(OS.L2.4-212.sols) 212 Mastery Stage Solutions

Tuesday, February 18, 2020 6:01 PM

OS.L2.4 | SHM Pendulums and Springs | Challenge Homework

Submit a digital copy (PDF, jpg, etc.) to gradescope.com. Every page should be labeled on the top left with the question code (e.g. GR.L1.4-01) and there should be only be one solution per page. The questions should be in order. If a solution takes more than one page, be sure to label that it is a continuation of the previous page's solution (e.g. GR.L1.4-01 continued). One question will be randomly selected and graded. Challenge homework for a given week are due the following week by Tuesday at midnight. If data is needed to complete a problem, be sure to cite the source you've acquired your data from. See the course website for further details.

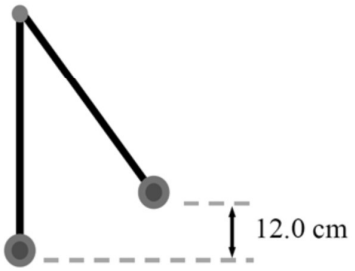
You will be asked to apply sense-making in most problems. Use the list below as a reference to the different sense-making techniques. More information about sense-making can be found on the BoxSand menu under Math Tools => Sense-making.

- *Sign:* Check the **sign** of each quantity makes sense.
- *Dimensionality:* Check the **dimensionality** and units of each quantity makes sense.
- *Order of Magnitude:* Check the **order of magnitude** of the final answer and other important quantities is within a a factor of 10 of what you think it should be.
- *Graphical Analysis:* Use a **graph** to see if the behavior of a solution makes sense.
- *Proportionality:* Using a symbolic solution, check the behavior of the answer when you change a given quantity on which it is dependent. Does the answer vary **proportionally** to what you expect?
- *Special Cases:* Check the behavior of a derived equation in limiting (**special**) cases makes sense, e.g. as x goes to 90 degrees in sin(x).
- *Self-consistency:* Check derived equations, functions, or values, are **self-consistent**, e.g. check that the slope of a derived position plot matches the values of the given velocity plot
- *Known Values:* Compare given or derived quantities with common well **known values**.
- *Related Quantities:* Compare the relative magnitude of two **related quantities**.

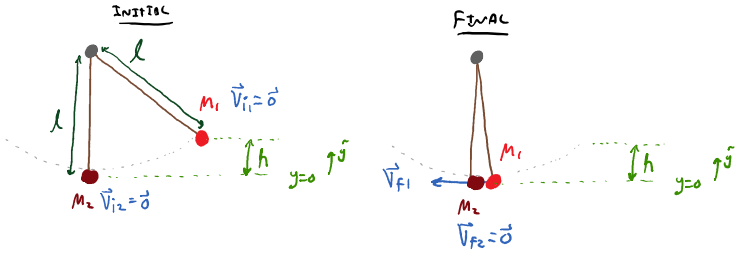
OS.L2.4-01

Two balls are connected to 60-cm-long strings of negligible mass. The other ends of the strings are fixed together as shown in the figure. One of the balls has a mass of 2.0 kg and is raised up to the right until it is 12.0 cm higher than the other ball, which has a mass of 3.0 kg. The upper ball is released from rest and sticks to the lower ball when they collide. The two balls which are now stuck together undergo simple harmonic motion. Find the...

(a) …frequency,
(b) maximum angular displacement,
(c) maximum height,
(d) and maximum speed of the subsequent motion after the collision.



STAGE 1: C.O.E. System $(m_1 + m_2 + \text{EARTH})$



Knowns
$m_1 = 2\,kg$
$m_2 = 3\,kg$
$\ell = 0.60\,m$
$h = 0.12\,m$

$$\Sigma E_i + \cancel{\Sigma W_{EXT}}^{0} = \Sigma E_f$$

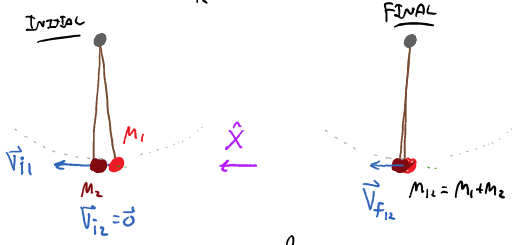
$$\cancel{KE_{i1}}^{0} + \cancel{KE_{i2}}^{0} + U^g_{i1} + \cancel{U^g_{i2}}^{0} = \cancel{KE_{f1}}^{0} + \cancel{KE_{f2}}^{0} + U^g_{f1} + \cancel{U^g_{f2}}^{0}$$

$$U^g_{i1} = KE_{f1}$$

$$\cancel{m_1} g h = \frac{1}{2}\cancel{m_1} V_{1f}^2$$

$$V_{1f} = \sqrt{2gh} \simeq 1.5344\ m/s$$

STAGE 2: C.O.M. System $(m_1 + m_2)$



$$\Delta \vec{P}_{12} = \cancel{\Sigma \vec{F}_{EXT_{12}}}^{0} \Delta t$$

$$\Delta \vec{P}_{12} = \vec{0}$$

$$\Delta P_{12x} = 0$$

$$P_{1ix} + \cancel{P_{2ix}}^{0} = P_{12fx}$$

$$m_1 V_{1ix} = (m_1 + m_2) V_{12fx}$$

$$m_1 \sqrt{2gh} = (m_1 + m_2) V_{12fx}$$

PART d

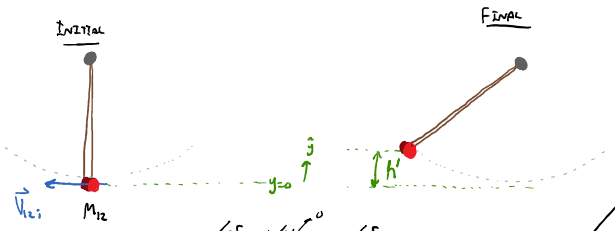
$m_1\sqrt{2gh}$ THIS occurs @ bottom of PENDULUM

$m_1\sqrt{2gh} = (m_1+m_2)V_{12fx}$ PART d

$V_{12fx} = \dfrac{m_1\sqrt{2gh}}{(m_1+m_2)}$ THIS OCCURS @ BOTTOM OF PENDULUM TRAJECTORY. THUS IS THE MAX SPEED.

$\approx \boxed{0.614\ m/s} \equiv V_{12\,max}$

STAGE 3: C.O.E. System (m_{12} + Earth)



$\Sigma E_i + \cancel{\Sigma W_{ENT}}^{0} = \Sigma E_f$

$KE_{12i} + \cancel{U^g_{12i}}^{0} = \cancel{KE_{12f}}^{0} + U^g_{12f}$

$\frac{1}{2}\cancel{m_{12}}V_{12i}^2 = \cancel{m_{12}}gh'$

$h' = \dfrac{V_{12i}^2}{2g}$

$h' = \dfrac{m_1^2\,2gh}{(m_1+m_2)^2 2g}$

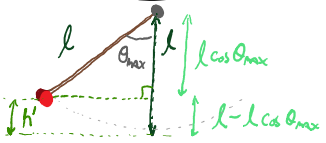
$h' = \dfrac{m_1^2}{(m_1+m_2)^2}h = \boxed{1.92\ cm}$ or $0.0192\ m$ PART C

a) FOR PENDULUMS...

$f = \frac{1}{2\pi}\sqrt{\frac{g}{\ell}}$

$\boxed{f \approx 0.644\ Hz}$

b) GEOMETRY OR ENERGY



$\ell - \ell\cos\theta_{max} = h'$

$\ell(1-\cos\theta_{max}) = h'$

$1-\cos\theta_{max} = \frac{h'}{\ell}$

$\cos\theta_{max} = 1 - \frac{h'}{\ell}$

$\theta_{max} = \cos^{-1}\left(1-\frac{h'}{\ell}\right)$

$\boxed{\theta_{max} \approx 14.5^\circ}$

$KE_{TOT} = U^g_{TOT}$

$\cancel{\frac{1}{2}m_{12}}V_{12max}^2 = \cancel{\frac{1}{2}m_{12}}g\ell\theta_{max}^2$

$V_{12\,max}^2 = g\ell\,\theta_{max}^2$

$\theta_{max} = \sqrt{\dfrac{V_{12max}^2}{g\ell}}$

$\theta_{max} = \dfrac{V_{12\,max}}{\sqrt{g\ell}}$

≈ 0.252922128 RAD

$\approx 14.5^\circ$