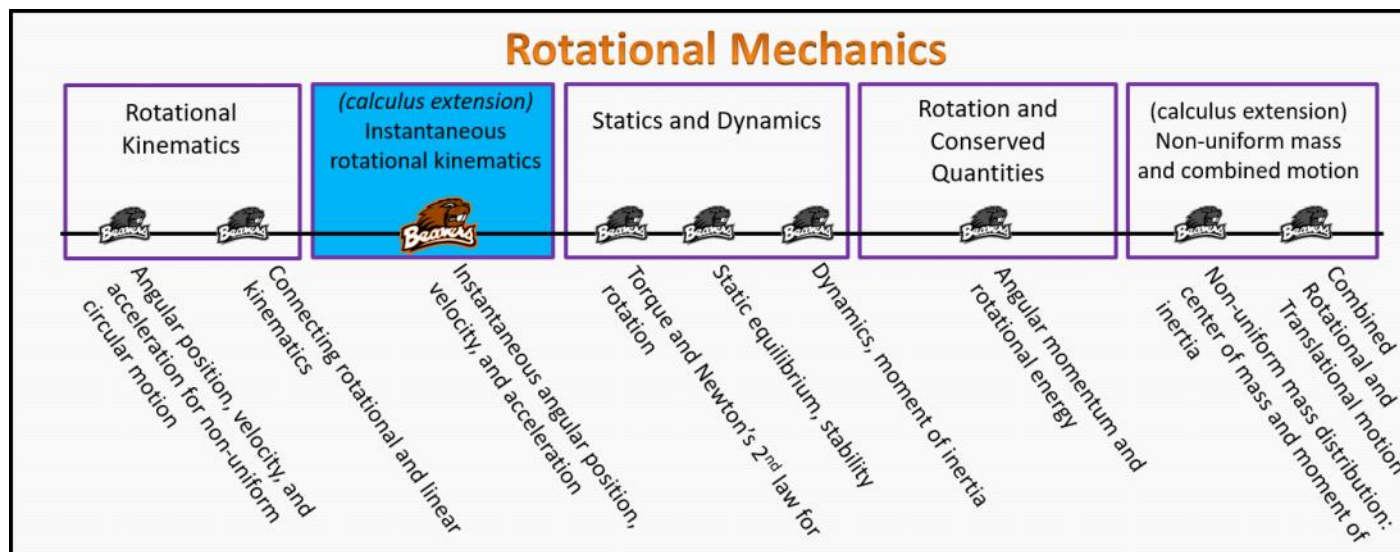


Instantaneous rotational kinematics

Foundation Stage (IR.2.L1)

lecture 1

Instantaneous θ, ω, α



Textbook Chapters (* Calculus version)

- **BoxSand** :: KC videos (N/A)
- **Giancoli** (Physics Principles with Applications 7th) :: 8-1
- **Knight** (College Physics : A strategic approach 3rd) :: N/A
- ***Knight** (Physics for Scientists and Engineers 4th) :: 4.4 ; 4.6

Warm up

IR.2.L1-1:

Description: Given a function, find min/max, derivative, and integral.

Learning Objectives: [?] - Can you identify the objectives from the previous lecture, and this lecture, that this question is relevant to?

Problem Statement: Below is a function, $g(z)$. We wish to take its derivative and integral.

(a) What is the derivative of g with respect to z ?

$$g(z) = -4z^2 + 7$$

$$\frac{dg}{dz} = -8z$$

(c) What is the integral of g with respect to z evaluated

(b) What is the max or min of this function? How do you know it's a max or min?

$$\frac{dg}{dz} = 0$$

$$-8z = 0$$

$$z = 0$$

$$g(z=0) = -4(0)^2 + 7$$

$$g_{\max} = 7$$

Why max?

OPTION 1

$$\frac{d^2g}{dz^2} = -8$$

OPTION 2

LOOK AT NATURE OF FUNCTION

$$g(z) = -8z^2 + 7$$

- PARABOLA
So

↑ JUST SHIFTS
PARABOLA UP



What is the integral of g with respect to z evaluated from 0 to z ?

$$\int_0^z g(z') dz' = \int_0^z (-4z'^2 + 7) dz'$$

$$= -\frac{4}{3} \int_0^z z'^2 dz' + 7 \int_0^z dz'$$

$$= -\frac{4}{3} z'^2 \Big|_0^z + 7 z' \Big|_0^z$$

$$= -\frac{4}{3} (z^2 - 0^2) + 7(z - 0)$$

$$\int_0^z g(z') dz' = -\frac{4}{3} z^2 + 7z$$

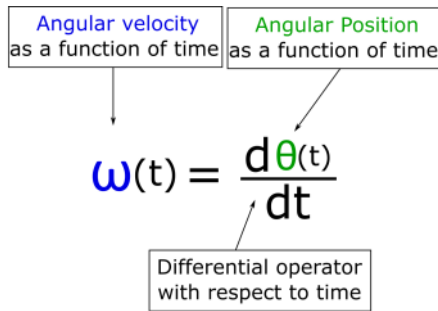
Selected Learning Objectives

- Coming soon to a lecture template near you.

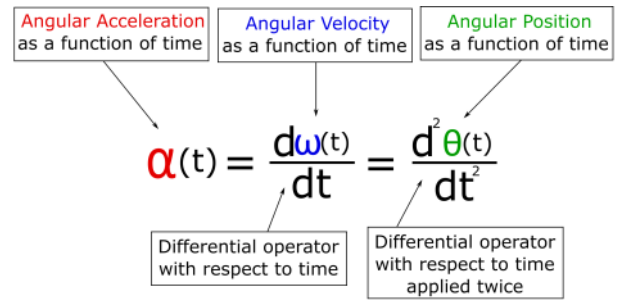
Key Terms

- Instantaneous angular velocity
- Instantaneous angular acceleration
- Fundamental theorem of calculus
- Derivative
- Integral

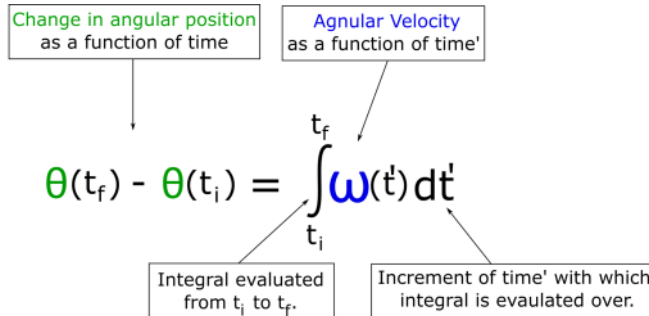
Key Equations



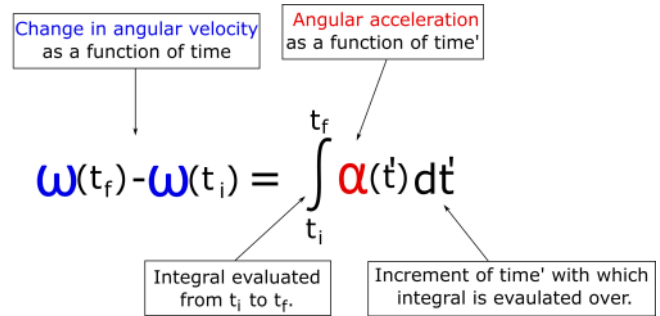
In words: The angular velocity as a function of time is equal to the time derivative of the angular position as a function of time.



In words: The angular acceleration as a function of time is equal to the time derivative of the angular velocity as a function of time. The angular acceleration as a function of time is also equal to the second derivative of angular position as a function of time.



In words: The change in angular position as a function of time is equal to the time' integral of angular velocity as a function of time' evaluated at t_i and t_f .



In words: The change in angular velocity as a function of time is equal to the time' integral of angular acceleration as a function of time' evaluated at t_i and t_f .

Key Concepts

- If given an angular position as a function of time equation, you can get the angular velocity as a function of time by taking the derivative with respect to time.
- If given an angular position as a function of time equation, you can get the angular acceleration as a function of time by taking the derivative with respect to time twice.
- If given an angular velocity as a function of time equation, you can get the angular acceleration as a function of time by taking the derivative with respect to time.
- If given an angular velocity as a function of time equation, you can get the change in angular position as a function of time by integrating with respect to time.
- If given an angular acceleration as a function of time equation, you can get the change in angular velocity as a function of time by integrating with respect to time.
- Critical points related to local maximums or minimums of a function are found by taking the derivative of the function and setting it equal to zero.

Act I: From angular position to angular velocity and angular acceleration

Questions

IR.2.L1-2:

Description: Given angular position function, find angular velocity, and angular acceleration functions. (2 minutes + 3 minutes + 3 minutes)

Learning Objectives: [1, 3, 6, 7]

Problem Statement: A certain electric motor starts from rest at $\theta = 0$. Its angular position for the first second of operation can be

modeled with function as shown below where t is measured in seconds.

(a) Which of the following most justifies the dimensions of the equation?

- F (1) The dimensions are already correct.
- F (2) The dimensions of the exponents must make the equation dimensionally correct.
- F (3) The dimensions when a function is given doesn't have to be correct.
- T (4) The value (2/3) must have the appropriate dimensions not shown in the function.

$$\theta(t) = \frac{2}{3} t^{3/2}$$

(DIMENSIONLESS)
 $[T]^{3/2}$
 $[T]^{3/2}$

(b) What is the angular displacement after the first second of operation?

$$\Delta\theta = \theta(t_f) - \theta(t_i)$$

$$= \frac{2}{3} (1)^{3/2}$$

$\Delta\theta = \frac{4}{9} \text{ RAD}$

(c) Construct a function that models this motors angular velocity as a function of time for the first second of operation.

$$\omega(t) = \frac{d\theta(t)}{dt}$$

$\omega(t) = t^{1/2}$

(d) Construct a function that models this motors angular acceleration as a function of time for the first second of operation.

$$\alpha(t) = \frac{d\omega(t)}{dt}$$

$\alpha(t) = \frac{1}{2} t^{-1/2}$

Act II: From Angular Velocity and Angular Acceleration to Angular Position

IR.2.L1-3:

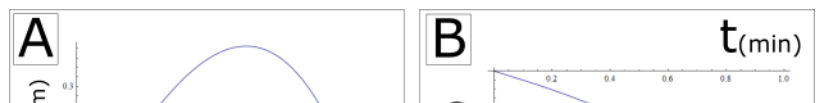
Description: Given angular velocity function, find angular displacement. (3 minutes + 8 minutes)

Learning Objectives: [4]

Problem Statement: A merry-go-round starts from rest, was given a push, then let to come back to rest. The angular velocity as a function of time was recorded as seen below were t is in minutes and ω is in rad/min.

$$\omega = t^3 - t$$

(a) Which graph correctly matches the angular velocity as a function of time?



(b) What was angular displacement after the maximum angular velocity is reached?

$$\Delta\theta(t) = \theta(t_{\text{max}}) - \theta(t=0)$$

$$= \theta(t) = \int_0^t \omega(t') dt'$$

$$= \int_0^t (t'^3 - t') dt'$$

$$= \frac{1}{4} t'^4 \Big|_0^t - \frac{1}{2} t'^2 \Big|_0^t$$

$$\theta(t) = \frac{1}{4} t^4 - \frac{1}{2} t^2 \quad \text{BUT @ WHAT } t \text{ IS } \omega = \omega_{\text{max}}!$$

$$\frac{d\omega(t)}{dt} = 0$$

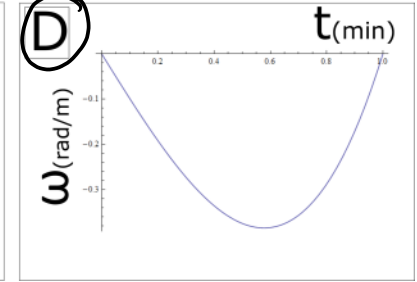
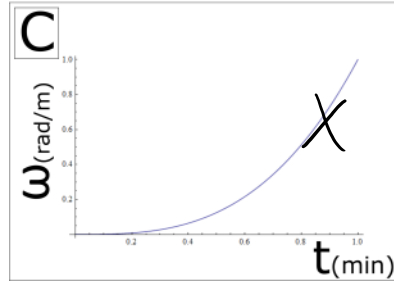
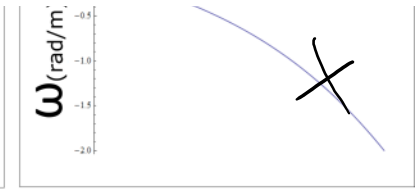
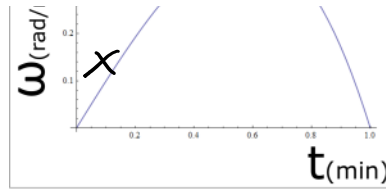
$$3t^2 - 1 = 0$$

$$t = +\sqrt{\frac{1}{3}}$$

$$\text{or } -\sqrt{\frac{1}{3}}$$

$$\theta(t) = -\frac{5}{36} \text{ RAD}$$

↑
BEHINDS MERRY-GO-ROUND!



1. If given an angular velocity as a function of time equation, can you find the final angular position as function of time?
 2. If given an angular acceleration as a function of time equation, can you find the final angular velocity as a function of time?
-

Hints

IR.2.L1-1: No hints.

IR.2.L1-2: No hints.

IR.2.L1-3: No hints.