(OS.L1.4-212.sols) 212 Mastery Stage Solutions

Tuesday, February 18, 2020 6:01 PM

OS.L1.4 | General Oscillations, SHM, Equation of Motion | Challenge Homework

Submit a digital copy (PDF, jpg, etc.) to gradescope.com. Every page should be labeled on the top left with the question code (e.g. GR.L1.4-01) and there should be only be one solution per page. The questions should be in order. If a solution takes more than one page, be sure to label that it is a continuation of the previous page's solution (e.g. GR.L1.4-01 continued). One question will be randomly selected and graded. Challenge homework for a given week are due the following week by Tuesday at midnight. If data is needed to complete a problem, be sure to cite the source you've acquired your data from. See the course website for further details.

You will be asked to apply sense-making in most problems. Use the list below as a reference to the different sense-making techniques. More information about sense-making can be found on the BoxSand menu under Math Tools => Sense-making.

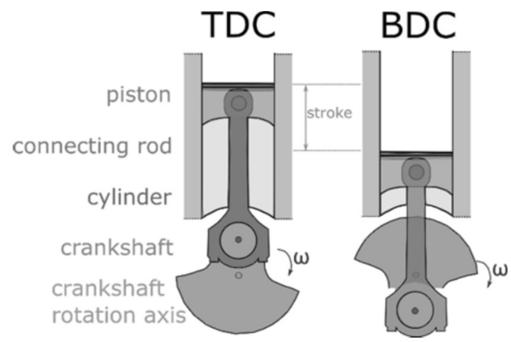
- *Sign:* Check the **sign** of each quantity makes sense.
- *Dimensionality:* Check the **dimensionality** and units of each quantity makes sense.
- *Order of Magnitude:* Check the **order of magnitude** of the final answer and other important quantities is within a a factor of 10 of what you think it should be.
- *Graphical Analysis:* Use a **graph** to see if the behavior of a solution makes sense.
- *Proportionality:* Using a symbolic solution, check the behavior of the answer when you change a given quantity on which it is dependent. Does the answer vary **proportionally** to what you expect?
- *Special Cases:* Check the behavior of a derived equation in limiting (**special**) cases makes sense, e.g. as x goes to 90 degrees in sin(x).
- *Self-consistency:* Check derived equations, functions, or values, are **self-consistent**, e.g. check that the slope of a derived position plot matches the values of the given velocity plot
- *Known Values:* Compare given or derived quantities with common well **known values**.
- *Related Quantities:* Compare the relative magnitude of two **related quantities**.

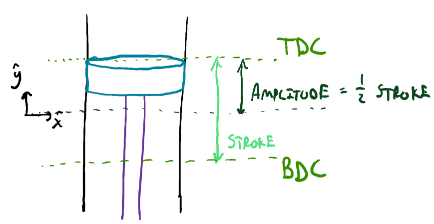
OS.L1.4-01

In an engine, a piston oscillates along a linear path between a maximum position called Top Dead Center (TDC) and minimum position called Bottom Dead Center (BDC). The linear distance that the piston travels from TDC to BDC is often referred to as the stroke of the engine. The stroke of the engine in my vehicle is 83.0 mm. The engine's maximum rotational frequency is 6,000 RPM. Let t = 0 seconds be the instant the piston is at TDC.

(a) Assume the piston undergoes simple harmonic motion at a constant frequency of 6,000 RPM. Construct an equation that describes the position as a function of time for the piston. Use SI units for all numbers that are used to construct the equation.

(b) Let's assume the piston has a mass of 0.750 kg. Ignoring all other forces (e.g. from the pressure difference on either end of the piston within the cylinder, friction, etc…) what is the maximum force that the connecting rods must apply to the pistons?





KNOWNS

AMPLITUDE = $\frac{1}{2}(83\ mm) \longrightarrow 0.0415\ m$

$f = 6\,000\ \frac{REV}{MIN} \times \frac{1\ MIN}{60\ SEC} \longrightarrow 100\ Hz$

@ t=0 y → TDC

a) $y(t) = y_{MAX} \begin{smallmatrix} SIN \\ OR \\ COS \end{smallmatrix} (\omega t)$

$$y(t) = 0.0415\ m\ \cos(200\pi\, t)$$

b) w/ $\Sigma \vec{F} = m\vec{a}$

MAX $\Sigma \vec{F}$ occurs @ $\vec{a}_{MAX}$

$a_{MAX} = \omega^2 X_{MAX}$

$= (200\pi)^2 (\tfrac{1}{2} STROKE)$

$|\Sigma \vec{F}_{MAX}| = m|\vec{a}_{MAX}|$

$|\Sigma \vec{F}_{MAX}| = m\,\omega^2 a_{MAX}$

$= (0.750\ kg)(200\pi\ \tfrac{RAD}{s})^2 (0.0415\ m)$

$= 1245\pi^2\ N$

$$|\Sigma \vec{F}_{MAX}| = 12300\ N$$

QUITE LARGE... CAN YOU NOW SEE WHY INTERNAL PARTS NEED TO BE STRONG AND PREFERABLY LIGHT