(OS.L3.4-212.sols) 212 Mastery Stage Solutions

Tuesday, February 18, 2020 6:01 PM

OS.L3.4 | Damped Oscillations | Challenge Homework

Submit a digital copy (PDF, jpg, etc.) to gradescope.com. Every page should be labeled on the top left with the question code (e.g. GR.L1.4-01) and there should be only be one solution per page. The questions should be in order. If a solution takes more than one page, be sure to label that it is a continuation of the previous page's solution (e.g. GR.L1.4-01 continued). One question will be randomly selected and graded. Challenge homework for a given week are due the following week by Tuesday at midnight. If data is needed to complete a problem, be sure to cite the source you've acquired your data from. See the course website for further details.

You will be asked to apply sense-making in most problems. Use the list below as a reference to the different sense-making techniques. More information about sense-making can be found on the BoxSand menu under Math Tools => Sense-making.

- *Sign:* Check the **sign** of each quantity makes sense.
- *Dimensionality:* Check the **dimensionality** and units of each quantity makes sense.
- *Order of Magnitude:* Check the **order of magnitude** of the final answer and other important quantities is within a a factor of 10 of what you think it should be.
- *Graphical Analysis:* Use a **graph** to see if the behavior of a solution makes sense.
- *Proportionality:* Using a symbolic solution, check the behavior of the answer when you change a given quantity on which it is dependent. Does the answer vary **proportionally** to what you expect?
- *Special Cases:* Check the behavior of a derived equation in limiting (**special**) cases makes sense, e.g. as x goes to 90 degrees in sin(x).
- *Self-consistency:* Check derived equations, functions, or values, are **self-consistent**, e.g. check that the slope of a derived position plot matches the values of the given velocity plot
- *Known Values:* Compare given or derived quantities with common well **known values**.
- *Related Quantities:* Compare the relative magnitude of two **related quantities**.

OS.L3.4-01

Watch the homework pendulum video on the course calendar on the BoxSand site and answer the following questions

(a) Find an equation for the angle as a function of time, that describes the motion of the pendulum – include damping.

(b) Use the equation to predict the position (positive or negative) and velocity (positive or negative) at t = 30 seconds. How well were you able to predict the position? Excluding human measurement error as a possibility, what could result in your predictions being off.

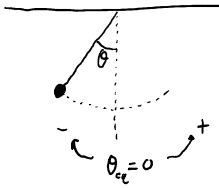
MEASUREMENTS FROM VIDEO

@ $t=0$ $\theta_{max} \approx 20^\circ \longrightarrow \theta_{max}(t=0) = 20^\circ$

10 OSCILLATIONS $\approx$ 18 SEC $\longrightarrow T = 1.8\ s \rightarrow f = \frac{5}{9}$ Hz

@ ≈ 48 SEC $\theta \approx 10^\circ \longrightarrow \theta(48s) = 10^\circ$

a)



$$\theta(t) = -\theta_{max}(t=0)\, e^{-t/\tau} \cos(\omega t)$$

BUT WHAT IS τ?

$$\theta_{max}(t) = \theta_{max}(t=0)\, e^{-t/\tau}$$

$$\theta_{max}(t=48) = \theta_{max}(t=0)\, e^{-48/\tau}$$

$$10 = 20\, e^{-48/\tau}$$

$$\frac{1}{2} = e^{-48/\tau}$$

$$\ln(\tfrac{1}{2}) = -\frac{48}{\tau}$$

$$\tau = \frac{-48}{\ln(\frac{1}{2})} \approx 69.25 \text{ SEC}$$

$$\theta(t) = -20^\circ\, e^{-t/69.25} \cos(2\pi f t)$$

$$\theta(t) = -20^\circ\, e^{-1.444\times10^{-2}t} \cos(3.49t)$$

OR

$$\theta(t) = -0.349 \text{ RAD}\, e^{-1.444\times10^{-2}t} \cos(3.49t)$$

b)

$\theta(t=30) \approx 6.7^\circ$

↑ + SO R.H.S. OF θ_{eq}

FOR $\mathcal{L}(t=30)$ LOOK AT $\theta(t)$ W/ t A LITTLE BEFORE & AFTER $t=30$ ... E.G. 29.9 & 30.1

$\theta(t=29.9) \approx 10^\circ$

$\theta(t=30.1) \approx 2.5^\circ$

θ IS ↓ AND PENDULUM IS ON RIGHT HAND SIDE

THUS $\mathcal{L}(t=30)$ MUST BE NEGATIVE