Case studies of traveling waves

Select LEARNING OBJECTIVES:

- Strengthen proportional reasoning skills.
- Be identify wave speed in specific cases: air and string.
- Understand the connection between waves and oscillations.
- Understand that the frequency of a wave is dependent only upon the source.
- Understand that the speed of the wave depends on the medium in which it travels.

TEXTBOOK CHAPTERS:

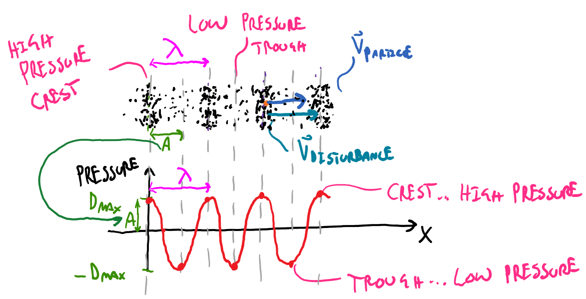
- Boxsand :: [Traveling waves]

WARM UP: Explain the difference between the speed of a transverse wave traveling along a string and the speed of a tiny piece of the string. ?

Both the mathematical model and terminology we developed in the traveling waves lecture are directly applicable to wave traveling on a string, sound waves, light waves, etc… In this lecture we will briefly look at a few specific cases: waves on a string, and sound waves.

Sound waves

Sound waves are longitudinal waves that propagate through the air (a gas/fluid). Below is an illustration of the density of air particles as the disturbance travels to the right, along with how pressure relates to each part.



Recall from our thermodynamics discussion, we found a room mean square speed of a gas particle is…

$$V_{RMS} = \sqrt{\frac{3k_BT}{m}}$$

As it turns out, the speed of a sound wave is roughly close to this value. The speed of sound in a gas is shown below.

$$V_{SOUND} = \sqrt{\frac{\gamma k_B T}{m}} \qquad ; \ \gamma \text{ IS A CONSTANT} \ldots \approx 1.67 \text{ FOR MONATOMIC GASES}$$

The details of origin of the above equation are important, but the features are. Note that it is roughly equal to the root mean square speed of the gas particles, and it also has the same functional dependence of temperature and mass of the particles. For standard temperatures of 20 °C, the seed of sound in air is about 343 m/s.

IN AIR

of temperature and mass of the particles. For standard temperatures of 20 °C, the seed of sound in air is about 343 m/s.

$V_{SOUND} \approx 343\ m/s$ IN AIR @ 20°C

PRACTICE: The sensitivity to sound for human ears is approximately 20 Hz - 20 kHz. At room temperature and pressure the speed of sound is about 343 m/s, what approximate wavelength range does this correspond to?

$V = f\lambda$

$\lambda = \frac{V}{f} = \frac{343\ m/s}{f}$

a. 10 mm - 10 m
b. 50 cm - 50 km
c. 20 mm - 20 m
d. 14 cm - 400 cm
e. 1.7 cm - 17 m

Waves on a string

Waves on a string are transverse waves that propagate through a string/rope. Take a picture of a wave on a string, put an x and y coordinate system on it and you have yourself a snapshot graph of a wave on a string. As it turns out, the speed of the disturbance that propagates through a string is…

$$V_{STRING} = \sqrt{\frac{|\vec{F}^T|}{\mu}}$$

WHERE $\mu = \frac{\text{MASS OF STRING}}{\text{LENGTH OF STRING}}$

LINEAR MASS DENSITY

PRACTICE: A wave is traveling on a string. If the same material string is used but it is twice as long, and the tension quadruples, what happens to the speed of the wave?

$\mu = $ CONST

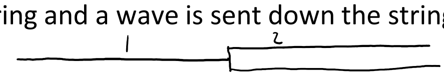
(a) Doubles.
(b) Halves.
(c) Increases by square root of 8.
(d) Decreases by square root of 8.
(e) Triples.

$V \propto \sqrt{|\vec{F}^T|}$

IF $|\vec{F}^T| \rightarrow 4|\vec{F}^T|$

$V \rightarrow 2V$

PRACTICE: Consider a situation where a light string is tied to a heavy string and a wave is sent down the string. Which of the following statements below are true?



(a) The tension in the light string is less than in the heavy string.
(b) The tension in the light string is greater than in the heavy string.
(c) The tension in the light string is equal to that in the heavy string.

FBD @ CONNECTION… w/ $\vec{a} = \vec{0}$ $|\vec{F}_1^T| = |\vec{F}_2^T|$

(d) The speed of the wave in the light string is less than in the heavy string.
(e) The speed of the wave in the light string is greater than in the heavy string.
(f) The speed of the wave in the light string is equal to that in the heavy string.

$V = \sqrt{\frac{|\vec{F}^T|}{\mu}}$ w/ $|\vec{F}^T|$ CONST…

$V \propto \frac{1}{\sqrt{\mu}}$ IF $\mu \downarrow V \uparrow$

(g) The frequency in the light string is less than in the heavy string.
(h) The frequency in the light sting is greater than in the heavy string.
(i) The frequency in the light string is equal to that in the heavy string.

BOUNDARY CONDITION… IF $f_1 \neq f_2$ THE ROPE WOULD BREAK APART

(j) The wavelength of the wave in the light string is less than in the heavy string.
(k) The wavelength of the wave in the light string is greater than in the heavy string.
(l) The wavelength of the wave in the light string is equal to that in the heavy string.

$V = f\lambda$ w/ $f =$ CONST

$V \propto \lambda$ $V_1 > V_2$ SO $\lambda_1 > \lambda_2$

PRACTICE: Consider a situation where a light string is tied to a heavy string that has a linear mass density that is twice as large. A 0.5 meter wavelength wave travels down the light string and into the heavy string. What is the wavelength of the wave in the heavy string?

$V = f\lambda$ $\quad V = \sqrt{\frac{|\vec{F}^T|}{\mu}}$

w/ $f =$ const

$\frac{V}{\lambda} =$ const

$\frac{v_1}{\lambda_1} = \frac{v_2}{\lambda_2}$

$\frac{\sqrt{\frac{|\vec{F}_1^T|}{\mu_1}}}{\lambda_1} = \frac{\sqrt{\frac{|\vec{F}_2^T|}{\mu_2}}}{\lambda_2}$

w $|\vec{F}_1^T| = |\vec{F}_2^T|$

$\frac{1}{\sqrt{\mu_1}\,\lambda_1} = \frac{1}{\sqrt{\mu_2}\,\lambda_2}$

$\lambda_2 = \sqrt{\frac{\mu_1}{\mu_2}}\,\lambda_1$

$\lambda_2 = \sqrt{\frac{\mu_1}{2\mu_1}}\ 5\ m$

$\lambda_2 = \frac{5}{\sqrt{2}}\ m$

$\lambda_2 \approx 3.54\ m$

QUESTIONS FOR DISCUSSION:

(1) Explain the difference between the speed of a transverse wave traveling along a string and the speed of a tiny piece of the string.