(WO.L1.4-212.sols) 212 Mastery Stage Solutions

Tuesday, February 18, 2020 6:01 PM

WO.L1.4 | Young's Double Slit | Challenge Homework

Submit a digital copy (PDF, jpg, etc.) to gradescope.com. Every page should be labeled on the top left with the question code (e.g. GR.L1.4-01) and there should be only be one solution per page. The questions should be in order. If a solution takes more than one page, be sure to label that it is a continuation of the previous page's solution (e.g. GR.L1.4-01 continued). One question will be randomly selected and graded. Challenge homework for a given week are due the following week by Tuesday at midnight. If data is needed to complete a problem, be sure to cite the source you've acquired your data from. See the course website for further details.

You will be asked to apply sense-making in most problems. Use the list below as a reference to the different sense-making techniques. More information about sense-making can be found on the BoxSand menu under Math Tools => Sense-making.

- *Sign:* Check the **sign** of each quantity makes sense.
- *Dimensionality:* Check the **dimensionality** and units of each quantity makes sense.
- *Order of Magnitude:* Check the **order of magnitude** of the final answer and other important quantities is within a a factor of 10 of what you think it should be.
- *Graphical Analysis:* Use a **graph** to see if the behavior of a solution makes sense.
- *Proportionality:* Using a symbolic solution, check the behavior of the answer when you change a given quantity on which it is dependent. Does the answer vary **proportionally** to what you expect?
- *Special Cases:* Check the behavior of a derived equation in limiting (**special**) cases makes sense, e.g. as x goes to 90 degrees in sin(x).
- *Self-consistency:* Check derived equations, functions, or values, are **self-consistent**, e.g. check that the slope of a derived position plot matches the values of the given velocity plot
- *Known Values:* Compare given or derived quantities with common well **known values**.
- *Related Quantities:* Compare the relative magnitude of two **related quantities**.

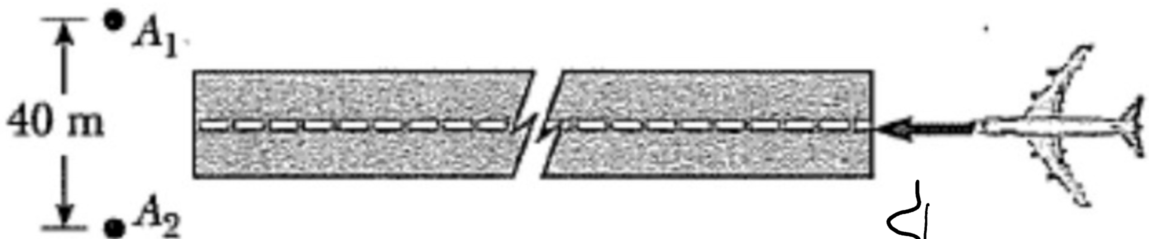
WO.L1.4-01

Young's double-slit experiment underlies the instrument landing system used to guide aircraft to safe landings when the visibility is poor. Although real systems are more complicated than the example described here, they operate on the same principles. A pilot is trying to align her plane with a runways as suggested in the figure.

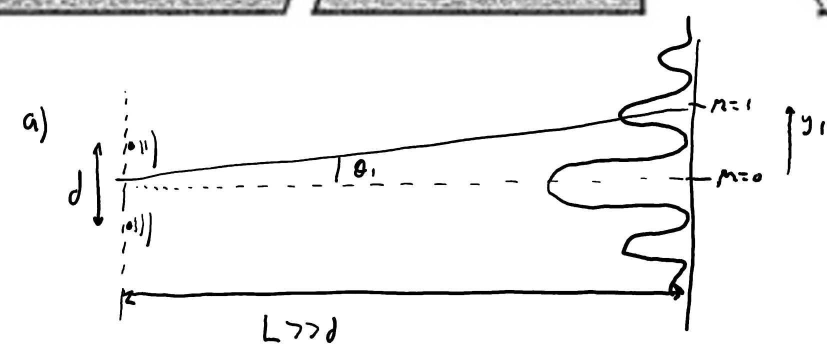
(a) Two radio antennas A1 and A2, separated by 40.0 m are positions adjacent to the runway. The antennas broadcast single frequency, in phase 20.0 MHz, coherent radio waves. You can assume the far field approximation is valid. **Find the wavelength of the waves** .

(b) The pilot "locks onto" the strong signal radiated along an interference maximum and steers the plane to keep the received signal strong. If she detects the central maximum, the plane will have the right heading to land when she reaches the runway. Suppose instead that the plane is flying along the first side maximum, one maxima from the central. **How far to the side of the runway centerline is the plane when it is 2.00 km from the antennas** ?

(c) It is possible to tell the pilot she is on the wrong maximum by sending out two signals from each antenna and equipping the aircraft with a two-channel receiver. The ratio of the two frequencies must not be the ratio of small integers (such as ¾). **Explain how this two-frequency system would work, and why it would not necessarily work if the frequencies were related by a small integer ratio.**



a)



$f = 20\ MHz = 20\times10^6\ Hz$

$\lambda = \frac{c}{f} = \frac{3\times10^8\ m/s}{20\times10^6\ 1/s} \approx$ **15 METERS**

b)

$d\sin\theta_1 = m_1\lambda$ $\qquad$ $L\tan\theta_1 = y_1$

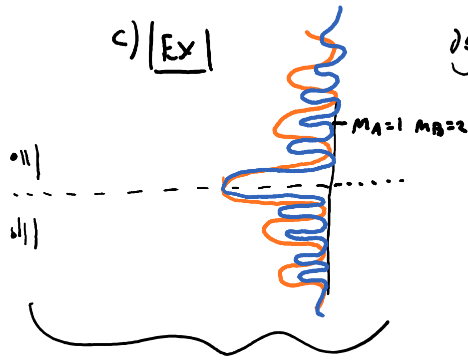
$40\ m\ \sin\theta_1 = 1\ (15\ m)$ $\qquad$ $(2000\ m)\tan\theta_1 = y_1$

$\theta_1 \approx 22.02°$

$y_1 \approx 809$ METERS

$y_1 \approx 809$ meters

c) Ex



$d\sin\theta_{m_A} = m_A\lambda_A$ $\quad$ $d\sin\theta_{m_B} = m_B\lambda_B$

Equal

$$m_A\lambda_A = m_B\lambda_B$$

$$\frac{\lambda_A}{\lambda_B} = \frac{m_B}{m_A}$$

If small ratio e.g. $\frac{1}{2}$ 2nd bright fringe lines up w/ 1st bright fringe

If large e.g. $\frac{10001}{1}$ then they don't line up until later & thus farther off course