| \frac{1609\ m}{1\ mi} = 3378.9\ m \quad |\vec{r}_f|$$

$$|\vec{r}_i| = \frac{1.33\ mi}{} \Big| \frac{1609\ m}{1\ mi} = 2139.97\ m \quad |\vec{r}_i|$$

$$|t| = \frac{5\ min}{} \Big| \frac{60\ s}{1\ min} = 300\ s \quad |t|$$

Knowns

$\vec{r}_i$

$\vec{r}_f$

$\vec{v}_i$

$\vec{v}_f$

t

Unknowns

$\vec{\Delta r}$

$\vec{\bar{v}}$

$\vec{\Delta v}$

$\vec{\bar{a}}$

equations: $\vec{\Delta r} = \vec{r}_f - \vec{r}_i$

$$\vec{\bar{v}} = \frac{\vec{\Delta r}}{\Delta t}$$

$$\vec{\bar{a}} = \frac{\Delta \vec{v}}{\Delta t}$$

Solve part a) average velocity

well we know that $\vec{\bar{v}} = \frac{\vec{\Delta r}}{\Delta t} = \frac{\vec{r}_f - \vec{r}_i}{\Delta t}$

$$\Rightarrow \vec{\bar{v}} = \frac{\langle 0, 3378.97\rangle m - \langle -2139.97, 0\rangle m}{300 s}$$

$$= \frac{\langle 2139.97, 3378.97\rangle m}{300 s} = \langle 7.13, 11.26\rangle \frac{m}{s}$$

sense making: dimensionality: $\frac{m}{s}$ expected since we want velocity ✓

sign/direction: $\vec{\bar{v}}$ points in direction of $\vec{\Delta r}$ and we know $\vec{\Delta r}$ points in $\langle +, + \rangle$ direction ✓

b) average acceleration

well we know that $\vec{\bar{a}} = \frac{\Delta \vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$

$$\Rightarrow \vec{\bar{a}} = \frac{\langle 2.23, 0\rangle \frac{m}{s} - \langle 11.17\cos 66, 11.17\sin 66\rangle \frac{m}{s}}{300 s}$$

$$= \frac{\langle 2.23 - 4.54, 0 - 10.2\rangle \frac{m}{s}}{300 s} = \langle -.0077, -.034\rangle \frac{m}{s^2}$$

sense making: sign/direction: $\vec{a}$ points in direction of $\vec{\Delta v}$ which has the same direction as answer ✓ $\vec{\Delta v}$ points in $\langle -, - \rangle$ direction

order of magnitude: acceleration is very small which should make sense as $\Delta \vec{v}$ is small and took 5 minutes to do so

Solve part C) distance and direction needed to return to initial reading which is really displacement, but opposite direction of displacement so we want $-\vec{\Delta r}$

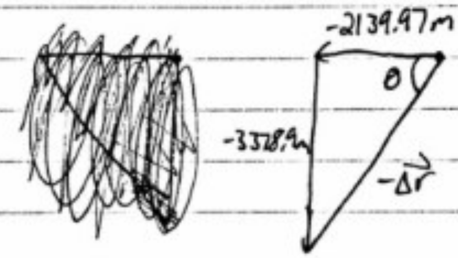
well, $\vec{\Delta r} = \vec{r}_f - \vec{r}_i$

and $-\vec{\Delta r} = -\vec{r}_f - -\vec{r}_i = \vec{r}_i - \vec{r}_f$

$= \langle -2139.97, 0 \rangle m - \langle 0, 3378.97 \rangle m$

$= \langle -2139.97, -3378.97 \rangle m$

distance and direction format wanted, not vector form



distance = hypotenuse = $\sqrt{(3378.9)^2 + (2139.97)^2} = 4000$ m or 4km

direction = θ $\tan(\theta) = \frac{3378.9}{2139.97}$

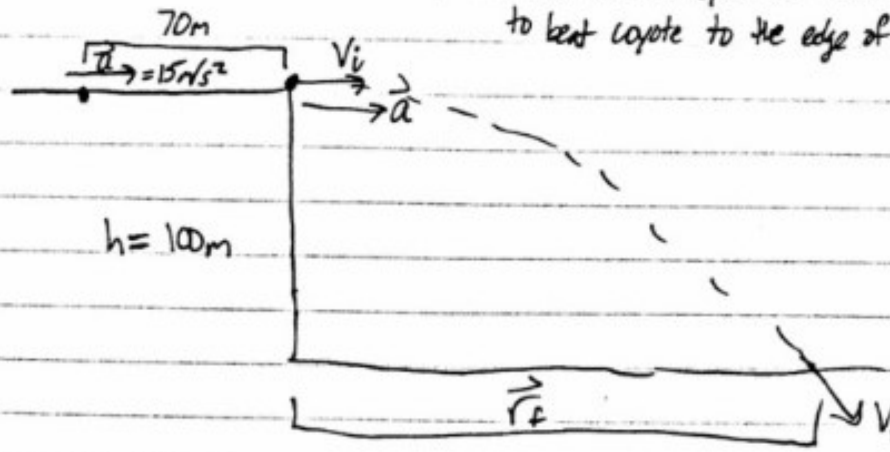
$\Rightarrow \theta = \tan^{-1}\left(\frac{3378.9}{2139.97}\right) = 57.65$ deg

so need to travel 4000m at 57.65 degrees S of E

Sensemaking: sign/direction: to get back to original position we need to head in opposite direction of $\vec{\Delta r}$ which is $\langle +,+ \rangle$ so we need to go $\langle -,- \rangle$ which we are ✓

order of magnitude: Since we have a right triangle we know that $\vec{\Delta r} > r_i$ and r_f which it is ✓

Challenge Homework Orientation: coyote starts at rest 70m away from cliff constant acceleration of 15.0 m/s². find constant speed of road runner to beat coyote to the edge of the cliff



coyote accelerates from rest 70m to cliff, then falls off cliff while still accelerating to the right due to boots. How far does the coyote land away from the base of the cliff? What is the coyote's impact velocity, or velocity as he reaches the ground $\vec{V_f}$?