(SW.L2.4-212.sols) 212 Mastery Stage Solutions

Wednesday, February 19, 2020 1:30 PM

SW.L2.4 | Beats and Two Source Interference | Challenge Homework

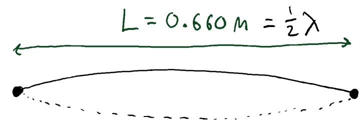
Submit a digital copy (PDF, jpg, etc.) to gradescope.com. Every page should be labeled on the top left with the question code (e.g. GR.L1.4-01) and there should be only be one solution per page. The questions should be in order. If a solution takes more than one page, be sure to label that it is a continuation of the previous page's solution (e.g. GR.L1.4-01 continued). One question will be randomly selected and graded. Challenge homework for a given week are due the following week by Tuesday at midnight. If data is needed to complete a problem, be sure to cite the source you've acquired your data from. See the course website for further details.

You will be asked to apply sense-making in most problems. Use the list below as a reference to the different sense-making techniques. More information about sense-making can be found on the BoxSand menu under Math Tools => Sense-making.

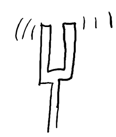
- *Sign:* Check the **sign** of each quantity makes sense.
- *Dimensionality:* Check the **dimensionality** and units of each quantity makes sense.
- *Order of Magnitude:* Check the **order of magnitude** of the final answer and other important quantities is within a a factor of 10 of what you think it should be.
- *Graphical Analysis:* Use a **graph** to see if the behavior of a solution makes sense.
- *Proportionality:* Using a symbolic solution, check the behavior of the answer when you change a given quantity on which it is dependent. Does the answer vary **proportionally** to what you expect?
- *Special Cases:* Check the behavior of a derived equation in limiting (**special**) cases makes sense, e.g. as x goes to 90 degrees in sin(x).
- *Self-consistency:* Check derived equations, functions, or values, are **self-consistent**, e.g. check that the slope of a derived position plot matches the values of the given velocity plot
- *Known Values:* Compare given or derived quantities with common well **known values**.
- *Related Quantities:* Compare the relative magnitude of two **related quantities**.

SW.L2.4-01

A huge spool of wire, 10,000 meters long, weighs 81.34N. You cut off a meter or so and tie it between two posts, 0.660 m apart. The tension in the wire is set to 52 N. When the string is plucked, at the same instant a 196 Hz tuning fork is also hit, what beat frequency is heard?



$f_1 = ?$



$f_2 = 196\ Hz$

Knowns

$\ell = 10\,000\ m$

$|\vec{F}^g_{wire}| = 81.34\ N$

$|\vec{F}^T| = 52\ N$

General: $v = f_1 \lambda$

String: $v = \sqrt{\frac{|\vec{F}^T|}{\mu}}$

$$f_1 \lambda = \sqrt{\frac{|\vec{F}^T|}{\mu}}$$

$L = \frac{1}{2}\lambda$

$$2 f_1 L = \sqrt{\frac{|\vec{F}^T|}{\mu}}$$

$\mu = \frac{m}{\ell}$, $m = \frac{|\vec{F}^g|}{g}$

$$2 f_1 L = \sqrt{\frac{|\vec{F}^T|}{\left(\frac{|\vec{F}^g_{wire}|}{g\ell}\right)}}$$

$$f_1 = \frac{1}{2L}\sqrt{\frac{g\ell|\vec{F}^T|}{|\vec{F}^g_{wire}|}} \approx 189.72$$

$$f_B = |f_1 - f_2|$$

$$\boxed{f_B \approx 6.28\ Hz}$$

SW.L2.4-02

Two speakers, 15 m apart, are facing each other producing identical 229 Hz sounds. You walk away from one speaker, towards the other, and you hear what sounds like beats at a frequency of 2.5 Hz. The questions below can be approached via a standing wave model or Doppler shift model.

Hints for standing wave model: Draw a standing wave pattern of multiple nodes and anti-nodes. The speakers are not necessarily at the nodes or anti-notes. A person would hear a loud sound at every anti-node and a soft sound at every node.

Hints for Doppler shift: As you walk from one speaker to the next, the two sources are Doppler shifted, thus you hear the beat frequency. The two frequencies in the beat frequency equation are thus then the Doppler shifted frequencies from the two speakers. Recall that beat frequency is the absolute value of the difference between two frequencies, so think carefully from which speaker you would hear a higher frequency and order the Doppler shifted frequencies such that you get a positive number when subtracted ?

(a) How fast are you walking?

(b) If the frequency of the sound emitted increased to 573 Hz and you continue to walk at the same speed, what frequency of beats will you hear?

(c) You can answer part (a) with either a standing wave model or as a Doppler shift effect. Answer part (a) again using the method you did not originally use.