(TW.L2.4-212.sols) 212 Mastery Stage Solutions

Tuesday, February 18, 2020 6:01 PM

TW.L2.4 | Sound, Light, Waves on Strings | Challenge Homework

Submit a digital copy (PDF, jpg, etc.) to gradescope.com. Every page should be labeled on the top left with the question code (e.g. GR.L1.4-01) and there should be only be one solution per page. The questions should be in order. If a solution takes more than one page, be sure to label that it is a continuation of the previous page's solution (e.g. GR.L1.4-01 continued). One question will be randomly selected and graded. Challenge homework for a given week are due the following week by Tuesday at midnight. If data is needed to complete a problem, be sure to cite the source you've acquired your data from. See the course website for further details.

You will be asked to apply sense-making in most problems. Use the list below as a reference to the different sense-making techniques. More information about sense-making can be found on the BoxSand menu under Math Tools => Sense-making.

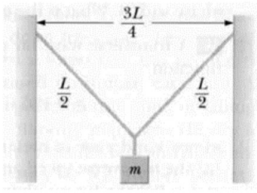
- *Sign:* Check the **sign** of each quantity makes sense.
- *Dimensionality:* Check the **dimensionality** and units of each quantity makes sense.
- *Order of Magnitude:* Check the **order of magnitude** of the final answer and other important quantities is within a a factor of 10 of what you think it should be.
- *Graphical Analysis:* Use a **graph** to see if the behavior of a solution makes sense.
- *Proportionality:* Using a symbolic solution, check the behavior of the answer when you change a given quantity on which it is dependent. Does the answer vary **proportionally** to what you expect?
- *Special Cases:* Check the behavior of a derived equation in limiting (**special**) cases makes sense, e.g. as x goes to 90 degrees in sin(x).
- *Self-consistency:* Check derived equations, functions, or values, are **self-consistent**, e.g. check that the slope of a derived position plot matches the values of the given velocity plot
- *Known Values:* Compare given or derived quantities with common well **known values**.
- *Related Quantities:* Compare the relative magnitude of two **related quantities**.

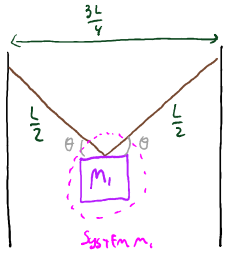
TW.L2.4-01

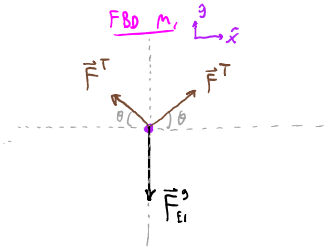
A small rope has a liner mass density of 8.00 g/m and is tied to two vertical walls that are separated by three fourths the length of the rope, as shown in the figure. A box of mass m is hanging from the rope halfway between its ends.

(a) Find an expression for the speed of a wave on the rope as a function of m?

(b) What must the mass of the box be to have a wave speed of 65.0 m/s?



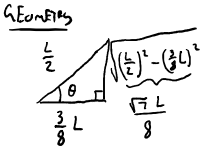




Knowns

$\mu = 8.00 \times 10^{-3}\ kg/m$

Geometry



$\sin\theta = \frac{\sqrt{7}L/8}{L/2}$

$\sin\theta = \frac{\sqrt{7}}{4}$

$\Sigma F_y = m_1 a_y^{\,0}$

$2|\vec{F}^T|\sin\theta - |\vec{F}_{E1}^{\,g}| = 0$

$2|\vec{F}^T|\sin\theta - m_1 g = 0$

$2|\vec{F}^T|\frac{\sqrt{7}}{4} - m_1 g = 0$

$\frac{\sqrt{7}}{2}|\vec{F}^T| = m_1 g$

$|\vec{F}^T| = \frac{2m_1 g}{\sqrt{7}}$

$v = \sqrt{\frac{|\vec{F}^T|}{\mu}}$

$v = \sqrt{\frac{2m_1 g}{\sqrt{7}\,\mu}} \rightarrow 30.4\sqrt{m_1}$

If $v = 65.0$ m/s

$65.0 = 30.4\sqrt{m_1}$

$m_1 \simeq 4.57$ kg