Inclined Planes

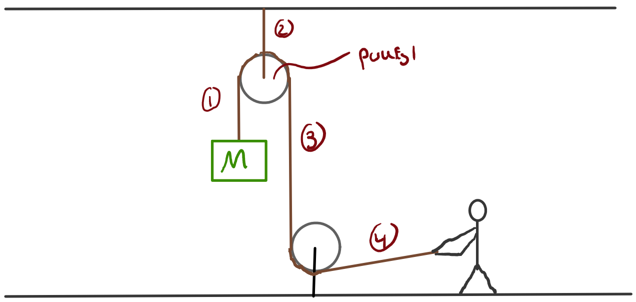
Select LEARNING OBJECTIVES:

i. Introduce the technique of rotating a coordinate system to help simplify the second law equations that will be written based off of the FBD.
ii. Be able to identify the direction of normal forces when the surfaces between two objects are not parallel to the ground.
iii. Demonstrate the ability to correctly identify any relevant angles in a pictorial representation on a FBD.
iv. Understand how to define a system for which to draw a FBD for.
v. Demonstrate the ability to draw a properly scaled free body diagram from an image of a scenario or a written description of a scenario.
vi. Understand the importance of including a coordinate system along with a FBD.
vii. Demonstrate the ability to translate a FBD into Newton's second law equations.
viii. Strengthen the ability to decompose vectors into components along the chosen coordinate system.
ix. Demonstrate the ability to quantitatively solve algebraic expressions, including quadratic equations.
x. Be able to determine the relative direction of the acceleration of an object based off of a properly scaled FBD.
xi. Understand that the two forces constituting a force pair will never show up on the same FBD.
xii. Be able to identify any internal forces for a system if applicable.
xiii. Further the development of problem solving strategies, which includes but is not limited to translating problems into different representations, determining knowns/unknowns, and identifying the relevant physics.

TEXTBOOK CHAPTERS:

- Giancoli (Physics Principles with Applications 7th) :: 4-8
- Knight (College Physics : A strategic approach 3rd) :: 5.4
- BoxSand :: Forces (Inclined Planes)

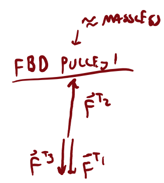
WARM UP: Rank the magnitude of tension in each segment of the massless ropes from least to greatest.



SECTIONS 1 3 + 4 ARE ALL THE SAME ROPE.

w/ MASSLESS/FRICTIONLESS ROPE APPROXIMATION

$$|\vec{F}^{T_1}| = |\vec{F}^{T_3}| = |\vec{F}^{T_4}| < |\vec{F}^{T_2}|$$

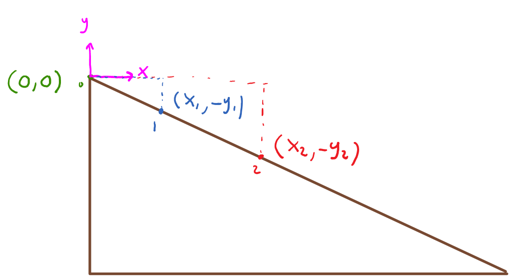


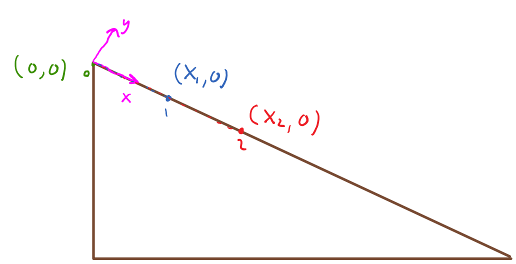
Often times the object we are trying to analyze sits on top of a surface that is not parallel to the horizontal ground. This slight variation adds some noteworthy complexities that this lecture will address. We will still limit ourselves to frictionless surfaces here.

The key to simplifying scenarios that contain inclined planes is the following:

1) Ask yourself if the object of interest is confined to move along a certain direction. (In the case of inclined planes, the object of interest can only move parallel to the incline plane.)
2) If the answer to the above question is yes, then rotate the coordinate system to align one of the axes along the direction of acceleration. (In the case of inclined planes, set one of the axes either up or down, parallel, the incline.)
3) If the object is on an incline and not accelerating either up or down the incline, then use a coordinate system that requires you to break down the least number of vectors into components.
4) Remember, you can choose your coordinate system anyway you which, so if you don't follow the above recommendations, you can still solve the problem properly. However, following the above recommendations typically makes the algebra a bit easier in the end.

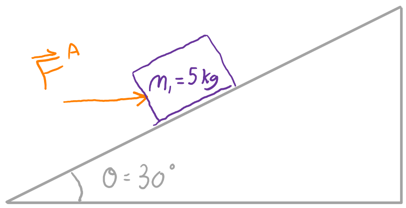
By following the above two steps, we simply set one of the components of the objects acceleration equal to zero, which makes the mathematical analysis just a tad bit easier than having a non-zero acceleration in both coordinate directions. Below is a comparison of a horizontal/vertical coordinate system compared to a rotated coordinate system to help illustrate why the acceleration in one direction is zero with the rotated axes.

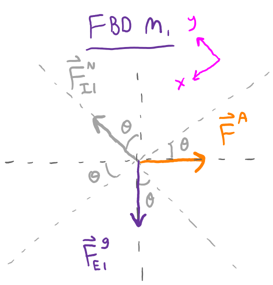




Notice how the rotate coordinate system on the right sets the y-component of the position of an object sliding down the incline to be zero. Since this y-component of position does not change, the velocity in the y-direction is a constant zero. And finally, since the y-component of velocity is constant, there is no acceleration in the y-direction.

EXAMPLE: Consider the figure below. Find the magnitude of acceleration of the 5 kg block down the frictionless incline if the magnitude of the applied force is 10 N.





$\Sigma F_y = m_1 a_y$

$F^N_{I1y} + F^A_y + F^g_{E1y} = m_1 \cancel{a_y}^{0}$

$|\vec{F}^N_{I1}| - |\vec{F}^A| Sin(\theta) - |\vec{F}^g_{E1}| Cos(\theta) = 0$

$|\vec{F}^N_{I1}| - |\vec{F}^A| S_\theta - m_1 g C_\theta = 0$ // 1 Eqn 1 unknown

$|\vec{F}^N_{I1}| = m_1 g C_\theta + |\vec{F}^A| S_\theta$

NICE, BUT WE ARE LOOKING FOR ACCELERATION.
LETS TRY THE X-DIRECTION ANALYSIS

$\Sigma F_x = m_1 a_x$

$F^A_x + F^g_{E1x} = m_1 a_x$

$-|\vec{F}^A| Cos(\theta) + |\vec{F}^g_{E1}| Sin(\theta) = m_1 a_x$

$-|\vec{F}^A| C_\theta + m_1 g S_\theta = m_1 a_x$ // 1 Eqn 1 unknown

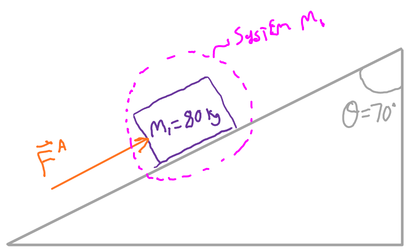
$a_x = \dfrac{m_1 g S_\theta - |\vec{F}^A| C_\theta}{m_1}$

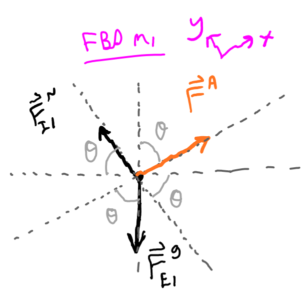
$a_x = \dfrac{(5)(9.8) Sin(30) - (10) Cos(30)}{5}\ m/s^2$

$a_x \approx 3.17\ m/s^2$

$\vec{a} = \vec{0}$

PRACTICE: A box is being pushed at a constant speed up an frictionless incline by some applied force that is parallel to the incline. The incline has an angle of 70 degrees with respect to the vertical. What is the magnitude of the applied force if the mass of the box is 80 kg?





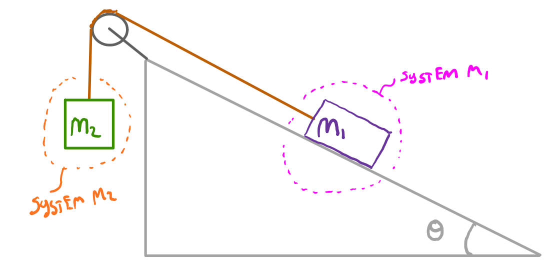
$\Sigma F_x = m_1 \cancel{a_{1x}}^{0}$

$|\vec{F}^A| - |\vec{F}^g_{E1}| Cos(\theta) = 0$

$|\vec{F}^A| - m_1 g Cos(\theta) = 0$ // 1 Eqn 1 Unknown

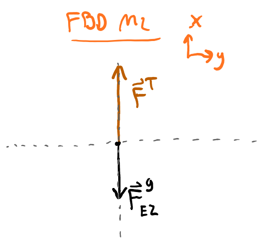
$|\vec{F}^A| = m_1 g Cos(\theta) \approx 268\ N$

PRACTICE: A box of mass 50 kg sits on top of a frictionless incline that makes an angle of 54.3 degrees with respect to the horizontal. A massless rope is attached to this 50 kg mass and hung over a massless/frictionless pulley where it then connects to a 10 kg mass. Find the acceleration of each mass.



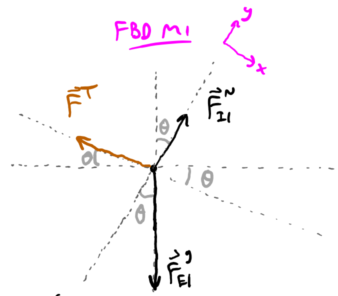
$m_1 = 50$ kg

$m_2 = 10$ kg



CONNECTIONS

$a_{1x} = a_{2x} = a_x$



$\Sigma F_x = m_2 a_{2x}$

$|\vec{F}^T| - |\vec{F}^g_{E2}| = m_2 a_x$

$|\vec{F}^T| - m_2 g = m_2 a_x$ 1 EQN 2 UNKNOWNS

$\Sigma F_x = m_1 a_{1x}$

$-|\vec{F}^T| + |\vec{F}^g_{E1}| \sin(\theta) = m_1 a_x$

$-|\vec{F}^T| + m_1 g \sin(\theta) = m_1 a_x$ 1 EQN 2 UNKNOWNS

2 EQNS 2 UNKNOWNS

$|\vec{F}^T| = m_2 a_x + m_2 g$

$-m_2 a_x - m_2 g + m_1 g \sin(\theta) = m_1 a_x$

$m_1 a_x + m_2 a_x = m_1 g \sin(\theta) - m_2 g$

$$a_x = \frac{g(m_1 \sin(\theta) - m_2)}{(m_1 + m_2)} \approx 5 \text{ m/s}^2$$

Conceptual questions for discussion

1) You have two choices:
    a. Lift a heavy box directly over your head to a height h above the ground.
    b. Push a heavy box up an incline to a height h above the ground.

    Which would you prefer to do and why?