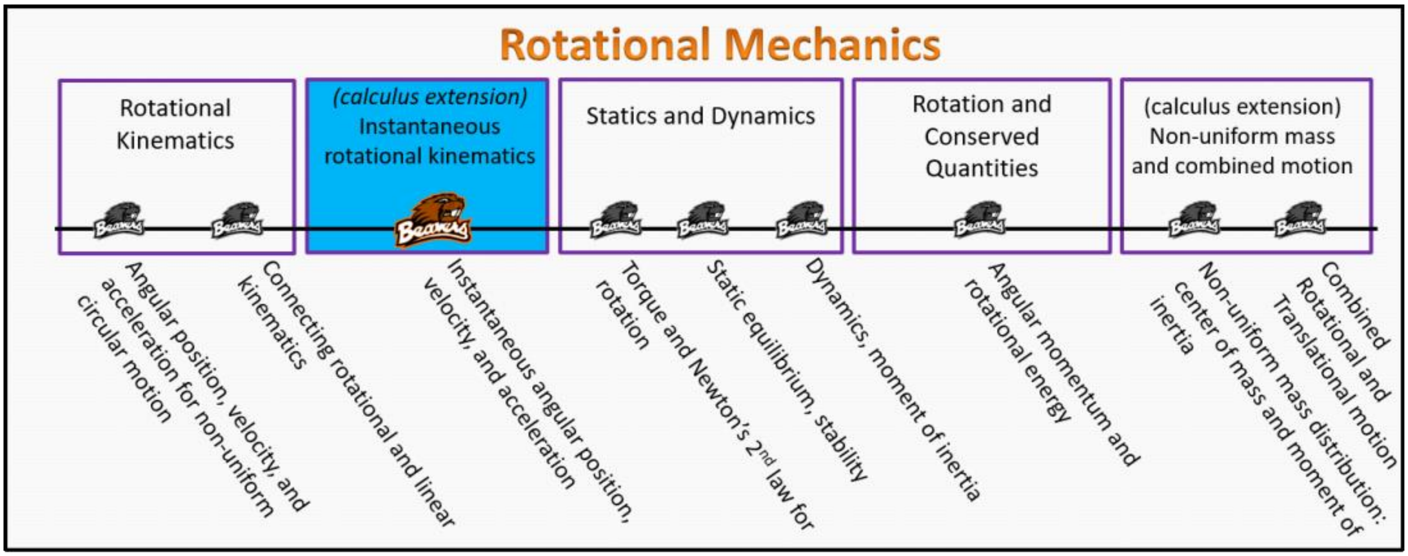
Instantaneous rotational kinematics
Foundation Stage (IR.2.L1)

lecture 1
Instantaneous θ, ω, α



Textbook Chapters (* Calculus version)

- **BoxSand** :: KC videos (N/A)
- **Giancoli** (Physics Principles with Applications 7th) :: 8-1
- **Knight** (College Physics : A strategic approach 3rd) :: N/A
- ***Knight** (Physics for Scientists and Engineers 4th) :: 4.4 ; 4.6

Warm up

IR.2.L1-1:

Description: Given a function, find min/max, derivative, and integral.

Learning Objectives: [?] - Can you identify the objectives from the previous lecture, and this lecture, that this question is relevant to?

Problem Statement: Below is a function, **g**(**z**). We wish to take its derivative and integral.

(a) What is the derivative of **g** with respect to **z**?

$$g(z) = -4\,z^2 + 7$$

(c) What is the integral of **g** with respect to **z** evaluated

(b) What is the max or min of this function? How do you know it's a max or min?

(c) What is the integral of **g** with respect to **z** evaluated from **0** to **z**?

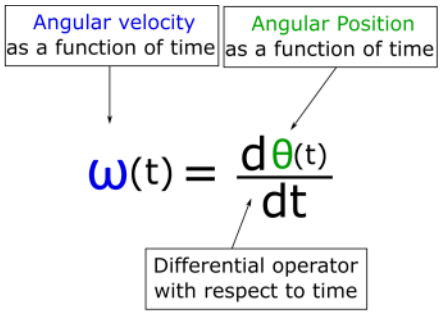
Selected Learning Objectives

1. Coming soon to a lecture template near you.

Key Terms

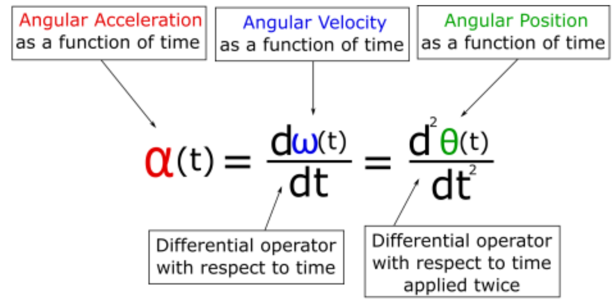
- Instantaneous angular velocity
- Instantaneous angular acceleration
- Fundamental theorem of calculus
- Derivative
- Integral

Key Equations



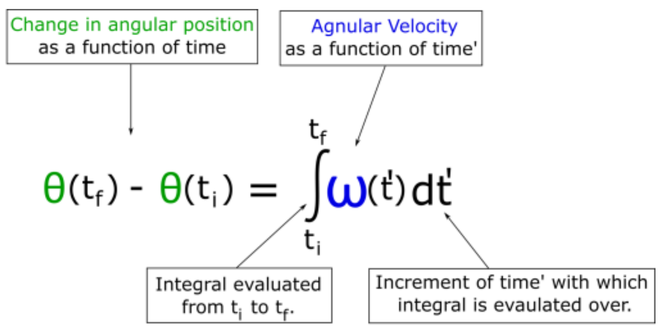
$$\omega(t) = \frac{d\theta(t)}{dt}$$

In words: The angular velocity as a function of time is equal to the time derivative of the angular position as a function of time.



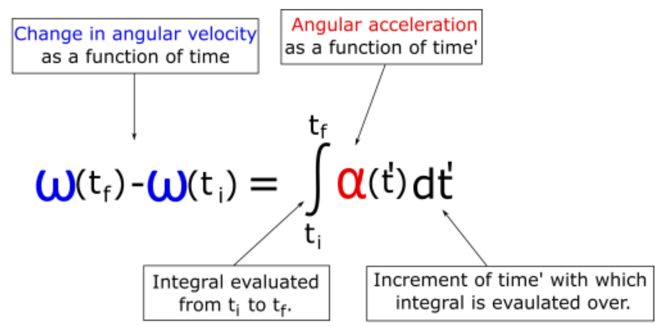
$$\alpha(t) = \frac{d\omega(t)}{dt} = \frac{d^2\theta(t)}{dt^2}$$

In words: The angular acceleration as a function of time is equal to the time derivative of the angular velocity as a function of time. The angular acceleration as a function of time is also equal to the second derivative of angular position as a function of time.



$$\theta(t_f) - \theta(t_i) = \int_{t_i}^{t_f} \omega(t')\,dt'$$

In words: The change in angular position as a function of time is equal to the time' integral of angular velocity as a function of time' evulated at t_i and t_f .



$$\omega(t_f) - \omega(t_i) = \int_{t_i}^{t_f} \alpha(t')\,dt'$$

In words: The change in angular velocity as a function of time is equal to the time' integral of angular acceleration as a function of time' evulated at t_i and t_f .

Key Concepts

- If given an angular position as a function of time equation, you can get the angular velocity as a function of time by taking the derivative with respect to time.
- If given an angular position as a function of time equation, you can get the angular acceleration as a function of time by taking the derivative with respect to time twice.
- If given an angular velocity as a function of time equation, you can get the angular acceleration as a function of time by taking the derivative with respect to time.
- If given an angular velocity as a function of time equation, you can get the change in angular position as a function of time by integrating with respect to time.
- If given an angular acceleration as a function of time equation, you can get the change in angular velocity as a function of time by integrating with respect to time.
- Critical points related to local maximums or minimums of a function are found by taking the derivative of the function and setting it equal to zero.

Act I: From angular position to angular velocity and angular acceleration

Questions

IR.2.L1-2:

Description: Given angular position function, find angular velocity, and angular acceleration functions. (2 minutes + 3 minutes + 3 minutes)

Learning Objectives: [1, 3, 6, 7]

Problem Statement: A certain electric motor starts from rest at **θ** = 0. It's angular position for the first second of operation can be modeled with function as shown below where **t** is measured in seconds.

$$\boldsymbol{\theta}(t) = \frac{2}{3}\boldsymbol{t}^{3/2}$$

(a) Which of the following most justifies the dimensions of the equation?

(1) The dimensions are already correct.
(2) The dimensions of the exponents must make the equation dimensionally correct.
(3) The dimensions when a function is given doesn't have to be correct.
(4) The value (2/3) must have the appropriate dimensions not shown in the function.

(b) What is the angular displacement after the first second of operation?

(c) Construct a function that models this motors angular velocity as a function of time for the first second of operation.

(d) Construct a function that models this motors angular acceleration as a function of time for the first second of operation.

Act II: From Angular Velocity and Angular Acceleration to Angular Position

IR.2.L1-3:

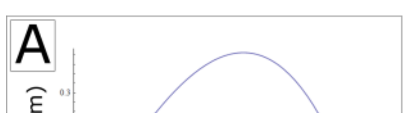
Description: Given angular velocity function, find angular displacement. (3 minutes + 8 minutes)

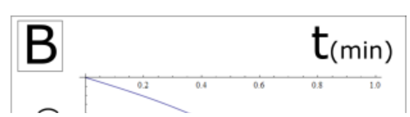
Learning Objectives: [4]

Problem Statement: A merry-go-round starts from rest, was given a push, then let to come back to rest. The angular velocity as a function of time was recorded as seen below were **t** is in minutes and **ω** is in rad/min.

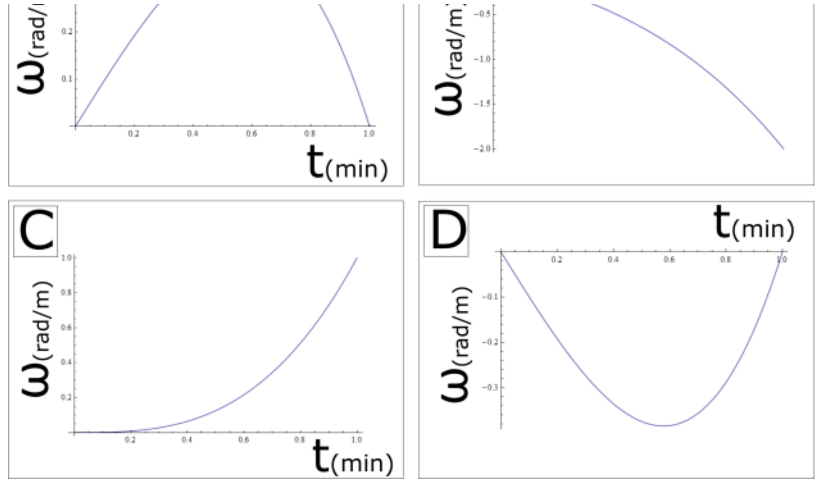
$$\omega = t^3 - t$$

(a) Which graph correctly matches the angular velocity as a function of time?





(b) What was angular displacement after the maximum angular velocity is reached?



Conceptual questions for discussion

1. If given an angular velocity as a function of time equation, can you find the final angular position as function of time?
2. If given an angular acceleration as a function of time equation, can you find the final angular velocity as a function of time?

Hints

IR.2.L1-1: No hints.

IR.2.L1-2: No hints.

IR.2.L1-3: No hints.