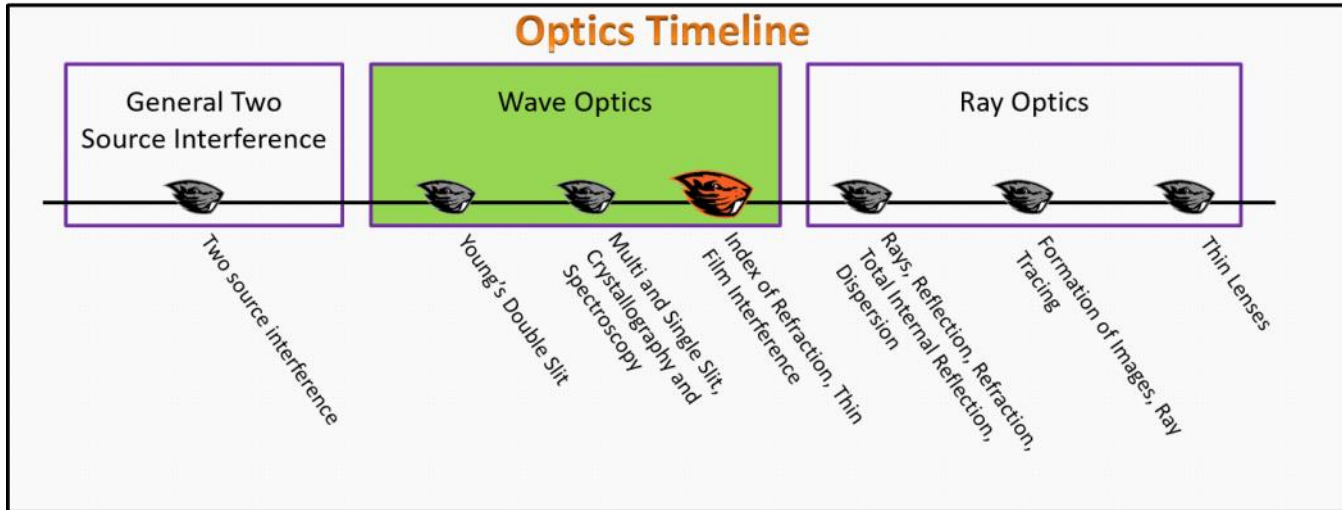


Wave Optics

Foundation Stage (WO.2.L3)

Lecture 3 Index of Refraction, Thin Film Interference



Textbook Chapters (* Calculus version)

- **BoxSand** :: KC videos ([Index of Refraction](#) ; [Thin Film Interference](#))
- **Knight** (College Physics : A strategic approach 3rd) :: 17.1 ; 17.4
- ***Knight** (Physics for Scientists and Engineers 4th) :: 16.5 ; 17.6
- **Giancoli** (Physics Principles with Applications 7th) :: 23-4 ; 24-8

Warm up

WO.2.L3-1:

Description: Conceptual question about interference.

Learning Objectives: [?] - Can you identify the objectives from the previous lecture, and this lecture, that this question is relevant to?

Problem Statement: Which of the following systems are examples of spatial interference? Which of the systems are examples of temporal interference?

- (1) Standing waves on a string.
- (2) Standing waves in a tube.
- (3) Beat frequencies.
- (4) General two source interference.
- (5) Young's double slit.
- (6) Diffraction grating.
- (7) Single slit.
- (8) Thin film interference.

Selected Learning Objectives

1. Coming soon to a lecture template near you.

Key Terms

- Index of refraction
- Speed of light (c)
- Effective speed
- Thin film

Key Equations

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Key Concepts

- Coming soon to a lecture template near you.

Questions

Act I: Index of Refraction

WO.2.L3-2:

Description: Conceptual question about index of refraction. (2 minutes + 2 minutes)

Learning Objectives: [?]

Problem Statement: Given the definition of index of refraction as the ratio of the speed of light in a vacuum over the effective speed in a medium we wish to study the index of refraction of water.

(a) Using your spidey sense, which of the following is most likely the index of refraction of water?

- (1) $n_w = 0.5$
- (2) $n_w = 1$
- (3) $n_w = 1.33$
- (4) $n_w = -2$
- (5) $n_w = 2 + 0.5 i$

$$n = \frac{c}{v}$$

V CANNOT BE GREATER THAN C
SO $n > 1$ AND +

(b) What is the effective speed of light in water? c is the speed of light in a vacuum.

- (1) $v = c$
- (2) $v = 1.33 c$
- (3) $v = 0.75 c$
- (4) $v = 0.5 c$

$$n = \frac{c}{v}$$

$$v = \frac{c}{n} = \frac{c}{1.33} = 0.75 c$$

WO.2.L3-3:

Description: Conceptual question about index of refraction. (3 minutes)

Learning Objectives: [?]

Problem Statement: Light goes from glass ($n_g = 1.52$) into water ($n_w = 1.33$). What happens to the effective speed of light as it moves from the glass to the water?

- (1) v increases.
- (2) v decreases.

(3) v stays the same because the speed of light is always constant.

$$\lambda = \frac{c}{v} \quad \text{w/ } c = \text{CONSTANT ALWAYS}$$

$$\lambda \propto \frac{1}{v} \quad n_2 \rightarrow n_1 \quad \lambda \downarrow \quad \text{SO } v \uparrow$$

WO.2.L3-4:

Description: Conceptual question about transmitted wave. (4 minutes)

Learning Objectives: [?]

Problem Statement: Orange light ($\lambda_{\text{vacuum}} = 611 \text{ nm}$) shines on a soap film ($n_{\text{film}} = 1.33$) that has air on either side of it. When the light travels from the air into the soap film, which features remain unchanged?

- (1) Wavelength.
- (2) Speed.
- (3) Wave number. $\sim k = \frac{2\pi}{\lambda}$
- (4) Amplitude.
- (5) Frequency.
- (6) Intensity.

WO.2.L3-5:

Description: Derivation question relating speed of traveling wave and definition of index of refraction and frequency at boundary to find how wavelengths across boundaries are related. (8 minutes)

Learning Objectives: [?]

Problem Statement: Recall that traveling waves have a wave speed of $v = f\lambda$. The index of refraction also relates the effective speed v to the speed of light in a vacuum: $n = c/v$. Knowing that frequency across a boundary from material 1 to material 2 remains constant, which of the following expressions can be derived from the above mathematical models?

(1) $n_1/\lambda_1 = n_2/\lambda_2$
 (2) $n_1\lambda_1 = n_2\lambda_2$
 (3) $c_1/(n_1\lambda_1) = c_2/(n_2\lambda_2)$
 (4) $f_1 = f_2$

$f_1 = f_2 = f$

$v_1 = f_1\lambda_1 \quad v_2 = f_2\lambda_2 \quad n_1 = \frac{c}{v_1} \quad n_2 = \frac{c}{v_2}$

$v_1 = f\lambda_1 \quad v_2 = f\lambda_2 \quad v_1 = \frac{c}{n_1} \quad v_2 = \frac{c}{n_2}$

$$v_1 = \dots \quad v_2 = \dots$$

$$\frac{v_1}{\lambda_1} = \frac{v_2}{\lambda_2}$$

$$\frac{\left(\frac{c}{n_1}\right)}{\lambda_1} = \frac{\left(\frac{c}{n_2}\right)}{\lambda_2}$$

$$\frac{c}{n_1 \lambda_1} = \frac{c}{n_2 \lambda_2}$$

$$n_1 \lambda_1 = n_2 \lambda_2$$

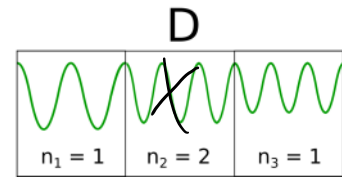
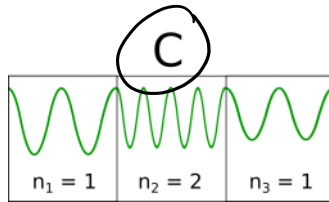
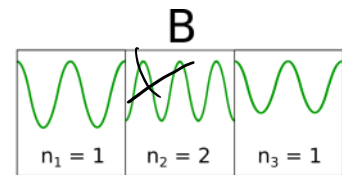
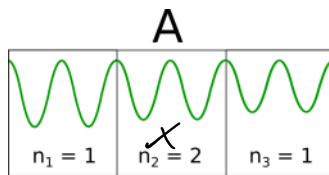
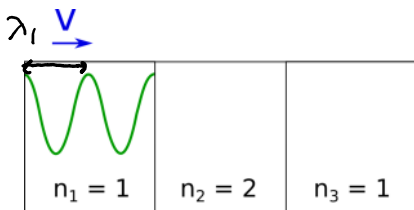
WO.2.L3-6:

Description: Conceptual question about wavelengths of transmitted wave. (4 minutes + 2 minutes)

Learning Objectives: [?]

Problem Statement: Consider a light wave traveling in air that then enters a medium with a higher index of refraction. The light then emerges back into the air on the other side.

(a) Which of the following physical representations best represents this system?



$$n_1 \lambda_1 = n_2 \lambda_2$$

$$1 \lambda_1 = 2 \lambda_2$$

(b) If the light has a wave length of 550 nm in a vacuum, what is it's wavelength in the medium with an index of refraction of 2?

- (1) 275 nm
- (2) 550 nm
- (3) 1100 nm
- (4) 137.5 nm
- (5) 2200 nm

$$n_v \lambda_v = n_2 \lambda_2$$

$$1 \lambda_v = 2 \lambda_2$$

$$\lambda_2 = \frac{\lambda_v}{2}$$

Act II: Thin Film Interference

WO.2.L3-7:

Description: Conceptual question about phase shifts upon transmission and reflection. (3 minutes)

Learning Objectives: [?]

Problem Statement: Which of the following situations causes a phase shift relative to the incident wave?

- (1) Light travels from a high index of refraction into a low index of refraction.
- (2) Light travels from low index of refraction into a high index of refraction.
- (3) Light from a high index of refraction reflects off of a low index of refraction.
- ④ Light from a low index of refraction reflects off of a high index of refraction.

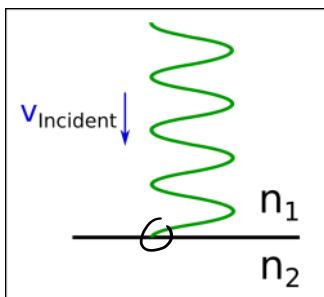
WO.2.L3-8:

Description: Conceptual question about phase shifts upon reflection and transmission. (2 minutes + 2 minutes)

Learning Objectives: [?]

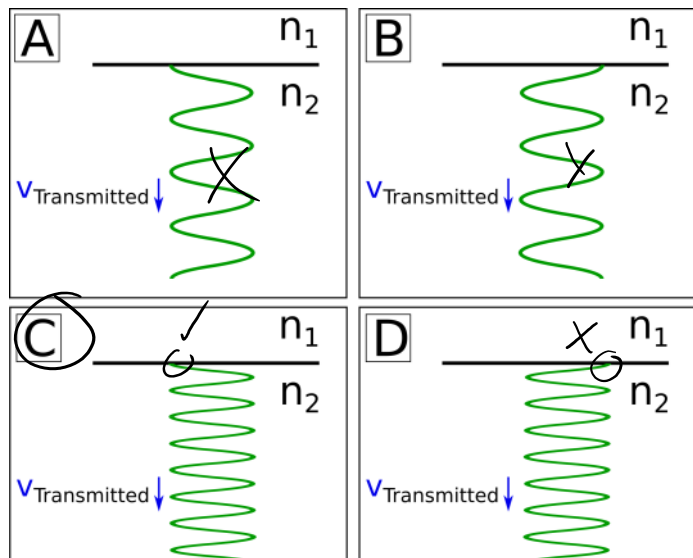
Problem Statement: Light is traveling from an index of refraction n_1 when it reaches a boundary of index n_2 where $n_1 < n_2$.

(a) Which of the following physical representations is a correct depiction of the transmitted wave, relative to the given incident wave, when transmitted into the material with the index of refraction n_2 ?



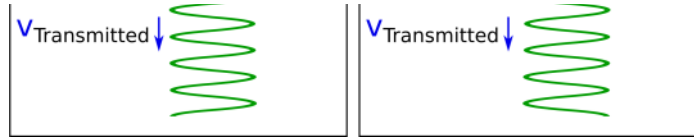
$$n_1 \lambda_1 = n_2 \lambda_2$$

$n_1 < n_2$
so $\lambda_1 > \lambda_2$

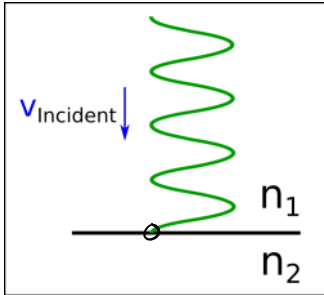


$n_1 < n_2$
 so
 $\lambda_2 \downarrow$

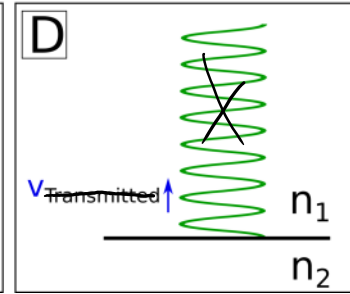
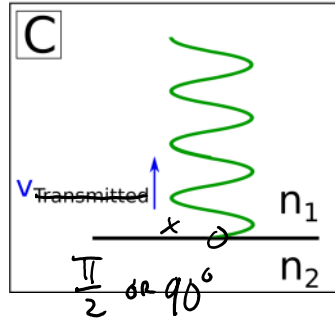
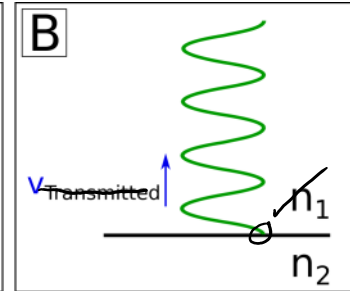
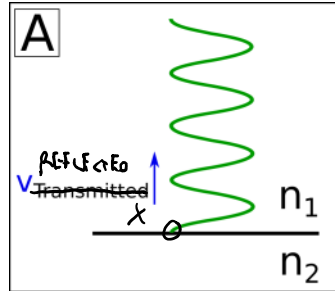
↳ NO PHASE SHIFT ON TRANSMISSION



(b) Which of the following physical representations is a correct depiction of a phase shift of π (180°) relative to the given incident wave upon reflection off of the material n_2 ?



REFLECTED BACK INTO
 n_1 so λ SAME



WO.2.L3-9:

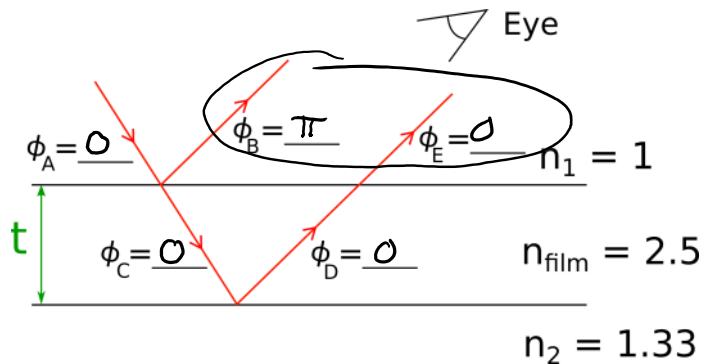
Description: Conceptual question about phase shifts upon reflection and transmission. (2 minutes + 1 minute + 1 minute + 1 minute + 1 minute + 1 minute)

Learning Objectives: [?]

Problem Statement: When light is normally incident on a thin film, the mathematical model which allows us to predict the thickness of the film is dependent on the phase changes upon reflection and transmission at the films boundaries. Below is a typical physical representation of the reflected and transmitted waves for this scenario. Note that the wave is not incident at an angle as seen in the physical representation, the rays are only drawn at an angle to provide space to label phases.

(a) What is the phase (in radians) of the original incident wave, ϕ_A ?

- (1) $\phi_A = 0$
 - (2) $\phi_A = \pi$
 - (3) $\phi_A = \pi/2$
 - (4) The phase is not necessarily known, so we can choose any radian value we wish.
- SO MIGHT AS WELL CHOOSE 0



(b) What is the phase (in radians) of the reflected wave, ϕ_B ?

(c) What is the phase (in radians) of the transmitted wave, ϕ_C ?

(d) What is the phase (in radians) of the reflected wave, ϕ_D ?

(e) What is the phase (in radians) of the transmitted wave, ϕ_E ?

(f) The waves labelled **B** and **E** in this physical representation are the two reference waves that can be conceptually thought of as "sources". Are these two reference waves in phase or out of phase with respect to each other?

- (1) In phase.
- (2) π out of phase.

WO.2.L3-10:

Description: Conceptual question about path length difference for speakers out of phase. (2 minutes + 4 minutes)

Learning Objectives: [?]

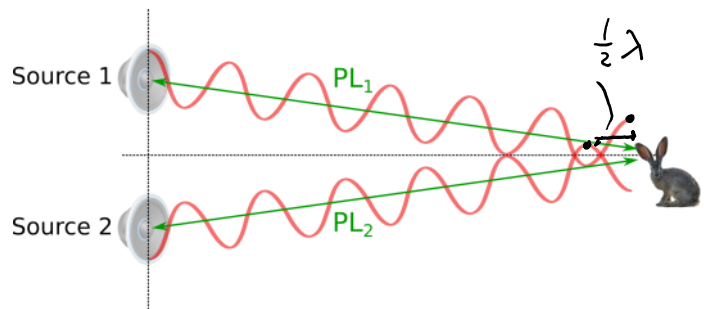
Problem Statement: Two loudspeakers emit coherent waves that are π out of phase with each other. hEAR the Hare is listening at a point along the perpendicular bisector to the vertical line which the speakers are located on as seen in the image below.

(a) Is hEAR at a location in space of constructive or destructive interference?

- (1) Constructive
- (2) Destructive

(b) What can be done to speaker 2 to if hEAR wishes to be at a location of constructive interference?

- (1) Do nothing to speaker 2 since hEAR is already at a location of constructive interference.
- (2) Move speaker 2 such that the path length 2 is changed by 1λ .
- (3) Move speaker 2 such that the path length 2 is changed by $1/2\lambda$.
- (4) Move speaker 2 such that the path length 2 is changed by $1/4\lambda$.



OR IN GENERAL FOR CONSTRUCTIVE

$$PLD = \left(m + \frac{1}{2}\right)\lambda \quad \text{w/ } m = 0, 1, 2, 3, \dots$$

WO.2.L3-11:

Description: Thin film problem with single wavelength. (3 minutes + 1 minute + 1 minute + 2 minutes + 3 minutes + 3 minutes)

Learning Objectives: [?]

Problem Statement: Orange light ($\lambda_{\text{vacuum}} = 611 \text{ nm}$) shines on a soap film ($n_{\text{film}} = 1.33$) that has air on either side of it. We wish to determine the minimum thickness, t , of the film that would cause the orange light to appear bright when viewing from the same side of the incident light.

(a) Determine if the two reference waves are in phase or out of phase?

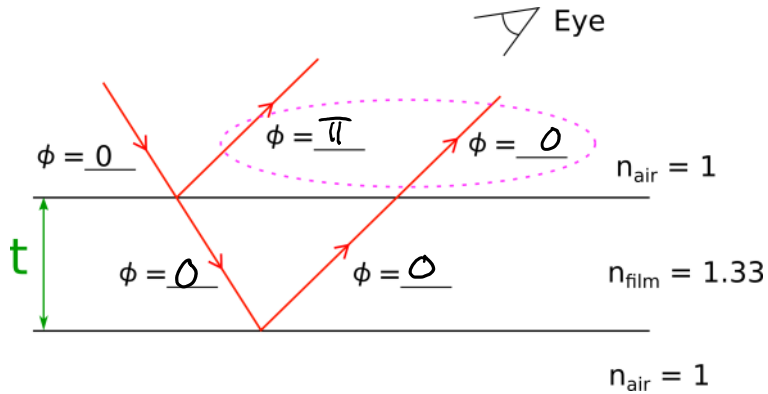
- (1) In phase.
- (2) π radians out of phase.

(b) Are we looking for constructive or destructive interference?

- (1) Constructive interference.
- (2) Destructive interference.

(c) What is the PLD between the first reflected and 2nd reflected wave?

- (1) $t/4$
- (2) $t/2$
- (3) t
- (4) $2t$
- (5) $4t$



(d) Which of the following models allows us to determine the thickness of the film given your answer to part (b)?

- (1) $2t = m \lambda_{\text{film}}$
- (2) $2t = (m + 1/2) \lambda_{\text{film}}$
- (3) $2t = m \lambda_{\text{vacuum}}$
- (4) $2t = (m + 1/2) \lambda_{\text{vacuum}}$

PLD = CONST
 $2t = (m + \frac{1}{2}) \lambda_f$ } π out of phase

(e) Orange light ($\lambda_{\text{vacuum}} = 611 \text{ nm}$) shines on a soap film ($n_{\text{film}} = 1.33$) that has air on either side of it. What is the wavelength of the orange light when in the film?

$\lambda_v \lambda_v = \lambda_f \lambda_f$

$\lambda_f = \frac{\lambda_v}{n_f} \lambda_v$

$\lambda_f = \frac{1}{1.33} (611 \text{ nm}) \approx 459 \text{ nm}$

(f) What is the minimum thickness of the film for this system?

$2t = (m + \frac{1}{2}) \lambda_f$

$t = \frac{1}{2} (m + \frac{1}{2}) \frac{1}{n_f} \lambda_v$

m	t
0	115 nm
1	345 nm
2	575 nm
...	...

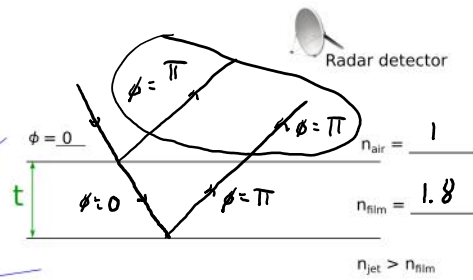
WO.2.L3-12:

Description: Thin film problem with one wavelength. (4 minutes + 1 minute + 5 minutes)

Learning Objectives: [?]

Problem Statement: The Decepticons are building a new top secret skin for their jets that makes them invisible to the Transformer's X-Band radar detectors. The X-Band operates at 12 GHz and the material they want to make the skin out of has an index of refraction of $n_{\text{film}} = 1.80$. The metal of the jet has a higher index of refraction than the film.

(a) Determine if the two reference waves are in phase or out of phase.



(b) Write out the mathematical model that would help the Decepticons determine the thickness of the film.

PLD = DEST.
 $2t = (m + \frac{1}{2}) \lambda_f$ } IN PHASE

(c) The Decepticons are building a new top secret skin for their jets that makes them invisible to the Transformer's X-Band radar detectors. The X-Band operates at 12 GHz and the material they want to make the skin out of has an index of refraction of $n_{film} = 1.80$. **What is the minimum thickness of the film that the Decepticons should apply to the surface of their jets?**

$$2t = (m + \frac{1}{2}) \lambda_f$$

$$2t = (m + \frac{1}{2}) \frac{\lambda_v}{n_f}$$

$$t = \frac{1}{2} (m + \frac{1}{2}) \frac{\lambda_v}{n_f}$$

$n_v \lambda_v = n_f \lambda_f$

m	t
0	0.347 cm
1	1.04 cm
2	1.74 cm
⋮	⋮
⋮	⋮

WO.2.13-13:

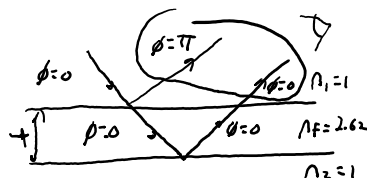
Description: Thin film problem with two wavelengths. (5 minutes + 3 minutes + 2 minutes)

Learning Objectives: [?]

Problem Statement: An art gallery contacted you to help solve their problem with glare from the thick glass ($n_g = 1.62$) that is in front of a valuable piece of artwork. After using a diffraction grating, you notice that the two primary wavelengths that are reflecting strongly off the glass are 460 nm and 690 nm (wave lengths are given in a vacuum). You decide that the best method would be to apply a thin layer of TiO_2 ($n_f = 2.62$) to the surface of the glass such that there would be minimum intensity reflections for both wavelengths. To help keep the integrity of the thin film, what is the 2nd thinnest coat you should apply to the glass? You view the reflections off the glass and thin film in air.

(a) Fill out the table below for the thickness for the 460 nm light. (b) Fill out the table below for the thickness for the 660 nm light.

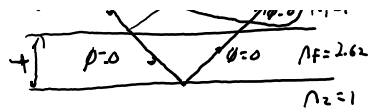
m	t (nm)
0	0
1	87.79
2	175.6



m	t (nm)
0	0
1	131.7
2	263.4

MIN THICKNESS

0	0
1	87.79
2	175.6
3	263.4
4	351.2
5	438.9
6	526.7



PLD = DE 63.

$2t = m\lambda_f$ OUT OF PHASE

$n_f \lambda_f = n_f \lambda_f$

$$2t = m \frac{\lambda_v}{n_f}$$

$$t = \frac{1}{2} m \frac{\lambda_v}{n_f}$$

0	0
1	131.7
2	263.4
3	395.0
4	526.7
5	658.4
6	790.1

MIN THICKNESS

~ 2ND MIN THICKNESS

(c) What is the 2nd minimum thickness of the film?

$$\approx \boxed{527 \text{ nm}}$$

Conceptual questions for discussion

- Coming soon to a lecture template near you.

Hints

- WO.2.L3-1: No hints.
- WO.2.L3-2: No hints.
- WO.2.L3-3: No hints.
- WO.2.L3-4: No hints.
- WO.2.L3-5: No hints.
- WO.2.L3-6: No hints.
- WO.2.L3-7: No hints.
- WO.2.L3-8: No hints.
- WO.2.L3-9: No hints.