(RO.L1.4-212.sols) 212 Mastery Stage Solutions

Tuesday, February 18, 2020 6:01 PM

RO.L1.4 | Rays, Reflection, TIR, Dispersion | Challenge Homework

Submit a digital copy (PDF, jpg, etc.) to gradescope.com. Every page should be labeled on the top left with the question code (e.g. GR.L1.4-01) and there should be only be one solution per page. The questions should be in order. If a solution takes more than one page, be sure to label that it is a continuation of the previous page's solution (e.g. GR.L1.4-01 continued). One question will be randomly selected and graded. Challenge homework for a given week are due the following week by Tuesday at midnight. If data is needed to complete a problem, be sure to cite the source you've acquired your data from. See the course website for further details.

You will be asked to apply sense-making in most problems. Use the list below as a reference to the different sense-making techniques. More information about sense-making can be found on the BoxSand menu under Math Tools => Sense-making.

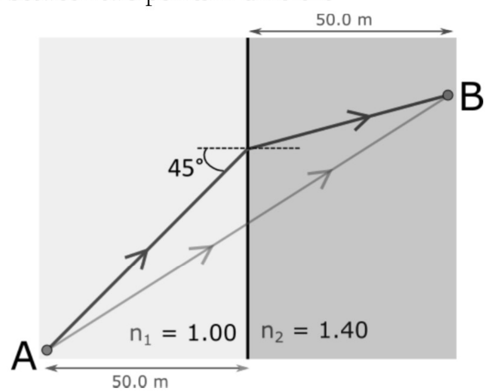
- *Sign:* Check the **sign** of each quantity makes sense.
- *Dimensionality:* Check the **dimensionality** and units of each quantity makes sense.
- *Order of Magnitude:* Check the **order of magnitude** of the final answer and other important quantities is within a a factor of 10 of what you think it should be.
- *Graphical Analysis:* Use a **graph** to see if the behavior of a solution makes sense.
- *Proportionality:* Using a symbolic solution, check the behavior of the answer when you change a given quantity on which it is dependent. Does the answer vary **proportionally** to what you expect?
- *Special Cases:* Check the behavior of a derived equation in limiting (**special**) cases makes sense, e.g. as x goes to 90 degrees in sin(x).
- *Self-consistency:* Check derived equations, functions, or values, are **self-consistent**, e.g. check that the slope of a derived position plot matches the values of the given velocity plot
- *Known Values:* Compare given or derived quantities with common well **known values**.
- *Related Quantities:* Compare the relative magnitude of two **related quantities**.

RO.L1.4-01

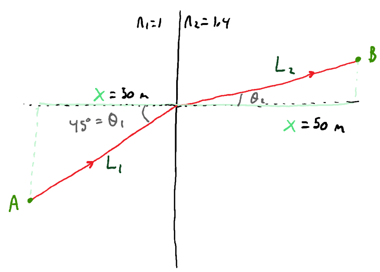
A beam of light propagates from point A in medium to point B in medium , as shown in the figure to the right. The index of refraction is different in these two media; therefore, the light follows a refracted path that obeys Snell's law.

(a) Calculate the time required for light to travel from A to B along the refracted path.

(b) Compare the time found in part (a) with the time it takes for light to travel from A to B along a straight line path. (Note that the time on the straight-line path is longer than the time on the refracted path. In general, the shortest time between two points in different media is along the path given by Snell's law.



a)



w/ ACCEL = 0

$$\text{TIME} = \frac{\text{DIST}}{\text{SPEED}}$$

$$\text{TIME} = t_1 + t_2$$

$$= \frac{L_1}{V_1} + \frac{L_2}{V_2} \quad n = \frac{c}{v}$$

$$= \frac{L_1}{c} + \frac{L_2}{(c/n_2)}$$

$$\text{TIME} = \frac{L_1}{c} + \frac{n_2 L_2}{c}$$

SNELL'S LAW

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

$$\theta_2 = \sin^{-1}\left(\frac{\sin\theta_1}{n_2}\right)$$

GEOMETRY

$$\cos\theta_1 = \frac{X}{L_1} \qquad \cos\theta_2 = \frac{X}{L_2}$$

$$L_1 = \frac{X}{\cos\theta_1} \qquad L_2 = \frac{X}{\cos\theta_2}$$

$$L_2 = \frac{X}{\cos\left(\sin^{-1}\left(\frac{\sin\theta_1}{n_2}\right)\right)}$$

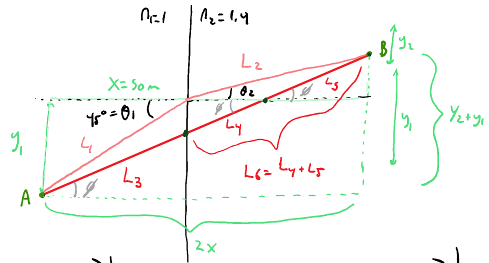
$$\text{TIME} = \frac{X}{c \cdot \cos\theta_1} + \frac{n_2 X}{c \cdot \cos\left(\sin^{-1}\left(\frac{\sin\theta_1}{n_2}\right)\right)}$$

$$\approx 2.357 \times 10^{-7}\ \text{SEC} + 2.7035 \times 10^{-7}\ \text{SEC}$$

($\sim t_1$, $\sim t_2$)

$$\boxed{\text{TIME} \approx 5.06 \times 10^{-7}\ \text{SEC}}$$

b)



$$TIME = t_3 + t_6$$

$$= \frac{L_3}{c} + \frac{L_6}{V_6} \quad n = \frac{c}{v}$$

$$TIME = \frac{L_3}{c} + \frac{n_2 L_6}{c}$$

L_3?

$$\sin\theta_1 = \frac{y_1}{L_1} \qquad \cos\theta_1 = \frac{x}{L_1}$$

$$y_1 = L_1 \sin\theta_1 \leftarrow L_1 = \frac{x}{\cos\theta_1}$$

$$y_1 = x\tan\theta_1 = 50\,m$$

$$\tan\phi = \frac{y_1 + y_2}{2x}$$

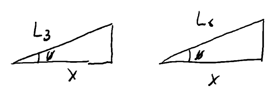
$$\tan\phi = \frac{x\tan\theta_1 + x\tan\theta_2}{2x}$$

$$\tan\phi = \frac{1}{2}(\tan\theta_1 + \tan\theta_2) = 38.4004446°$$

$$\cos\phi = \frac{x}{L_3}$$

$$L_3 = \frac{x}{\cos\phi} \simeq 63.80084847\,m$$

L_6?



$$L_3 = L_6$$

$$\therefore L_6 = \frac{x}{\cos\phi}$$

$$TIME = \frac{L_3}{c} + \frac{n_2 L_6}{c}$$

$$= \frac{x}{c \cdot \cos\phi} + \frac{n_2 x}{c\cos\phi} = \frac{x}{c\cos\phi}(1 + n_2)$$

$$\simeq 2.126694949 \times 10^{-7}\,sec + 2.97737372424 \times 10^{-7}\,sec$$

TIME ≃ 5.10×10^{-7} SEC

TIME B − TIME A = 4×10^{-9} SEC

TIME B ≃ 4 ns SLOWER THAN TIME A