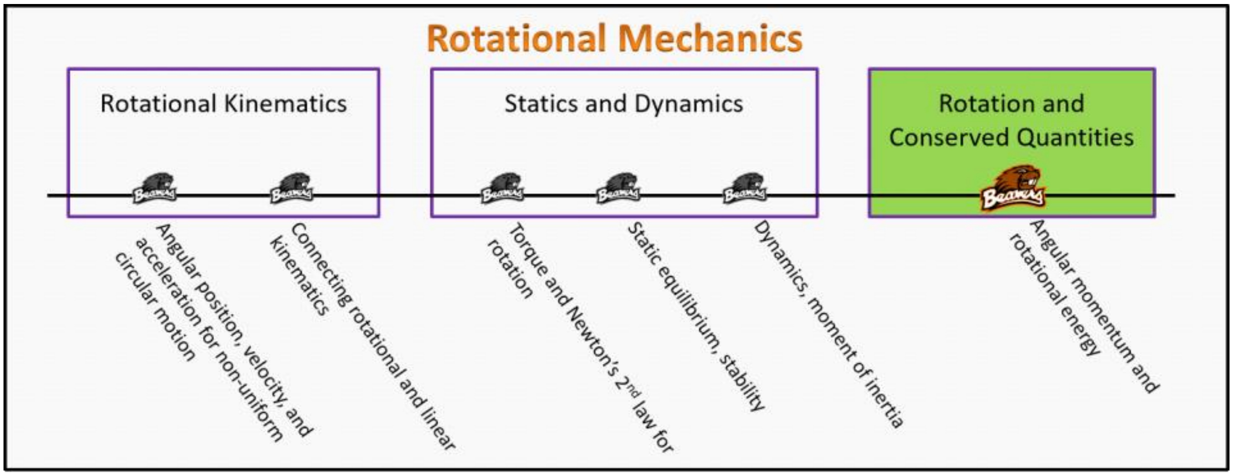
Rotation and Conserved Quantities
Foundation Stage (RC.2.L1)

lecture 1
Angular momentum and rotational energy



Textbook Chapters (* Calculus version)

- **BoxSand** :: KC videos (angular momentum ; rotational energy)
- **Knight** (College Physics : A strategic approach 3rd) :: 9.7 ; 10.3
- ***Knight** (Physics for Scientists and Engineers 4th) :: 12.3 ; 12.11
- **Giancoli** (Physics Principles with Applications 7th) :: 8-7 ; 8-8

Warm up

RC.2.L1-1:

Description: Given forms and values of energy for a generic system, find one unknown final energy value.

Learning Objectives: [?] - Can you identify the objectives from the previous lecture, and this lecture, that this question is relevant to?

Problem Statement: A system initially has the following forms of energy: $\mathbf{E^{th}}$ = 30 J , $\mathbf{E^{chem}}$ = 10 J , $\mathbf{U^{g}}$ = 20 J , $\mathbf{KE_T}$ = 15 J , $\mathbf{KE_R}$ = 5 J , $\mathbf{U^{s}}$ = 2 J. During some amount of time there was negative 10 J of external work on the system. After the external work is done it is found that system now has the following forms of energy: $\mathbf{E^{th}}$ = 30 J , $\mathbf{E^{chem}}$ = 2 J , $\mathbf{U^{g}}$ = 25 J , $\mathbf{KE_T}$ = 5 J , $\mathbf{KE_R}$ = ? J , $\mathbf{U^{s}}$ = 0 J. How much rotational kinetic energy ($\mathbf{KE_R}$) is in the final state?

(1) 0 J
(2) 5 J
(3) 10 J
(4) 20 J

$$E^{th}_i + E^{chem}_i + U^g_i + KE_{Ti} + KE_{Ri} + U^s_i + W_{EXT} = E^{th}_f + E^{chem}_f + U^g_f + KE_{Tf} + KE_{Rf} + U^s_f$$

$$30 + 10 + 20 + 15 + 5 + 2 - 10 = 30 + 2 + 25 + 5 + KE_{Rf} + 0$$

$$72 = 62 + KE_{Rf}$$

$$\boxed{KE_{Rf} = 10\ J}$$

Selected Learning Objectives

1. Coming soon to a lecture template near you.

Key Terms

- Rotational kinetic energy
- Angular momentum
- Conservation of angular momentum
- Zero angular impulse approximation

Key Equations

Angular momeuntum | Moment of inertia | Angular velocity

$$\vec{L}_o = I_o \vec{\omega}_o$$

Reference axis o

In words: The angular momentum of an object about reference axis o is equal to the moment of inertia of the object about reference axis o multiplied by the angular velocity of the object about reference axis o.

Chang in angular momeuntum | Moment of inertia | Change in angular velocity

$$\Delta\vec{L}_o = I_o \Delta\vec{\omega}_o$$

Reference axis o

In words: The change in angular momentum of an object about reference axis o is equal to the moment of inertia of the object about reference axis o multiplied by the change in angular velocity of the object about reference axis o.

Average torque external to the system | Change in time | Change in angular momeuntum of the system

$$\sum \vec{\bar{\tau}}_{ext,o} \Delta t = \Delta \vec{L}_{system,o}$$

Net (i.e. add up all of the __)

In words: Angular impulse, which is defined as the average net torque external to the system about reference axis o multiplied by the change in time that the average net torque is present, is equal to the change in angular momentum of the system about reference axis o.

Rotational Kinetic Energy | Moment of inertia | angular speed | Angular momeuntum | angular speed

$$KE_R = \frac{1}{2} I \omega^2 = \frac{1}{2} L \omega$$

In words: The rotational kinetic energy of an object is equal to one half the moment of inertia of the object multiplied by the angular speed of the object squared. The rotational kinetic energy of an object is also equal to one half of the angular momentum of the object multipled by the angular speed.

Key Concepts

- An angular momentum analysis (angular impulse angular momentum theorem, and/or conservation of angular momentum) follows the same procedures as a linear momentum analysis.
- The change in angular momentum is proportional to the net external torque, which is proportional to angular acceleration, which is proportional to change in angular velocity. Therefore, all the above vector quantities point in the same direction (e.g. have the same sign CCW/CW).
- The change in time seen in the definition of angular impulse is the time interval that the external torques are acting on the system.
- The angular momentum of a system with more than one object is the summation of all of the individual angular momentum of each object within the system.
- An energy analysis with objects that have rotational kinetic energy follows the same procedures as an energy analysis without rotation, except for the extra rotational energy term.

- If there are multiple objects within a system, you must include the kinetic energy of each individual object in an energy analysis.

Questions

Act I: Angular momentum

RC.2.L1-2:

Description: Calculate angular momentum of a system of multiple point particles. (4 minutes)

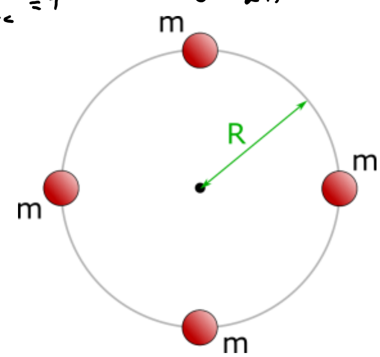
Learning Objectives: [1, 12, 13]

Problem Statement: Four point particles each of mass m are fixed to a negligible mass wire bent into a circle of radius R as shown below. If the masses are spinning clockwise around the center at a constant 60 RPM, what is the angular momentum of the 4-mass-system?

$60\,\frac{REV}{MIN} \times \frac{1\,MIN}{60\,SEC} = 1\,\frac{REV}{SEC} = f \longleftrightarrow \omega = 2\pi f$

(1) 0 $m \cdot R^2$
(2) -4 $m \cdot R^2$
(3) -25.1 $m \cdot R^2$
(4) 60 $m \cdot R^2$
(5) -240 $m \cdot R^2$
(6) 1510 $m \cdot R^2$

SYSTEM : 4 MASSES



$L_{sys,o} = \Sigma L_o$

$= L_{1,o} + L_{2,o} + L_{3,o} + L_{4,o}$

$= -I_{1,o}\omega - I_{2,o}\omega - I_{3,o}\omega - I_{4,o}\omega$

$= -mR^2\omega - mR^2\omega - mR^2\omega - mR^2\omega$

$\Sigma L_o = -4mR^2\omega = -4mR^2\left(2\pi\left(1\,\frac{REV}{S}\right)\right) \approx -25.1\,mR^2$

*RECALL $\vec{P}_{sys}$

$= \Sigma\vec{P} = \vec{P}_1 + \vec{P}_2 + \vec{P}_3 + \dots$

$= m_1\vec{v}_1 + m_2\vec{v}_2 + m_3\vec{v}_3$

RC.2.L1-3:

Description: Identify the angular impulse angular momentum mathematical representation. (1 minute)

Learning Objectives: [1, 12, 13]

Problem Statement: Recall the impulse-momentum theorem: $\Sigma\vec{\mathbf{F}}_{ext}\,\Delta t = \Delta\vec{\mathbf{p}}_{sys}$. Which of the following expressions could be angular impulse - angular momentum theorem?

(1) $\Sigma\vec{\mathbf{F}}_{ext}\,\Delta t = \Delta\vec{\mathbf{p}}_{sys}$

(2) $\Sigma\vec{\boldsymbol{\tau}}_{ext,o}\,\Delta t = \Delta\vec{\mathbf{p}}_{sys}$

(3) $\Sigma\vec{\boldsymbol{\tau}}_{ext,o}\,\Delta t = \Delta\vec{\boldsymbol{\omega}}_{sys,o}$

(4) $\Sigma\vec{\boldsymbol{\tau}}_{ext,o}\,\Delta t = \Delta\vec{\mathbf{L}}_{sys,o}$

RC.2.L1-4:

Description: Identify if conservation of angular momentum is valid. Explain why angular velocity changes if moment of inertia changes. (1 minute + 5 minutes)

Learning Objectives: [1, 12, 13]

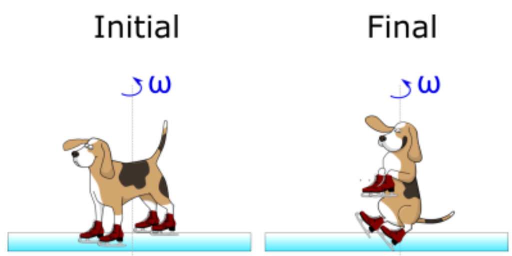
Problem Statement: Dizzy the dog is ice skating spinning in circles when she stands up on her hind legs. Assume frictionless.

(a) With the figure dog-skater as the system, is there any net external torque?

$\cancel{\sum \tau_{EXT,o}}^{0} \Delta t = \Delta L_{sys,o}$

$0 = \Delta L_{sys,o}$

(1) Yes
(2) No



I_i I_f

MORE MASS FURTHER AWAY FROM REF. AXIS.

(b) When the dog stands up, her angular velocity ______________ because her ______________ .

(1) increases ; mass decreases
(2) decreases ; moment of inertia increases
(3) increases ; moment of inertia decreases
(4) increases ; moment of inertia increases

$L_i = L_f \rightarrow L = \text{CONSTANT}$

$I_i \omega_i = I_f \omega_f \qquad L = I\omega$

$L(\text{const}) = (I\downarrow)(\omega\uparrow)$

RC.2.L1-5:

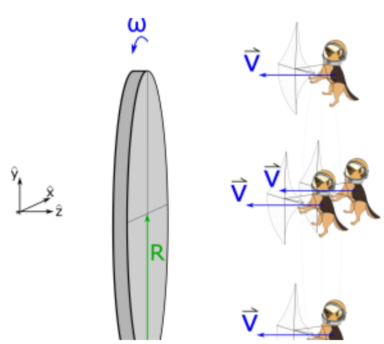
Description: Scaffold conservation of angular momentum and momentum. (1 minute + 1 minute + 4 minutes + 2 minutes + 6 minutes + 8 minutes + 4 minutes)

Learning Objectives: [1, 12, 13]

Problem Statement: A uniform 600 kg disk with a radius of 5 meters is in space rotating counter-clockwise at 1 RPM about the **z**-axis. Four 30-kg space-parachuting dogs are floating perpendicular to the face of the disk and have velocities of 20m/s in the $-\hat{z}$ direction as shown below. The four dogs land simultaneously on the rotating space disk. There is sufficient friction between the dogs and the disk that when they land they rotate with the disk and do not slide relative to the disk's surface.

(a) For a system of just the space disk, is there a net external torque about the z-axis as the dogs land?

$\boxed{\text{YES}}$ FROM DOGS



(b) For a system of disk+dogs, is there any net external torque about the z-axis as the dogs land?

$\boxed{\text{NO}}$ ISOLATED

A uniform 600 kg disk with a radius of 5 meters is in space rotating counter-clockwise at 1 RPM about the **z**-axis. Four 30-kg space-parachuting dogs are floating perpendicular to the face of the disk and have velocities of 20m/s in the $-\hat{\mathbf{z}}$ direction as shown below. The four dogs land simultaneously on the rotating space disk. There is sufficient friction between the dogs and the disk that when they land they rotate with the disk and do not slide relative to the disk's surface.

$$1\frac{REV}{MIN} \times \frac{1MIN}{60SEC} = \frac{1}{60}Hz \equiv f$$

(c) What is the angular moment about the z-axis of the disk+dogs system before the dogs land?

$$L_{i\,sys,z} = \cancel{L_{i\,Dogs,z}}^{\,0} + L_{i\,DISK,z}$$

$$= I_{Disk,z}\,\omega_z$$

$$= \frac{1}{2}M_{DISK}R^2\,2\pi f$$

$$\boxed{\Sigma L_{i,z} = 785.4\ \frac{kg\,m^2}{s}}$$

(d) How does the angular moment of the disk+dogs system before and after the dogs land compare?

(1) $\mathbf{L}_{i,z} > \mathbf{L}_{f,z}$

(2) $\mathbf{L}_{i,z} < \mathbf{L}_{f,z}$

(3) $\mathbf{L}_{i,z} = \mathbf{L}_{f,z}$

$$\cancel{\Sigma\tau_{EXT,z}}^{\,0}\Delta t = \Delta L_{sys,z}$$

$\therefore$

$$L_{sys\,i,z} = L_{sys\,f,z}$$

$$\Sigma L_{i,z} = \Sigma L_{f,z}$$

(e) What is the moment of inertia of the system about the z-axis after the dogs land?

* RECALL $\Sigma\vec{P}_i = \Sigma\vec{P}_f$

$$m_1\vec{V}_{1i} + m_2\vec{V}_{2i} + \ldots = m_1\vec{V}_{1f} + m_2\vec{V}_{2f} + \ldots$$

(f) What is the final rotational rate (in RPM) after the dogs land?

$$\Sigma L_{i,z} = \Sigma L_{f,z}$$

$$785.4\ \frac{kg\,m^2}{s} = 4L_{Dog\,f,z} + L_{DISK\,f,z}$$

$$= 4M_{Dog}R^2\omega_f + \frac{1}{2}M_{DISK}R^2\omega_f$$

$$= 3000\,\omega_f + 7500\,\omega_f$$

$$785.4 = 10500\,\omega_f$$

$$\omega_f = 0.0748\ \frac{RAD}{s}$$

$$\omega = 2\pi f$$

$$f = \frac{0.0748}{2\pi} = 0.01190\ Hz$$

$$\boxed{f = 0.714\ RPM}$$

(g) The center of mass of the disk is initially stationary. What happens to the center of mass of the rotating disk after the dogs land?

F (1) The center of mass velocity doesn't change because linear momentum is conserved..

F (2) The center of mass velocity doesn't change because kinetic energy is conserved.

T (2) The center of mass begins to move in the $-\hat{\mathbf{z}}$ direction because there is a net external force acting on it when the dogs land.

F (3) The center of mass begins to move in the $-\hat{\mathbf{z}}$ direction because there is a net external torque acting on it when the dogs land.

SYS | DOGS + DISK

$$\Sigma\cancel{F_{EXT\,z}}^{\,0}\Delta t = \Delta P_{sys\,z}$$

$$\cancel{\sum F_{EXT\,z}}^{0}\,\Delta t = \Delta P_{sys\,z}$$

$$\sum P_{iz} = \sum P_{fz}$$

$$\cancel{P_{i\,DISK\,z}}^{0} + 4P_{i\,Dog\,z} = P_{f(DISK+Dogs)\,z}$$

$$-4M_{Dog}V_{i\,Dog} = (4M_{Dog} + M_{DISK})V_{fz}$$

$$V_{fz} = \frac{-4M_{Dog}}{(4M_{Dog} + M_{DISK})} V_{i\,Dog}$$

$$= \frac{-4(30\,kg)}{(4(30\,kg) + 600\,kg)}\, 20\,\frac{m}{s}$$

$$\boxed{V_{fz} = -3.33\,m/s}$$

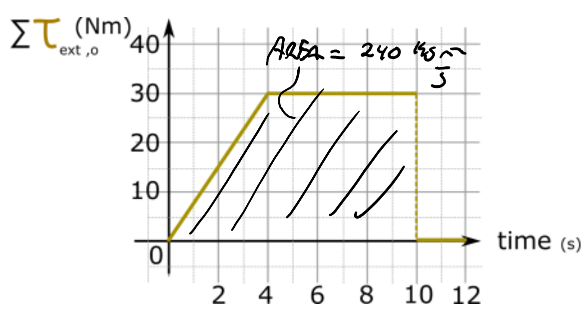
RC.2.L1-6:

Description: Given a net torque vs time graph and initial conditions, calculate final angular momentum and angular velocity. (5 minutes + 3 minutes)

Learning Objectives: [1, 12, 13]

Problem Statement: A metal smith using a bench grinder applies a net torque as a function of time from their newly forged knife on a grinding wheel shown by the graph below. The grinder is not plugged in so it's spinning freely.

(a) If the grinder's wheel started with an initial angular momentum of -300 kg·m²/s , what is its final angular moment after 10 seconds?



$$AREA = \Delta L$$

$$240\,\frac{kgm^2}{s} = L_f - L_i$$

$$\boxed{L_f = -60\,\frac{kgm^2}{s}}$$

(b) The moment of inertia is a constant 0.550 k·gm^2 , what is the final angular velocity of the grinder wheel?

$$L_f = I_f \omega_f$$

$$-60 = (0.55)\omega_f$$

$$\boxed{\omega_f \approx 109\,\frac{RAD}{s}}$$

Act II: Rotational energy

RC.2.L1-7:

Description: Identify which graph is related to rotational work. (2 minutes + 2 minutes)

Learning Objectives: [1, 12, 13]

Problem Statement: We wish to explore the graphical representation of rotational work.

(a) The rotational work due to a torque is the area under a

(1) force vs position graph.
(2) position vs force graph.
(3) torque vs angular position graph.
(4) torque vs time graph.

(b) Consider your answer to part (a). What can this area also represent?

(1) Rotational momentum.
(2) Change in rotational momentum.
(3) Rotational kinetic energy.
(4) Change in rotational kinetic energy.
(5) Change in energy.

RC.2.L1-8:

Description: Identify energy transformations and transfers with energy flow diagrams. (3 minutes + 3 minutes + 3 minutes + 5 minutes)

Learning Objectives: [1, 12, 13]

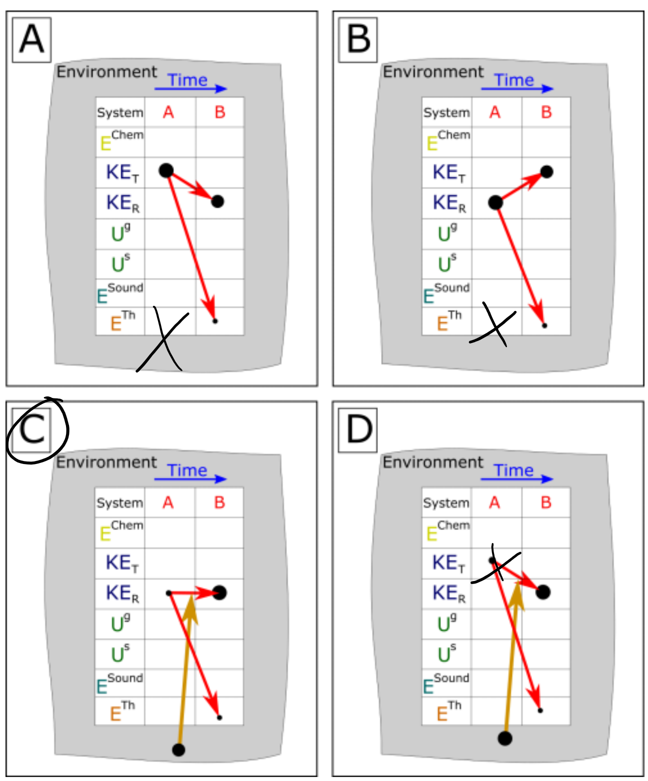
Problem Statement: Match the following energy flow diagrams with the given scenario.

(a) While fishing, you hook into a killer Oregon steelhead and it begins taking line, swimming directly away from you. Snapshots were taken when the fish was at the following locations:

A: The moment after the fish bit and slowly begins swimming away.

B: Some time later when the fishy is still hooked and swimming away quickly.

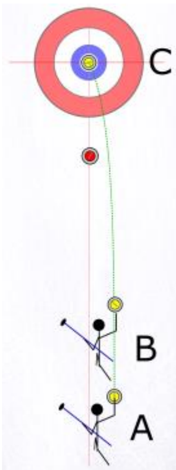
System: fishing reel

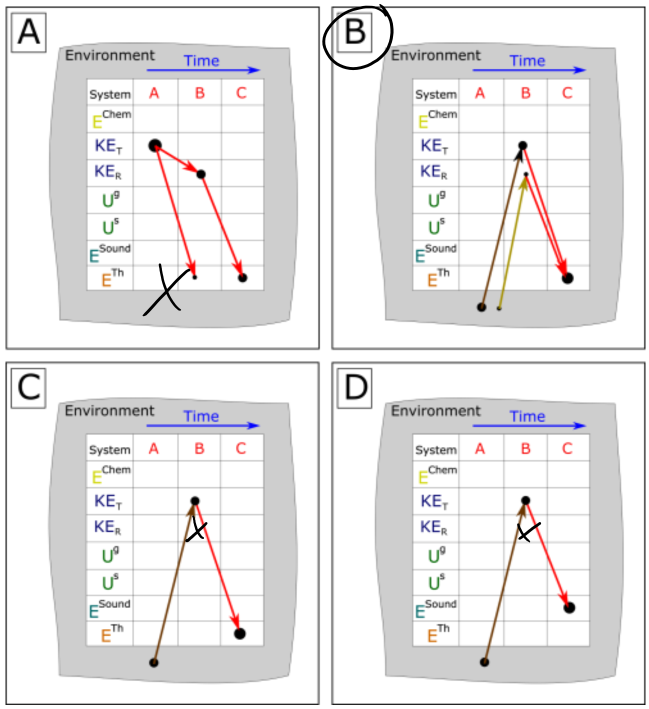


(b) Consider the game of curling. Three snapshots are taken when the stone is at locations **A**, **B**, and **C**. The dashed green line shows the trajectory of the stone's center of mass. Snapshots are taken when the stone is at the following locations:

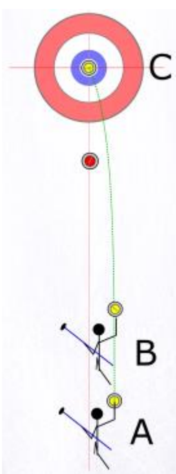
- **A:** The person and the stone are at rest.
- **B:** The stone has just left the person's hand rotating counter-clockwise.
- **C:** The stone has stopped on the button.

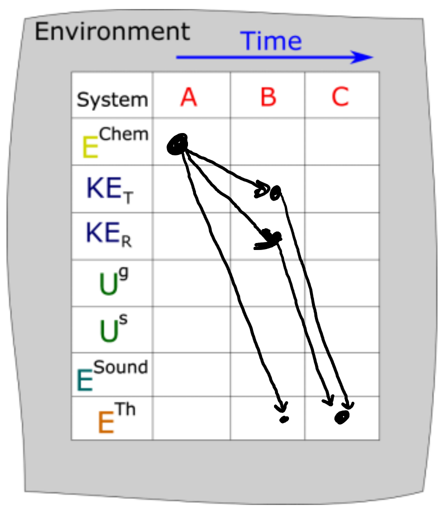
System: stone + surface





(c) Consider the same scenario as part (b), but this time the system is stone+surface+person. Fill out the energy flow diagram below.

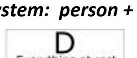


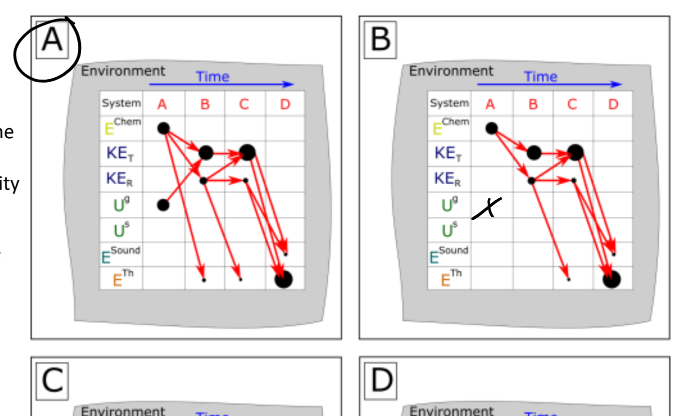


(d) A round of bowling begins with a bowling ball in a person's hand raised backwards above the level ground. Snapshots are taken when the ball is at the following locations:

- **A:** The ball and person are at rest with the ball at some height above the ground cocked backwards.
- **B:** The ball is two thirds down the lane with the velocity of the center of mass and angular velocity about the center of mass given.
- **C:** The ball is about to hit the pins with the velocity of the center of mass and angular velocity about the center of mass given.
- **D:** The ball and pins are at rest.

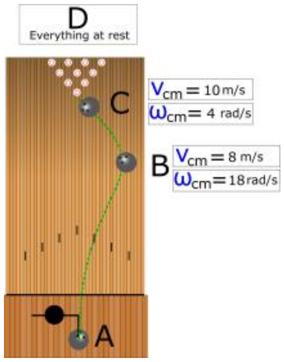
System: person + ball + lane + earth + pins + atmosphere



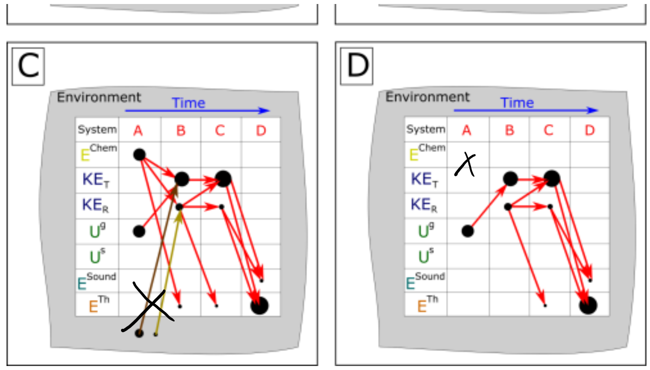


D: The ball and pins are at rest.

System: person + ball + lane + earth + pins + atmosphere



$\omega \downarrow + V \uparrow$



RC.2.L1-9:

Description: Conceptual application of conservation of energy involving rotational kinetic energy. (4 minutes)

Learning Objectives: [1, 12, 13]

Problem Statement: A solid sphere rolls without slipping along a track shaped as shown below. It starts from rest at location **A** and is moving vertically when it leaves the track at location **B**. At its highest point in the air, the sphere will be __________ location **A**.

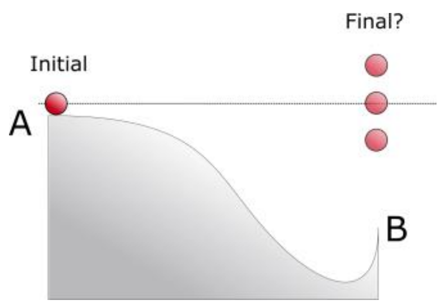
(1) above
(2) below
(3) at the same height as

Sys: EARTH + BALL

$$\cancel{KE_{Ti}}^{0} + \cancel{KE_{Ri}}^{0} + U^g_i + \cancel{W_{ext}}^{0} = \cancel{KE_{Tf}}^{0} + KE_{Rf} + U^g_f$$

$$U^g_i = \underbrace{KE_{Rf} + U^g_f}$$

So $U^g_i > U^g_f$

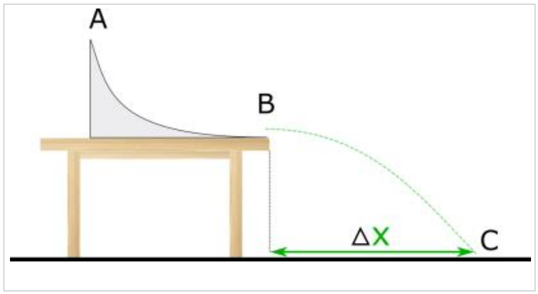


RC.2.L1-10:

Description: Conceptual application of conservation of energy involving rotational kinetic energy. (8 minutes)

Learning Objectives: [1, 12, 13]

Problem Statement: A solid sphere, solid disk, and hollow ring of equivalent mass and radius are rolled without slipping down a ramp on a table. They both start from rest at **A**, then fly horizontally off the edge of the table at **B**. Rank the horizontal distance, Δ**x**, each travels during their flight in the air to the moment they land at **C**.



$\underline{B \to C}$

$\Delta x = V_{ix}\Delta t + \cancel{\tfrac{1}{2}a_x\Delta t^2}^{\,0}$

$\Delta x = V_{ix}\Delta t$

$\Delta x \propto V_{ix}$

$\underline{B \to C}$

$\Delta y = \cancel{V_{iy}}^{\,0}\Delta t + \tfrac{1}{2}a_y\Delta t^2$

$\Delta y = \tfrac{1}{2}g\Delta t^2$

$\left.\begin{matrix}\text{SAME } g \\ \text{SAME } \Delta y\end{matrix}\right\}$ SAME Δt

Sols. coming soon

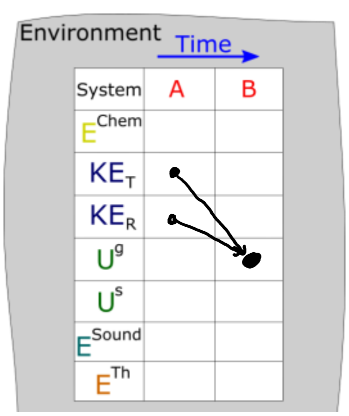
RC.2.L1-11:

Description: Conservation of energy application for hoop rolling up hill. (3 minutes + 2 minutes + 5 minutes + 4 minutes)

Learning Objectives: [1, 12, 13]

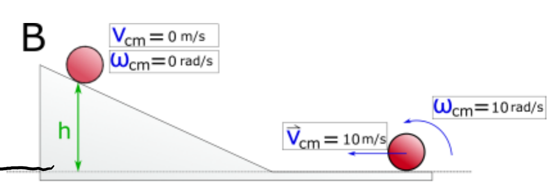
Problem Statement: A thin hoop with a radius of 2 meters is moving so that its center of mass is initially moving at 20 m/s while also rolling without slipping at 10 rad/s along a horizontal surface. It rolls up an incline, coming to rest as shown below.

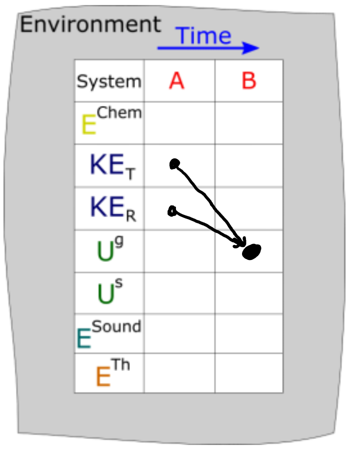
(a) Fill out an energy flow diagram for the hoop+earth system. Ignore friction.

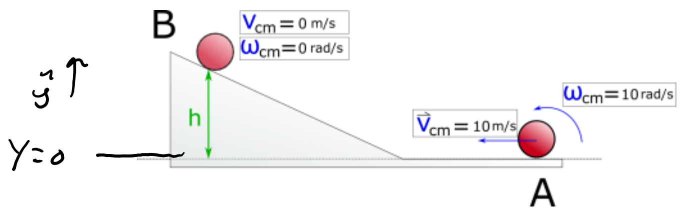


$y \uparrow$

$Y=0$







(b) Below is the work-energy equation with all of the forms of energy we have discussed up to this point. Which of the following energy terms are zero?

$$\cancel{\Delta E^{Chem}}^{0} + \Delta KE_T + \Delta KE_R + \Delta U^g + \cancel{\Delta U^s}^{0} + \cancel{\Delta E^{Sound}}^{0} \cancel{\Delta E^{Th}}^{0} = \cancel{W_{ext}}^{0}$$

$$KE_{Ti} + KE_{Ri} + \cancel{U^g_i}^{0} = \cancel{KE_{Tf}}^{0} + \cancel{KE_{Rf}}^{0} + U^g_f$$

$$KE_{Ti} + KE_{Ri} = U^g_f$$

(c) A thin 20 gram hoop with a radius of 2 meters is moving so that its center of mass is initially moving at 20 m/s while also rolling without slipping at 10 rad/s along a horizontal surface. **Use your simplified work-energy equation from part (b) to find the vertical height up the incline the hoop reaches when it stops.**

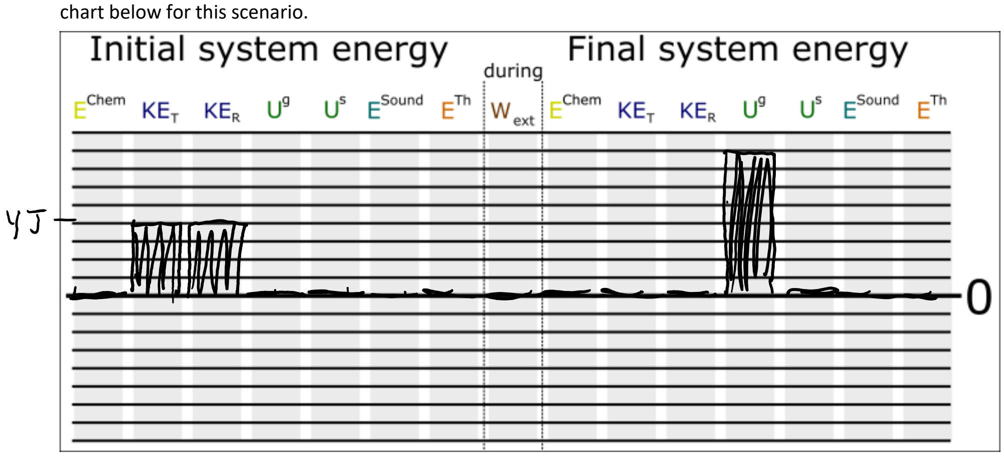
$$\frac{1}{2} m V_{cm}^2 + \frac{1}{2} I_{cm} \omega_{cm}^2 = m g y_f$$

$$\frac{1}{2} \cancel{m} V_{cm}^2 + \frac{1}{2} (\cancel{m} R^2) \omega_{cm}^2 = \cancel{m} g y_f$$

$$\frac{1}{2} V_{cm}^2 + \frac{1}{2} R^2 \omega_{cm}^2 = g y_f$$

$$\boxed{y_f = 40.8 \text{ m}}$$

(d) Another useful physical representation to show energy transformations and transfers is an energy bar chart. Fill in the energy bar chart below for this scenario.



$\frac{1}{2} m v_i^2 = 4$ J

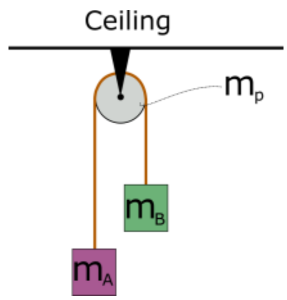
$\frac{1}{2} I \omega^2 = 4$ J

RC.2.L1-12:

Description: Conservation of energy application for multiple moving objects. (4 minutes + 3 minutes + 10 minutes)

Learning Objectives: [1, 12, 13]

Problem Statement: Two unequal masses are connected across a solid disk pulley. A few moments after releasing them from rest, the speed of one of the masses is recorded.



(a) If the pulley is replaced with one of a smaller radius but equivalent mass, and the experiment is repeated, the same hanging mass be going _______________ after the same amount of time elapses?

(1) faster
(2) slower
(3) the same speed

$U^g \rightarrow KE_{TA} + KE_{TB} + KE_{Rp}$

$KE_{Rp} = \frac{1}{2} I_p \omega^2$

$= \frac{1}{2}\left(\frac{1}{2} m_p r_p^2\right)\omega^2$

If $r_p \downarrow$ $KE_{Rp} \downarrow$ Thus $KE_T \uparrow$

(b) Below shows semi-simplified work-energy equations. Which one is the correct simplification for this scenario?

F (1) $U_i^g = KE_{T,f} + KE_{R,f} + U_f^g$
F (2) $U_{iB}^g = KE_{T,fA} + KE_{R,fp} + U_{fA}^g$
F (3) $U_{iB}^g = KE_{T,fA} + KE_{T,fB} + KE_{R,fp} + U_{fA}^g$
T (4) $0 = \Delta U_{iB}^g + \Delta U_{iA}^g + KE_{T,fA} + KE_{T,fB} + KE_{R,fp}$

$y \uparrow$

(c) What is the final speed of one of the hanging masses after it goes through a magnitude of displacement of 0.50 meters? The radius of the solid disk pulley is 0.20 meters and has a mass of 0.100 kg. Mass A = 1 kg, Mass B = 2 kg

Connections

$v_{fA} = v_{fB} \equiv v_f$

$v_f = \omega_f r_p$

$0 = m_B g \Delta y_B + m_A g \Delta y_A + \frac{1}{2} m_A v_{fA}^2 + \frac{1}{2} m_B v_{fB}^2 + \frac{1}{2} I_p \omega_f^2$

$0 = m_B g \Delta y_B + m_A g \Delta y_A + \frac{1}{2} m_A v_f^2 + \frac{1}{2} m_B v_f^2 + \frac{1}{2}\left(\frac{1}{2} m_p r_p^2\right)\left(\frac{v_f}{r_p}\right)^2$

$0 = g\left(m_B \Delta y_B + m_A \Delta y_A\right) + \frac{1}{2} v_f^2 \left(m_A + m_B\right) + \frac{1}{4} m_p v_f^2$

$0 = g\left(m_B \underset{(-)}{\Delta y_B} + m_A \underset{(+)}{\Delta y_A}\right) + \frac{1}{2} v_f^2 \left(m_A + m_B + \frac{1}{2} m_p\right)$ -4.9

$\boxed{v_f \approx 1.79 \text{ m/s}}$

RC.2.L1-13:

Description: Conceptual application of conservation of energy and energy transformations. (4 minutes)

Learning Objectives: [1, 12, 13]

Problem Statement: A figure skater stands on one spot on ice (assumed frictionless) and spins around with her arms extended. When she pulls in her arms, she reduces her moment of inertia and her angular speed increases. Compared to her initial rotational kinetic energy, her rotational kinetic energy after she has pulled in her arms must be:

(1) the same because no external work is done on her.
(2) larger because she's rotating faster.
(3) smaller because her moment of inertia is smaller.

$\cancel{\sum \tau_{ext,cm}}^{\,0} \Delta t = \Delta L_{cm}$

$0 = \Delta L_{cm}$

$\underbrace{I_{cm i} \omega_i}_{L_i} = \underbrace{I_{cm f} \omega_f}_{L_f}$

$L = \text{CONST}$

$L = I\omega$

$KE_R = \frac{1}{2} \underbrace{I \omega^2}_{\text{BOTH CHANGE}}$

$KE_R = \frac{1}{2} L \omega$

$\uparrow$ const

$KE_R \propto \omega$

$\underline{\text{SO SINCE } \omega \uparrow \quad KE_R \uparrow}$

WHAT? WHERE DID THIS ENERGY COME FROM?

$E^{CHEM} \rightarrow KE_R$

Conceptual questions for discussion

1. Can a point particle traveling in a straight line have angular momentum? Support your answer with examples or an explanation.
2. Use your knowledge of Newton's 2nd law for rotation and moment of inertia to explain why it is harder to do a sit-up when your arms are behind your head compared to your arms crossed on your chest.
3. Do you agree with the following statement: If there is no external work on a system, then the rotational kinetic energy of that system will remain constant because of conservation of energy. Support your answer with examples or an explanation.
4. What happens to the rotational rate of Earth's about its rotational axis, if anything, when tall building are built near the equator?
5. *Challenge problem: If in outer space you are initially at rest facing in the galactic north direction. Which one of the following actions is possible? (Hint: There is one possible scenario. Your body is not rigid (i.e. you can move your upper body separate from your lower body).
   i. Move your center of mass towards the galactic north direction.
   ii. Change the momentum of your center of mass.
   iii. Rotate your body so you are now facing the galactic south direction.
   iv. Change the angular momentum of your body about its center of mass.

Hints

RC.2.L1-1: No hints.

RC.2.L1-2: Recall that the linear momentum of a system of objects is the sum of all the individual momentum of each object within the system. Angular momentum works the same way.

RC.2.L1-3: No hints.

RC.2.L1-4: Start from angular impulse - angular momentum theorem.

RC.2.L1-5: No hints.

RC.2.L1-6: Recall that areas always give a change in a quantity, and that a change in any quantity is "final minus initial".

RC.2.L1-7: No hints.

RC.2.L1-8: No hints.

RC.2.L1-9: In the mathematical representation, apply conservation of energy without plugging in any functional forms of energies (e.g. don't use 1/2 $\mathbf{m}\ \mathbf{v}^2$, rather use $\mathbf{KE}_T$)

RC.2.L1-10: Start with stage **B** to **C** and apply kinematics to determine what the horizontal distance is proportional to (remember there are two components, x and y). Then apply a conservation of energy analysis.

RC.2.L1-11: No hints.

RC.2.L1-12: Remember if there is more than one moving object in a system, then include kinetic energy terms for each object individually.

RC.2.L1-13: No hints.