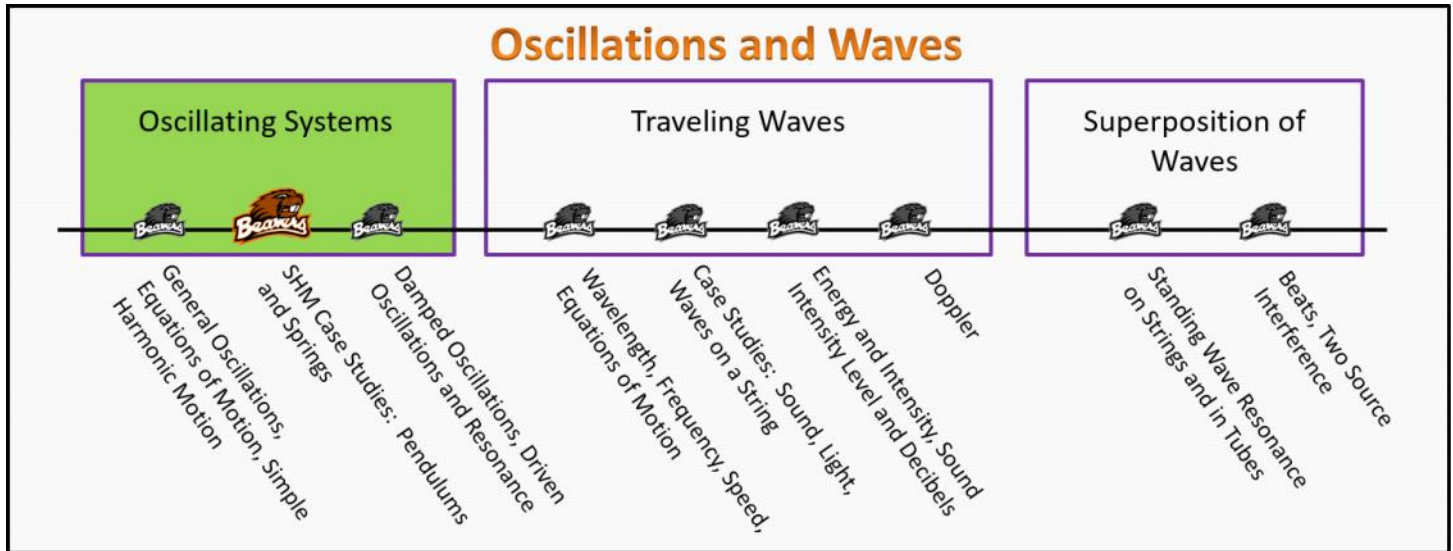


Oscillating Systems

Foundation Stage (OS.2.L2)

Lecture 2

SHM Case Studies: Pendulums and Spring



Textbook Chapters (* Calculus version)

- **BoxSand** :: KC videos ([Springs & Pendulums](#))
- **Knight** (College Physics : A strategic approach 3rd) :: 14.2 ; 14.4 ; 14.5
- ***Knight** (Physics for Scientists and Engineers 4th) :: 15.4 ; 15.5 ; 15.6
- **Giancoli** (Physics Principles with Applications 7th) :: 11-1 ; 11-2 ; 11-3

Warm up

OS.2.L2-1:

Description: Conceptual question about how frequency is related to the oscillating system's parameters.

Learning Objectives: [?] - Can you identify the objectives from the previous lecture, and this lecture, that this question is relevant to?

Problem Statement: Congratulations! You've been selected to be the first person to go to Mars. Unfortunately NASA is on a budget and can't afford a stopwatch, so you will have to take with you a cheap spring-mass system and a pendulum.

(a) Which SHO could you use to keep track of time while on the flight from Earth to Mars in deep space?

(b) Which SHO could you use to keep track of time while on the surface of Mars?

SPRING-MASS

$$T = 2\pi \sqrt{\frac{M}{K}}$$

PENDULUM

$$T = 2\pi \sqrt{\frac{l}{g}}$$

as g is not needed space

EITHER. SO MIGHT AS WELL USE BOTH.

$$\sqrt{k}$$

NO g IN DEEP SPACE.

Selected Learning Objectives

- Coming soon to a lecture template near you.

Key Terms

- Spring-mass oscillator
- Pendulum (i.e. simple pendulum)

Key Equations

Springs	Pendulums
$\omega = \sqrt{\frac{k}{m}}$	$\omega = \sqrt{\frac{g}{\ell}}$
$v_{\max} = \omega x_{\max}$	$\Omega_{\max} = \omega \theta_{\max}$
$a_{\max} = \omega v_{\max} = \omega^2 x_{\max}$	$\alpha_{\max} = \omega \Omega_{\max} = \omega^2 \theta_{\max}$
$E_{\text{total}} = \frac{1}{2} k x_{\max}^2 = \frac{1}{2} m v_{\max}^2$	$E_{\text{total}} = \frac{1}{2} m g \ell \theta_{\max}^2 = \frac{1}{2} m \ell^2 \Omega_{\max}^2$

Key Concepts

- Coming soon to a lecture template near you.

Questions

Act I: Comparing Features of Spring-Mass Oscillators and Pendulums

OS.2.L2-2:

Description: Proportional reasoning for SHM with spring constant and frequency. Proportional reasoning for SHM with period and free-fall acceleration. Proportional reasoning for SHM with spring constant and mass and max speed. (2 minutes + 2 minutes + 5 minutes)

Learning Objectives: [1, 12, 13]

Problem Statement: Did someone say proportional reasoning?

(a) If the frequency of a spring-mass system is doubled and the mass remains constant, by how much has the spring constant changed?

(1) Decreased by a factor of 4

(2) Decreased by a factor of 2

$$f = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$$

(a) If the frequency of a spring-mass system is doubled and the mass remains constant, by how much has the spring constant changed?

- (1) Decreased by a factor of 4
- (2) Decreased by a factor of 2
- (3) Remained the same
- (4) Increased by a factor of 2
- (5) Increased by a factor of 4

$$F = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$$

$$F \propto \sqrt{k}$$

$$k \propto F^2$$

$$\text{IF } F \rightarrow 2F$$

$$\text{NEW } k \rightarrow 4k$$

(b) If we take a pendulum to planet Zorgon 3, the period doubles. What is the free-fall acceleration on Zorgon 3?

- (1) $1/4 g$
- (2) $1/2 g$
- (3) g
- (4) $2 g$
- (5) $4 g$

$$l = \text{CONST}$$

$$T = 2\pi \sqrt{\frac{l}{g}}$$

$$g \propto \frac{1}{T^2}$$

$$T \propto \frac{1}{\sqrt{g}}$$

$$\text{IF } T \rightarrow 2T$$

$$\text{NEW } g \rightarrow \frac{1}{4}g$$

(c) A spring-mass system is switched out with one that has a spring constant that is 4 times as large, and a mass that is a one fourth as large. The amplitude of the oscillation is also decreased by a factor of 2. By what factor has the maximum speed changed in the oscillation?

- (1) $1/4$
- (2) $1/2$
- (3) 1
- (4) 2
- (5) 4

$$\omega = \sqrt{\frac{k}{m}}$$

$$V_{\max} = \omega X_{\max}$$

$$V_{\max} = \sqrt{\frac{k}{m}} X_{\max}$$

$$k \rightarrow 4k$$

$$m \rightarrow \frac{1}{4}m$$

$$X_{\max} \rightarrow \frac{1}{2} X_{\max}$$

$$V_{\max} \rightarrow \sqrt{\frac{4}{(\frac{1}{4})}} \cdot \frac{1}{2} V_{\max}$$

$$\rightarrow 4 \cdot \frac{1}{2}$$

$$\rightarrow 2$$

OS.2.L2-3:

Description: Conceptual question to compare features of spring-mass and pendulum systems. (4 minutes)

Learning Objectives: [1, 12, 13]

Problem Statement: Which of the following is the same for both spring-mass oscillators and pendulums with small amplitudes?

- T (1) Frequency is independent of amplitude.
- F (2) Frequency is independent of energy.
- F (3) Frequency is dependent on mass.
- F (4) Frequency is dependent on the free-fall acceleration g .
- F (5) Both amplitudes are measured in meters.

SHO

Act II: Spring-Mass Oscillator

OS.2.L2-4:

Description: Extract SHM quantities from a graphical representation of a spring-mass oscillator. (1 minutes + 3 minutes + 3 minutes + 3 minutes + 3 minutes)

Learning Objectives: [1, 12, 13]

Problem Statement: A plot of displacement from equilibrium vs time for a spring-mass system is shown below. The mass attached to the ideal spring has a mass of 7 kg.

(a) What is the period of oscillation?

$$\boxed{T = 4 \text{ sec.}}$$

(b) What is the maximum speed the mass reaches?

$$\begin{aligned} v_{\max} &= \omega X_{\max} \\ &= \frac{2\pi}{T} X_{\max} = \frac{2\pi}{4} (3) \text{ m/s} \\ &= \boxed{4.71 \text{ m/s}} \end{aligned}$$

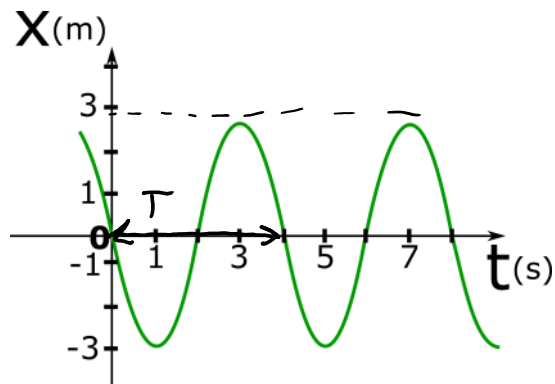
(c) What is the spring constant of the spring?

$$\begin{aligned} \omega &= \sqrt{\frac{k}{m}} \quad \rightarrow \quad \frac{2\pi}{4} = \sqrt{\frac{k}{7}} \\ \frac{2\pi}{T} &= \sqrt{\frac{k}{m}} \quad \rightarrow \quad \boxed{k = 17.3 \text{ N/m}} \end{aligned}$$

(d) What is the total energy of this system?

$$\begin{aligned} E_{\text{total}} &= \frac{1}{2} m v_{\max}^2 \quad \text{OR} \quad \frac{1}{2} k X_{\max}^2 \\ &= \frac{1}{2} (7) (4.71)^2 \quad \text{OR} \quad \frac{1}{2} (17.3) (3)^2 \\ &= \boxed{77.6 \text{ J}} \quad \text{OR} \quad \boxed{77.8 \text{ J}} \end{aligned}$$

↔
Rounding Errors



(e) Construct the mathematical equation that represents the displacement of this system as a function of time. USE SI units.

$$\boxed{X(t) = -3 \text{ m} \sin\left(\frac{\pi}{2} t\right) \text{ meters}}$$

OS.2.L2-5:

Description: Extract SHM quantities from a mathematical representation of a spring-mass oscillator. (1 minute + 2 minutes + 1 minute + 3 minutes + 2 minutes + 2 minutes + 2 minutes).

Learning Objectives: [1, 12, 13]

Problem Statement: A classmate of yours was studying the motion of a spring-mass oscillator with a spring that has a spring constant of 25 N/m. Your classmate constructed an equation in SI units that represented the displacement of the mass as seen below. From this equation, determine the following quantities.

$$x(t) = \frac{\pi}{18} \cos\left(\frac{\pi}{2} t\right)$$

(a) Amplitude.

$$\frac{\pi}{18} \text{ RAD} \approx \boxed{0.175 \text{ RAD}}$$

OR

$$\frac{\pi}{18} \text{ RAD} \times \frac{180^\circ}{\pi \text{ RAD}} = \boxed{10^\circ}$$

(b) Period.

$$\omega = \frac{\pi}{2}$$

$$\frac{2\pi}{T} = \frac{\pi}{2}$$

$$\boxed{T = 4 \text{ SEC}}$$

(c) Frequency.

$$f = \frac{1}{T}$$

$$\boxed{f = 0.25 \text{ Hz}}$$

(d) Mass of the oscillator.

$$\omega = \sqrt{\frac{k}{m}}$$

$$2\pi f = \sqrt{\frac{k}{m}}$$

$$m = \frac{k}{4\pi^2 f^2} = \frac{25}{4\pi^2 (0.25)^2} = \boxed{10.1 \text{ kg}}$$

(e) Maximum speed of the mass.

$$\begin{aligned} v_{\text{max}} &= \omega X_{\text{max}} \\ &= 2\pi f X_{\text{max}} \\ &= 2\pi (0.25) \left(\frac{\pi}{18}\right) \\ &= \boxed{0.274 \text{ m/s}} \end{aligned}$$

(f) Maximum acceleration of the mass.

$$\begin{aligned} a_{\text{max}} &= \omega^2 X_{\text{max}} \\ &= 4\pi^2 f^2 X_{\text{max}} \\ &= 4\pi^2 (0.25)^2 \left(\frac{\pi}{18}\right) \\ &= \boxed{0.431 \text{ m/s}^2} \end{aligned}$$

(g) Total energy of oscillation.

$$\begin{aligned} \frac{1}{2} k X_{\text{max}}^2 &= \boxed{0.381 \text{ J}} \\ \text{OR} \\ \frac{1}{2} m v_{\text{max}}^2 &= \boxed{0.381 \text{ J}} \end{aligned}$$

Act II: Pendulums**OS.2.L2-6:**

Description: Conceptual question about period of pendulum oscillator and changing mass. (3 minutes)

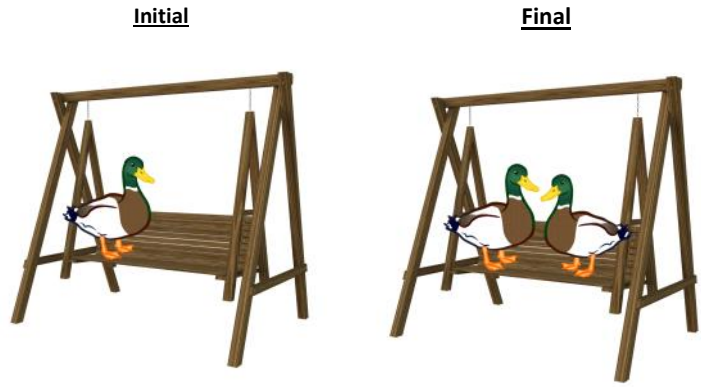
Learning Objectives: [1, 12, 13]

Problem Statement: After losing the big game to the beavers, a sad duck is on a swing and it is observed that the period is 3.0 seconds. A second duck jumps on the swing in efforts to comfort it's friend. With the two ducks on the swing, the period is

- (1) 6.0 s.
- (2) greater than 3.0 s but not necessarily 6.0 s.
- ③ 3.0 s.
- (4) less than 3.0 s but not necessarily 1.5 s.
- (5) 1.5 s.

$$T = 2\pi \sqrt{\frac{l}{g}}$$

↑
NO MASS



OS.2.L2-7:

Description: Conceptual question about period for pendulum and changing mass distribution. (3 minutes)

Learning Objectives: [1, 12, 13]

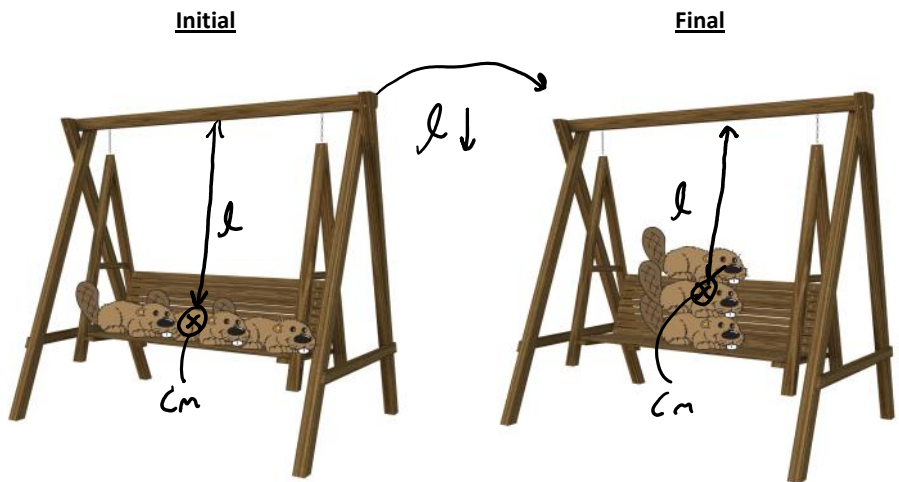
Problem Statement: After hearing the news about the big game against the ducks, three happy beavers initially on a swing side by side decide to do a beaver pile. The new natural frequency of the swing is

- ① greater.
- (2) less.
- (3) the same.

$$f = \frac{1}{2\pi} \sqrt{\frac{g}{l}}$$

$$f \propto \frac{1}{\sqrt{l}}$$

If $l \downarrow$
then $f \uparrow$

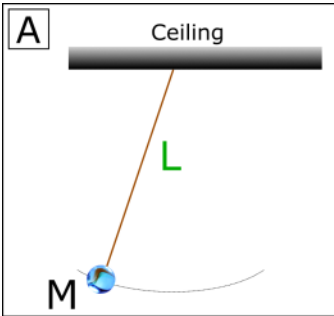


OS.2.L2-8:

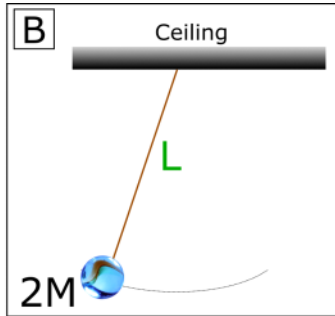
Description: Ranking question for frequency of pendulums with different mass and lengths. (4 minutes)

Learning Objectives: [1, 12, 13]

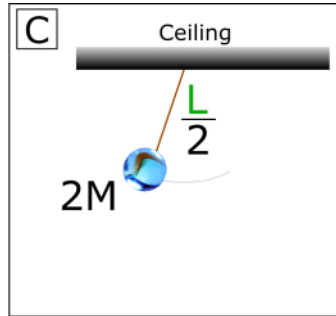
Problem Statement: A simple pendulum shown in case **A** consists of a mass **M** attached to a massless string of length **L**. If the mass is pulled to one side a small distance and released, it will swing back and forth undergoing SHM. Cases **B**, **C**, and **D** are variations of this system. Rank the oscillation frequency of each case.



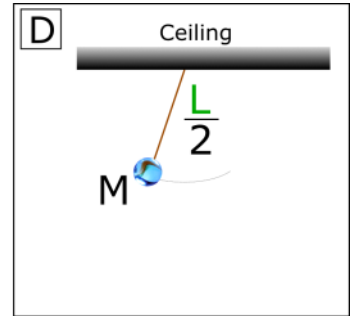
f



f



$\sqrt{2} f$



$\sqrt{2} f$

$$f = \frac{1}{2\pi} \sqrt{\frac{g}{L}}$$

$$f \propto \frac{1}{\sqrt{L}}$$

$$f_A = f_B < f_C = f_D$$

OS.2.L2-9:

Description: Activity to find g using a pendulum. (20 minutes)

Learning Objectives: [1, 12, 13]

Problem Statement: Each group should have a piece of string and a mass.

(a) Use *only* these two items (i.e. no cell phones, no rulers, etc..) to *estimate* g , the free-fall acceleration in units of m/s^2 .

$$T = 2\pi \sqrt{\frac{l}{g}} \rightarrow g = \frac{l}{T^2 4\pi^2}$$

\swarrow MEASURE
 \uparrow MEASURE
 \nwarrow ESTIMATE $4 \times 3 \times 3$

(b) Use these items, and anything else at your disposal to estimate g . Do not change your answer until prompted.

OS.2.L2-10:

Description: Write-out question for a pendulum. (6 minutes + 6 minutes + 5 minutes)

Learning Objectives: [1, 12, 13]

Problem Statement: Penelope the pelican is 4.0 kg and strapped to a harness via a light 3-m-long string that forms a simple pendulum. The pelican pendulum system is placed in an evacuated air chamber; don't panic, Penelope is wearing a high tech space suit.

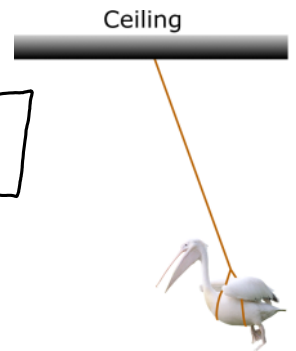
(a) If the pelican pendulum system is released from an angle of 12° , how many oscillations are made in one day?

$$f = \frac{1}{2\pi} \sqrt{\frac{g}{L}}$$

$$f = \frac{1}{2\pi} \sqrt{\frac{9.8}{3}}$$

$$f \approx 0.2876554072 \frac{1}{s}$$

$$\frac{0.2876554072 \text{ osc}}{1s} \times \frac{60s}{1min} \times \frac{60min}{1hr} \times \frac{24hr}{1Day} \approx \boxed{24853 \frac{\text{osc}}{\text{Day}}}$$



(b) The 4.0 kg pelican pendulum system (3-m-long string) is then transported (via pelican express) to the moon, where gravity is one sixth the value on Earth. If it is raised up to 10° and released, how long will it take to undergo the same number of oscillations as it did on Earth in one day?

$$f = \frac{1}{2\pi} \sqrt{\frac{g}{L}}$$

$$f \propto \sqrt{g}$$

$$g \rightarrow \frac{1}{6}g$$

$$\text{so } f \rightarrow \frac{1}{\sqrt{6}} f$$

$$f = \frac{1}{\sqrt{6}} (0.2876554072) \text{ Hz}$$

$$f \approx 0.117438282 \frac{1}{s}$$

$$T = \frac{1}{f}$$

$$T = 8.515361372 \text{ s}$$

$$\frac{8.515361372 \text{ s}}{1 \text{ osc}} \times 24853 \text{ osc} \approx \boxed{211632 \text{ s}}$$

or $\boxed{2.5 \text{ Days}}$

(c) What length could you make the string on the moon to have the answers to part (a) and (b) be equal?

$$f = \frac{1}{2\pi} \sqrt{\frac{g}{L}}$$

$$F = \frac{1}{2\pi} \sqrt{\frac{g}{l}}$$

SINCE $g \rightarrow \frac{1}{8}g$

FOR F SO BE CONST

I WOULD HAVE l SO $l \rightarrow \frac{1}{8}l$

$$\text{SO } l_{\text{max}} = \frac{1}{8}(3)m = \boxed{\frac{1}{2}m}$$

Conceptual questions for discussion

1. Coming soon to a lecture template near you

Hints

OS.2.L2-1: No hints.

OS.2.L2-2: No hints.

OS.2.L2-3: No hints.

OS.2.L2-4: No hints.

OS.2.L2-5: No hints.

OS.2.L2-6: No hints.

OS.2.L2-7: No hints.

OS.2.L2-8: No hints.

OS.2.L2-9: No hints.

OS.2.L2-10: No hints.