Density

Select LEARNING OBJECTIVES:

- Demonstrate the ability to characterize materials based off of their density.
- Introduce the approximation of incompressible fluids.
- Use proportional reasoning to relate mass, volume, and density.

TEXTBOOK CHAPTERS:

- Giancoli (Physics Principles with Applications 7th) :: 10-2
- Knight (College Physics : A strategic approach 3rd) :: 13.1
- BoxSand :: Density

WARM UP: Suppose you measure the mass of an object, 200 g, then divided this mass by the volume of the object 100 cm^3. How do you interpret the number 200/100 ? Basically, what does this number mean to you?

$\frac{200g}{100\,cm^3} = 2\,\frac{g}{cm^3}$ # OF GRAMS IN 1 CM^3 OF THE OBJECT

Consider comparing the differences between gold and Styrofoam (polystyrene foam). Perhaps you wish to compare the weight of the two materials. Well, you can obtain one pound of gold and one pound of Styrofoam, so perhaps weight is not a good quantity when trying to compare the two materials. But now that you have a pound of each material, you notice that pound of gold takes up much less volume than the pound of Styrofoam. Cleverly, you define a new quantity called mass density, which is the mass per unit volume of each material. Mathematically this is written as…

$$\rho = \frac{m}{V}$$

It turns out, that the mass density is a material property. One pound of gold has the same mass density as 2 pounds, 1,000 pounds, 1/100th of a pound, etc..

*Note: There are other types of density (e.g. energy density, charge density, number density, area density, etc…). Since we will be working with mass density in fluid mechanics, we often drop the mass and just call it density.

We must be a little bit more careful though. Consider water, if we place a container of water under enough pressure, we can slightly change the density. Likewise, if we change the temperature of a container of water, the density also slightly changes. Thus, the density of any material is dependent on temperature and pressure. However, for solids and liquids, the density is very nearly constant for a wide range of temperatures and pressure, thus throughout our fluid mechanics studies, we will ignore these small density variations due to temperature and pressure changes. Another way to state this approximation is to say that we will assume fluids are incompressible (i.e. their densities are constant for any temperature or pressure).

Many of the empirical observation within the field of fluid mechanics lead to equations or laws that are dependent on the mass of the fluid or object being observed. Often times we cannot directly measure the mass but know information about the volume of the fluid or object. Thus the density of a fluid is an important quantity when modeling fluids.

PRACTICE: The density of air at 20 °C is about 1.20 kg/m³. What is the mass of a living room with dimensions of 4.27 m x 3.96 m x 2.62 m?

(a) 44.3 kg
(b) 53.2 kg
(c) 0.0271 kg
(d) 0.0390 kg
(e) 36.9 kg

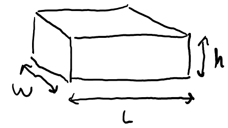
$\rho = \frac{m}{V}$

$m = \rho V$

$V = ?$

$V = L \cdot w \cdot h$

$V \approx 44.3\ m^3$



$m = (1.20\ \frac{kg}{m^3})(44.3\ m^3)$

$m = 53.2\ kg$

PRACTICE: A 100 mL beaker contains 175 g of liquid. What is the liquid's density in SI units?

(a) 1750 kg/m³
(b) 1.75 x 106 g/m³
(c) 1.75 g/m³
(d) 1.75 g/mL

SI

$1\ mL = 1\ cm^3$

$100\ mL \times \frac{1\ cm^3}{1\ mL} \times \left(\frac{1\ m}{100\ cm}\right)^3 = 1 \times 10^{-4}\ m^3$

$175\ g \times \frac{1\ kg}{1000\ g} = 0.175\ kg$

$\rho = \frac{m}{V}$

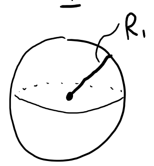
$\rho = \frac{0.175\ kg}{1 \times 10^{-4}\ m^3}$

$\rho = 1750\ \frac{kg}{m^3}$

PRACTICE: A gas is a compressible substance. If gas of volume V1 is trapped inside a sphere of radius R1, what will be the density of the gas in terms of the original density if the sphere's radius is halved?

(a) $\rho_2 = \rho_1$
(b) $\rho_2 = 1/2\ \rho_1$
(c) $\rho_2 = 1/4\ \rho_1$
(d) $\rho_2 = 1/8\ \rho_1$
(e) $\rho_2 = 2\ \rho_1$
(f) $\rho_2 = 4\ \rho_1$
(g) $\rho_2 = 8\ \rho_1$

i



$V_1 = \frac{4}{3}\pi R_1^3$

$\rho_1 = \frac{m_1}{V_1}$

f

$V_2 = \frac{4}{3}\pi R_2^3$

$\rho_2 = \frac{m_2}{V_2}$

w/ $m_1 = m_2$

$\rho_1 V_1 = \rho_2 V_2$

AND w/ $V \propto R^3$

$\rho_1 R_1^3 = \rho_2 R_2^3$

w/ $R_2 \rightarrow \frac{1}{2} R_1$

$\rho_1 R_1^3 = \rho_2 (\frac{1}{2} R_1)^3$

$\rho_1 = \frac{1}{8} \rho_2$

So... $\rho_2 = 8 \rho_1$

QUESTIONS FOR DISCUSSION:

(1) Is density a microscopic or macroscopic quantity?
(2) Is it always true that a material with a higher density will have heavier molecules than a material with a lower density.