Buoyancy

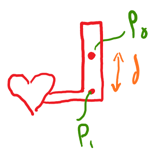
Select LEARNING OBJECTIVES:

- Understand that the buoyant force is a result of a pressure gradient within a fluid.
- Demonstrate the ability to analyze a scenario involving a buoyant force by constructing FBDs and applying Newton's laws of motion.
- Strengthen the ability to perform proportional reasoning

TEXTBOOK CHAPTERS:

- Ginacoli ((Physics Principles with Applications 7th) :: 10-7
- Knight (College Physics : A strategic approach 3rd) :: 13.4
- Boxsand :: Buoyancy

WARM UP: When blood pressure is measured, the arm cuff is typically measured at the heart level. Will you get the same readings if you raised your arm above heart level? Explain.

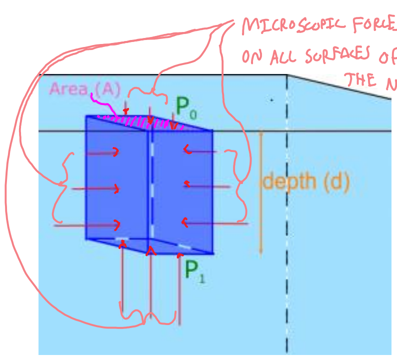


$P_1 = P_0 + \rho_{blood}\, g\, d$

↑ IF YOU TAKE BLOOD PRESSURE NOT AT HEART LEVEL IT WILL GIVE A DIFFERENT READING. ∴ PRESSURE AT A DEPTH.

In this section we will explore Archimedes principle which relates an upward force acting on an object that is fully or partially submerged within a fluid to the weight of the fluid that the object displaces. We will also build upon our understanding of density, pressure, and hydrostatics to quantify the buoyant force.

Did you ever notice that lifting a partially or fully submerged object in water feels easier to do than lifting the same object when it is outside the water? This effect is a consequence of the pressure gradient that the object experiences when in the water. Recall that pressure gradients cause an associated net force. The net force caused by a pressure gradient within a fluid is what we refer to as the buoyant force, and it points in the upward direction which is why it feels easier to lift objects in water. From a microscopic view, the net force associated with collisions from particles from above is less than the net force associated with collisions from below. The difference between these two net forces defined at the buoyant force. Below is a figure to help visualize the buoyant force.

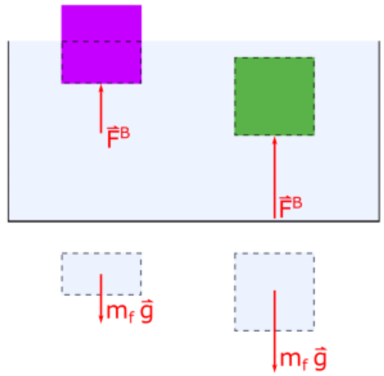


MICROSCOPIC FORCES FROM PARTICLES ON ALL SURFACES OF VOLUME ELEMENT.

THE NET EFFECT ON EACH SURFACE IS... $\underbrace{\Sigma\vec{F}_L,\ \Sigma\vec{F}_R}$, $\underbrace{\Sigma\vec{F}_{TOP},\ \Sigma\vec{F}_{BOTTOM}}$

W/NO $\vec{a}$... $\Sigma\vec{F}_L = \Sigma\vec{F}_R$ AND $\boxed{\Sigma\vec{F}_{BOTTOM} - \Sigma\vec{F}_{TOP} = \vec{F}^B}$ BUOYANT FORCE

The discovery of how the buoyant force can be related other physical quantities is attributed to Archimedes.. Archimedes' principle can be stated as ***"when an object is partially or fully submerged in a fluid which is at rest, the pressure gradient within the fluid causes an associated net force in the upwards direction which is equal to the weight of the fluid that the object displaced"***. Below is a physical representation to help illustrate Archimedes' principle which was stated in the descriptive representation above.



Notice that both the purple and green objects have the same dimensions, thus the same volume. However, the purple object is displacing a smaller volume of water than the green object. The volume of water that each object is displacing is shown below the container. Archimedes' principle tells us that the buoyant force that each object experiences is equal to the weight of the displaced fluid, thus the green object experiences a larger buoyant force.

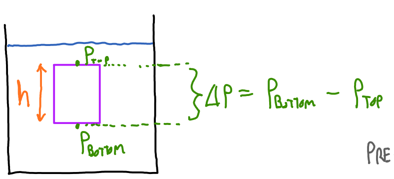
We can now quantify the buoyant force that any object experiences when either partially or fully submerged in a fluid as shown in the equation below.

$$|\vec{F}^B| = \underbrace{\overbrace{m_F}^{\text{MASS OF DISPLACED FLUID}}\, g}_{\text{WEIGHT OF DISPLACED FLUID}}$$

However it is often much easier to work with volumes and densities instead of mass, so if we use our definition for mass, m = rho V and substitute this into the equation above, we get a much more handy expression for the magnitude of the buoyant force as seen below.

$$\boxed{|\vec{F}^B| = \overbrace{\overbrace{\underbrace{\rho_F}_{\text{DENSITY OF FLUID}}\,\underbrace{V_{dF}}_{\text{VOLUME OF DISPLACED FLUID}}}^{\text{MASS OF DISPLACED FLUID}}\, g}^{\text{WEIGHT OF DISPLACED FLUID}}}$$

Above is the buoyant force as per Archimedes' principle. We know that pressure changes a function of depth below the surface and that pressure differences cause an associated net force. Thus is Archimedes' principle consistent with our knowledge about pressure at a depth and pressure differences? To answer this consider an object fully submerged in a fluid. Let's look at the associated net force caused by the pressure difference between the top and the bottom of the object within the fluid.



PRESSURE DIFFERENCES $\Longrightarrow \Sigma\vec{F}$

$$\Delta P = \frac{\Sigma\vec{F}}{A}$$

PRESSURE AT A DEPTH.

$$\rho_F\, g\, h = \frac{\Sigma\vec{F}}{A}$$

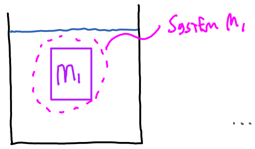
$$\rho_F\, g\, \underbrace{hA}_{\text{VOLUME OF DISPLACED FLUID}} = \Sigma\vec{F}$$

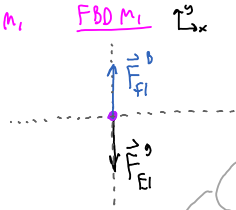
$$\boxed{\rho_F V_{Fd}\, g = \Sigma\vec{F}}$$

BUOYANT FORCE!!

Float or sink?

If you consider an object fully submerged in a fluid and apply Newton's laws of motion, you can construct the conditions that must be satisfied for an object to sink or float.





$\Sigma F_y = m_1 a_{1y}$

$|\vec{F}^B_{F1}| - |\vec{F}^g_{E1}| = m_1 a_{1y}$

$\rho_F V_{dF}\, g - m_1 g = m_1 a_{1y}$

FULLY SUBMERGED ... $V_{dF} = V_0$

$\rho_F V_0 g - \bar{\rho}_0 V_0 g = \bar{\rho}_0 V_0 a_y$

$a_y = \dfrac{\rho_F V_0 g - \bar{\rho}_0 V_0 g}{\bar{\rho}_0 V_0}$

$\bar{\rho}_0$ ≡ AVERAGE DENSITY OF OBJECT

$a_y = g\left(\dfrac{\rho_F}{\bar{\rho}_0} - 1\right)$

∴

IF $\rho_F > \bar{\rho}_0$ THEN $a_y (+)$ ... FLOAT

IF $\rho_F < \bar{\rho}_0$ THEN $a_y (-)$ ... SINK

IF $\rho_F = \bar{\rho}_0$ THEN $a_y = 0$ ... NEUTRAL BUOYANT

PRACTICE: Two equal-sized action figures that have different masses are held by strings so that they are submerged in water at different depths. The pressure exerted on the bottom surface of the 3 kg action figure by the water is ____________ the pressure on the bottom surface of the 1 kg action figure.

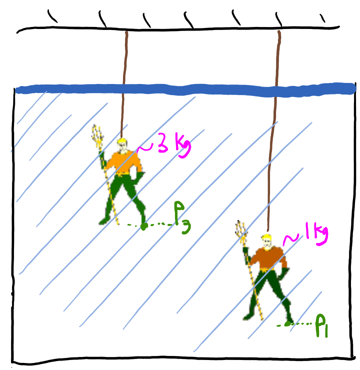
(a) greater than
(b) less than
(c) equal to

PRESSURE @ A DEPTH ... $P_{Bottom} = P_{Top} + \rho_F g d$

... SAME ρ_F, SAME P_{Top} ....

$d_3 < d_1$

$\therefore\ P_3 < P_1$



The buoyant force exerted by the water on the 3 kg action figure is ________ the buoyant force on the 1 kg action figure.

(a) greater than
(b) less than
(c) equal to

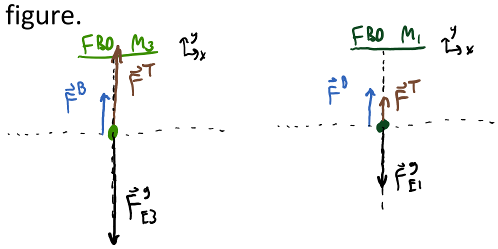
$|\vec{F}^B| = \rho_F V_{dF}\, g$

w/ ρ_F SAME + g SAME

$|\vec{F}^B| \propto V_{dF}$ .... BOTH DISPLACE SAME VOLUME OF FLUID SO EQUAL

The tension in the string holding the 3 kg action figure is _______ the tension in the string holding the 1 kg action figure.

(a) greater than
(b) less than
(c) equal to



.... SAME $|\vec{F}^B|$

... $m_3 > m_1$ ... $|\vec{F}^g_{E3}| > |\vec{F}^g_{E1}|$

$\therefore\ |\vec{F}^T_3| > |\vec{F}^T_1|$

PRACTICE: In each case, a block hanging from a string is suspended in a liquid. All of the blocks are the same size, but they have different masses (labeled MB) because they are made of different materials. All of the containers have the same volume of liquid, but the masses of these liquids vary (labeled ML) since the liquids are different. The volume of the blocks is one-sixth the volume of the liquids. Rank the buoyant forces on the blocks.

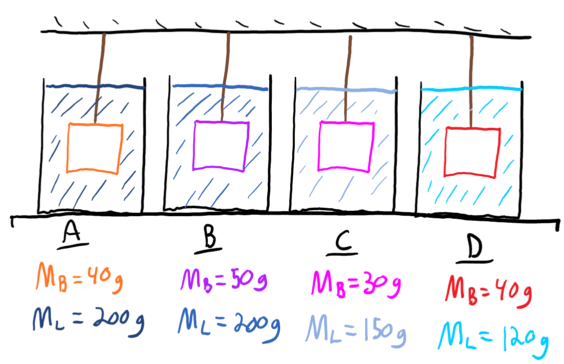
$|\vec{F}^B| = \rho_F V_{dF}\, g$

SAME FOR ALL CASES...

$\therefore\ |\vec{F}^B| \propto \rho_F$

w/ $\rho_F = \dfrac{M_F}{V_F}$ AND V_F = SAME FOR ALL CASES..

THEN $\rho_F \propto M_F$ AND $|\vec{F}^B| \propto M_F$

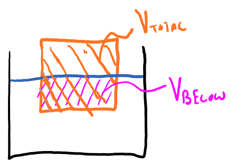


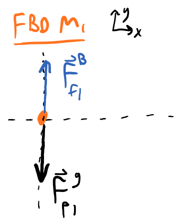
w/ $m_{fA} = m_{fB} > m_{fC} > m_{fD}$

$$\boxed{|\vec{F}^B_A| = |\vec{F}^B_B| > |\vec{F}^B_C| > |\vec{F}^B_D|}$$

PRACTICE: On a distant planet the acceleration due to gravity is greater than it is on earth. Would you float more easily in water on this planet than on earth?

(a) More easily.
(b) Less easily.
(c) Float the same.





$\Sigma F_y = m a_y^{\,0}$

$|\vec{F}^B_{f1}| - |\vec{F}^g_{P1}| = 0$

$\rho_F V_{DF} g_p - m_1 g_p = 0$

$\rho_F V_{BELOW} - \rho_1 V_{TOTAL} = 0$

$\rho_F V_{BELOW} = \rho_1 V_{TOTAL}$

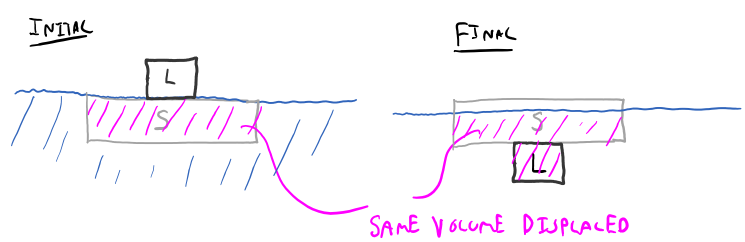
$$\boxed{\frac{V_{BELOW}}{V_{TOTAL}} = \frac{\rho_1}{\rho_F}}$$

* ρ IS MATERIAL PROPERTY ... SAME ON BOTH PLANETS

SO % V_{BELOW} IS CONSTANT.

PRACTICE: A piece of lead is fastened on top of a large solid piece of Styrofoam that floats in a container of water. Because of the weight of the lead, the waterline is flush with the top surface of the Styrofoam. If the piece of Styrofoam is turned upside down so that the piece of lead is now suspended underneath it,

(a) the arrangement sinks,
(b) the waterline is below the top surface of the Styrofoam.
(c) the waterline is still flush with the top surface of the Styrofoam.



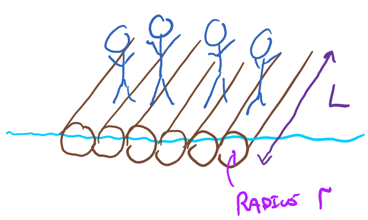
The water level in the container…
(a) rises.
(b) drops.
(c) remains the same.

SAME TOTAL OBJECT WEIGHT ....

...SO STILL SAME WEIGHT OF DISPLACED FLUID...

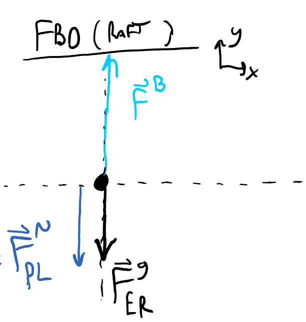
∴ THUS SAME DISPLACED VOLUME.

PRACTICE: What is the smallest number of whole logs (ρ_{log}= 725 kg/m^3 ; radius = 0.08 m ; length = 3.0 m) that can be used to build a raft that will carry four people, each whom has a mass of 80 kg?



*SMALLEST # .... MAX VOLUME DISPLACED TO FIND MIN #

$4m_p\vec{g} = \vec{F}^N_{PL}$



$\Sigma F_y = m_L a_y^{\,0}$

$|\vec{F}^B| - |\vec{F}^N_{PL}| - |\vec{F}^g_{ER}| = 0$

$\rho_F V_{FD} g - 4m_p g - m_R g = 0$ # of LOGS

$\rho_F V_R - 4m_p - N m_L = 0$

$N \rho_F V_L - 4m_p - N \rho_L V_L = 0$

$N V_L (\rho_F - \rho_L) = 4 m_p$

$$N = \frac{4M_p}{V_L(\rho_F - \rho_L)}$$

$$\boxed{N = \frac{4M_p}{\pi r^2 L(\rho_F - \rho_L)}} \approx 19.29 \text{ LOGS}$$

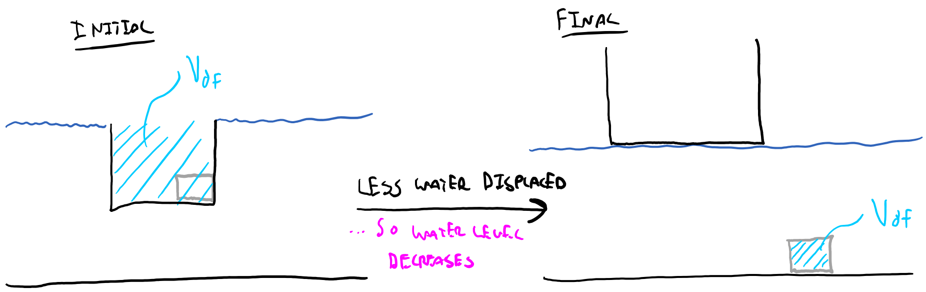
$\dots$ $\boxed{\text{SO 20 WHOLE LOGS}}$

PRACTICE: Two mobsters drop a suspiciously large brick of concrete out of a boat in a large lake. Does the level of the water in the lake, relative to the shore…

(a) increase.
(b) decrease.
(c) stay the same.
(d) Depends on who's asking, are you asking?

LOOK @ EXTREME CASES, …

… LET $M_{BOAT} \approx 0$ & INITIALLY AT MAX CAPACITY



QUESTIONS FOR DISCUSSION:

(1) Two cups are filled to the same level with water. One of the two cups has ice cubes floating in it. When the ice cubes melt, which cup is the level of the water higher?
(2) When don't ships made out of iron sink?