<u>Hydrostatics</u>

Select LEARNING OBJECTIVES:

- Apply pressure at a dpeth principles while solving problems.
- Relate Pascal's Law to hydraulic applications.
- Identify mechanical advantage in hydraulic systems.
- Construct FBDs and e-FBDs in parallel with a hydrostatics analysis to analyze systems.

TEXTBOOK CHAPTERS:

- Ginacoli ((Physics Principles with Applications 7th) :: 10-3, 10-5, 10-6
- Knight (College Physics : A strategic approach 3rd) :: 13.2, 13.3
- Boxsand :: Hydrostatics

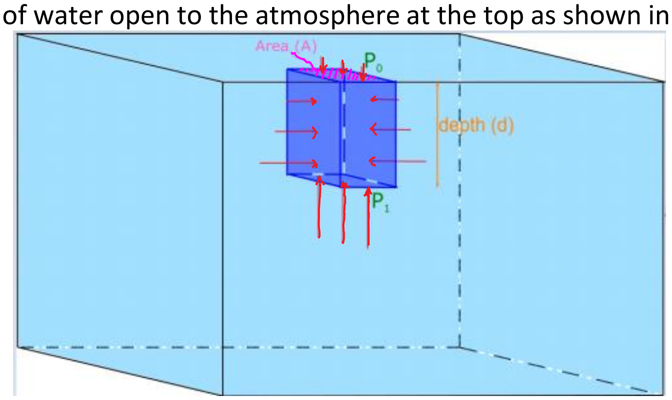
WARM UP: Suppose you measure the mass of an object, 200 g, then divided this mass by the volume of the object 100 cm^3. How do you interpret the number 200/100 ? Basically, what does this number mean to you?

When a body of fluid is near a massive object such as the Earth, a pressure gradient within the fluid is formed. In this lecture we will quantify these pressure gradients by finding an expression for pressure at some depth below the surface of an incompressible fluid. We will also explore applications of Pascal's law which tells us how a change in pressure within an incompressible fluid is transmitted to other locations within the fluid.

<u>Pressure at a depth</u>

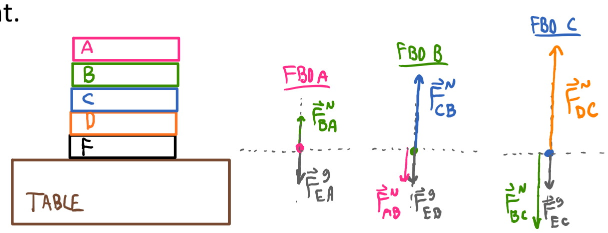
Have you ever gone swimming in a really deep body of water? If so, you are probably familiar with the sensation you get when diving down to very deep depths below the surface of the water. This sensation is a result of your inner ears letting you know the pressure of the water around you is increasing. How can we mathematically show this pressure increase as we go deeper into the water? This phenomena is also referred to as "pressure at a depth"?

Consider a container of water open to the atmosphere at the top as shown in the figure below.

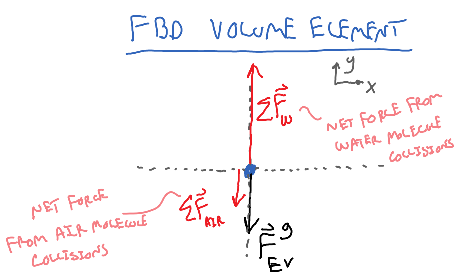


Let the density of the water be ρ_f. Now imagine a smaller rectangular volume element of water in the container as indicated by slightly darker blue. The downward force on this volume element of water at the top of the surface is due to the atmospheric pressure P_0 multiplied by the area A of the top of the volume element. You can think about this force from a microscopic view; air molecules are constantly colliding with the water's surface, which causes the air molecules to change momentum and thus the net impulse from all of these collisions (on the order of 10^{24}) result in a force from the atmosphere on the water. Since this volume of water is stationary, (i.e. it is not accelerating), we know that there must be an upward force at the bottom of the volume element which is greater than the top force because it must also support the weight of the volume element. Again, a microscopic view at the bottom of the volume element allows us to think about the water molecules colliding with the bottom "surface" of the volume element. This statement is analgous to the following situation in the figure to the right.

5 books are stacked on top of each other, the top book A has a normal force acting on it upwards from book B. This normal force between A and B is equal to the weight of the top book only. As we work our way down the stack, the normal force holding the books up increases because it needs to support the total weight of the books above.



Going back to our water volume element analysis, there are also forces from the pressure on either side (left, and right) of the volume element. These horizontal forces must be equal to each other because the volume element is not accelerating in either direction. The descriptive representation of this scenario is nicely visualized with an accompanying FBD for the volume element of water as shown below. (The horizontal forces are not shown since they all cancel out.)



$$\Sigma F_y = m_v a_y^{\nearrow 0}$$

$$|\Sigma \vec{F}_w| - |\Sigma \vec{F}_{air}| - |\vec{F}^g_{Ev}| = 0$$

$$P_1 A - P_0 A - m_v g = 0 \qquad \rho_F = \frac{m_v}{V}$$

$$P_1 A - P_0 A - \rho_F V g = 0 \qquad V = Ad$$

$$P_1 \cancel{A} - P_0 \cancel{A} - \rho_F \cancel{A} d g = 0$$

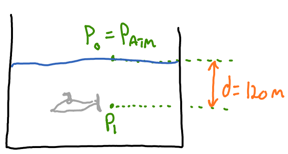
$$\boxed{P_1 = P_0 + \rho_F g d}$$

Using Newton's second law we can translate our physical representation (FBD) into a mathematical representation as seen above.

Notice the end result for the pressure at the bottom of the volume element of water is not dependent on the area or volume, it only depends on the depth below the surface d, the density of the fluid ρ_f, the acceleration due to gravity caused by the planet the body of fluid is near "g", and the pressure above the surface of the fluid P_0.

*NOTE: Our analysis of pressure at a depth used the assumption that the fluid is incompressible (i.e. the fluid has a constant density). This is a good assumption for liquids like water that do not compress much. Compare this to the atmosphere which is still considered a fluid, however it is easily compressible, thus our pressure at a depth analysis would only be valid for small depths below an atmosphere where the density is roughly constant.

PRACTICE: When a submarine dives down to a depth of 120 m, to how large a total pressure is its exterior surface subjected? The density of seawater is 1030 kg/m^3.



$$P_1 = P_0 + \rho_F g d$$

$$= (101325\ \text{Pa}) + (1030\ \tfrac{\text{kg}}{\text{m}^3})(9.8\ \tfrac{\text{m}}{\text{s}^2})(120\ \text{m})$$

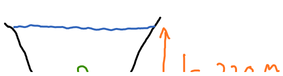
$$= 1312605\ \text{Pa}$$

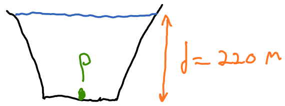
$$P_1 = 1.31 \times 10^6\ \text{Pa} \quad \text{or} \quad 1312605\ \text{Pa} \times \frac{1\ \text{atm}}{101325\ \text{Pa}} \approx 13\ \text{atm}$$

PRACTICE: The depth of water at the Hoover Dam is roughly 220m. What is the water pressure at the base of the dam?

$$P = \rho_F g d$$

$$= (1000\ \tfrac{\text{kg}}{\text{m}^3})(9.81\ \tfrac{\text{m}}{\text{s}^2})(220\ \text{m})$$





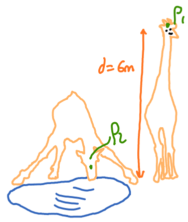
$P = \rho_F g d$

$= (1000 \frac{kg}{m^3})(9.81 \frac{m}{s^2})(220 m)$

$= 2158200$ Pa

$\boxed{P = 2.16 \times 10^6 \text{ Pa}}$ or.. 21 atm

PRACTICE: Calculate the difference in pressure that the blood vessels in a giraffe's head have to accommodate as the head is lowered from a full upright position to the ground level for a drink. The height of an average giraffe is about 6 m.



$P_2 = P_1 + \rho_F g d$

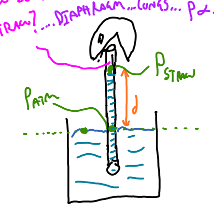
$\Delta P = \rho_F g d$ DENSITY of BLOOD.

$= (1050 \frac{kg}{m^3})(9.81 \frac{m}{s^2})(6 m)$

$\boxed{\Delta P = 61740 \text{ Pa} \quad \text{or} \quad 0.61 \text{ ATM}}$

PRACTICE: Steve is selling a vibranium straw that is 5.0 m long for a price of \$20. Steve's good pal is also selling a straw that is made out of uru metal for \$20 but is 20 m long. Assuming you plan to use these straws in a vertical direction on the surface of the earth, which straw would you buy and why? Neither of these straws can be cut into smaller pieces.

How Do HUMANS REMOVE THE AIR FROM THE STRAW? ... DIAPHRAGM...LUNGS... $P \propto \frac{1}{V}$



0 FOR MAX HEIGHT

$P_{ATM} = \cancel{P_{STRAW}} + \rho_F g d$

$\boxed{d = \frac{P_{ATM}}{\rho_F g}} \approx \frac{101325 \text{ Pa}}{(1000 \frac{kg}{m^3})(9.81 \frac{m}{s^2})} \approx 10 \text{ m}$

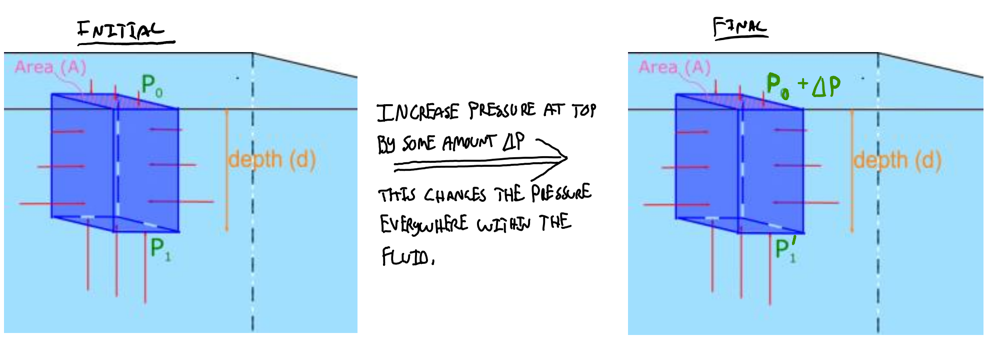
MAX HEIGHT WE CAN GET WATER TO...

∴ BUY STEVE'S STRAW SINCE IT'S LESS THAN 10 m LONG

Pascal's law

Pascal's law is formally stated as follows, ***"for an incompressible and enclosed fluid, a change in pressure at one location results in the same change in pressure, without a diminish in magnitude, at all locations within the enclosed fluid and walls"***.

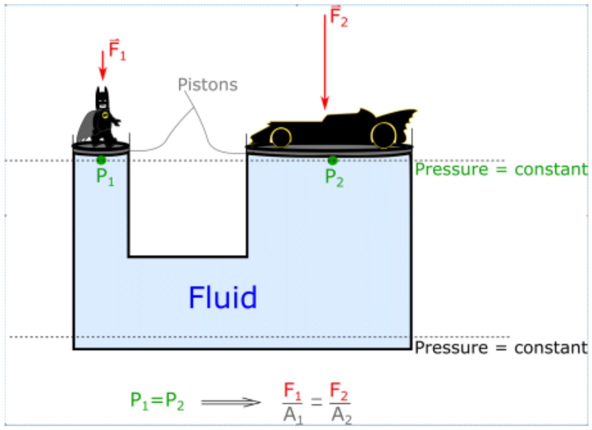
Consider the analysis we did in the "pressure at a depth" section above. If the pressure P_0 above the surface of the water increases by some amount ΔP so that the new pressure above the surface is now $P'_0 = P_0 + \Delta P$, then the pressure at all locations within the fluid also increase by the same amount ΔP. Thus the pressure at the bottom of the volume element would be $P_1 + \Delta P = P_0 + \rho_f g d + \Delta P$. This is illustrated in the figure below.



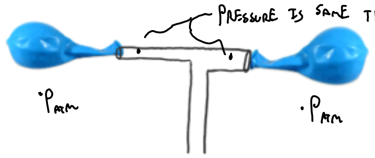
$P_1 = P_0 + \rho_F g d$

$P_1' = P_1 + \Delta P = P_0 + \rho_F g d + \Delta P$

Hydraulic brakes and lifts generally work by applying a force over an area at one location which increases the pressure at that location along with all other locations in the system. Thus hydraulics are an interesting application of Pascal's law. Below is a figure illustrating a simplified hydraulic lift.



PRACTICE: Two identical uninflated balloons are connected to a T-shaped tube as shown below.



PRESSURE IS SAME THROUGHOUT ENCLOSED SYSTEM.... PASCAL.

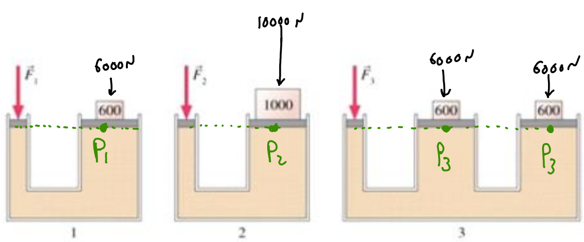
W/ SAME PRESSURE DIFFERENCE
SAME ΣF TO "STRETCH" BALLOONS

When air is blown into the open end of the tube,

.... WHAT IS DIFFERENT?...
RATE @ WHICH THEY INFLATE
∴ AFTER EQ... BALLOONS ARE SAME SIZE

(a) the balloon on the left inflates more.
(b) the balloon on the right inflates more.
(c) both balloons inflate equally.

PRACTICE: Rank the magnitudes of the forces required to balance the masses. The masses are all in kilograms and the area of each weight holding the piston is the same.



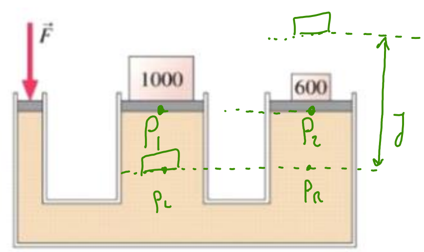
$P = \frac{F_\perp}{A}$ .... w/ $F_{\perp_2} > F_{\perp_1}$ and $A_1 = A_2$ } $P_2 > P_1$

w/ $F_{\perp_3} = 2F_{\perp_1}$ and $A_3 = 2A_1$ } $P_1 = P_3$

} $P_2 > P_1 = P_3$

∴ $\boxed{|\vec{F}_2| > |\vec{F}_1| = |\vec{F}_3|}$

PRACTICE: Two different masses (in kg) are initially placed on top of equal area low friction pistons. The pistons are connected to a third piston where a force F is applied such that it doesn't move. What happens to the other two pistons?



1. The 1000 kg and 600 kg masses move upward.
2. The 1000 kg mass moves upwards while the 600 kg mass moves downward.
3. The 1000 kg and 600 kg masses move downward.
4. The 1000 kg mass moves downward while the 600 kg mass remains still.
5. The 1000 kg mass moves downward while the 600 kg mass moves upward.
6. Both the 1000 kg and 600 kg masses remain at rest.

$P_1 = \frac{10000\,N}{A}$ $P_2 = \frac{6000\,N}{A}$

w/ $A_1 = A_2$

$P_1 > P_2$ .... NOT IN EQ...

SO 1000 kg ↓ 600 kg ↑ UNTIL $P_L = P_R$

rest.

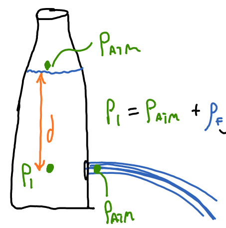
P_1 / P_2 ....

So 1000 kg ↓ 600 kg ↑ until $P_L = P_R$

$$\frac{10000\,N}{A} = \frac{6000\,N}{A} + \rho_F g d$$

PRACTICE: When a hole is made in the side of a container holding water, water flows out and follows a parabolic trajectory. If the container is dropped in free fall, the water flow

(a) diminishes.
(b) stops altogether.
(c) goes out in a straight line.
(d) curves upwards.



$P_1 = P_{ATM} + \rho_F g d$

PRESSURE @ A DEPTH IS CAUSED BY GRAVITY ∴

∴ FREE-FALL EXPERIENCES NO EFFECTIVE GRAVITY

QUESTIONS FOR DISCUSSION:

(1) Can you drink through a straw while on the moon?
(2) A plastic bottle with a hole in it contains water and is in space very far from any massive object. Will the water leak out of the hole? If it does, can you do anything to the system to stop the leak? If it does not leak, can you do anything to the system to make it leak?
(3) When blood pressure is measured, the arm cuff is typically measured at the heart level. Will you get the same readings if you raised your arm above heart level? Explain.