Energy, intensity, and sound level intensity

Select LEARNING OBJECTIVES:

- Understand the relationship between intensity and power.
- Understand the origin of the $1/r^2$ nature of Intensity drop-off over distance for many sources.
- Be able to use the decibel equation to calculate intensities at various locations.
- Use intensity and geometric information to calculate the power output of an object.

TEXTBOOK CHAPTERS:

- Boxsand :: Energy & Intensity

WARM UP: A traveling wave transports…
(a) both energy and matter.
(b) matter but not energy.
(c) energy but not matter.

We started our discussion of traveling waves with a billiard ball example to help illustrate that energy is transferred from one point in space to another without the actual individual oscillators traveling between the two locations. The billiard ball example highlighted an example of translational kinetic energy being transferred from one location to the next. The billiard balls had no restoring forces on them so after the energy was transferred the billiard balls were at rest. However, by connecting springs to the billiard balls we were able to set up a situation where the billiard balls could vibrate back and forth in SHM sending vibrational energy from one point in space to another. *In general, waves transport vibrational energy from one location in space to another. As the wave travels through the medium, vibrational energy is transferred from one particle of the medium to the next particle of the medium.*

Energy and Power

Recall the practice problem form "*Lecture 19: Simple harmonic oscillators*" where a mass connected to a spring was hanging at rest from the ceiling. There was no vibrational energy present. You then pulled the mass down a distance from its equilibrium location (putting energy into the system) before letting it go and setting the system into simple harmonic motion. Note that the objective of that question was to highlight that the mass-spring system had no vibrational energy until you put energy into the system by doing work on it. I want to emphasize this because it helps highlight an important feature of oscillations; the magnitude of the vibrational energy was determined by you, the source. Traveling waves contain a collection of oscillators, so the energy they transfer from one location to another is also determined by the source. Think about it, you get to decide how loud you want to talk; you are the source of the energy that gets transferred to your friend as you talk to them.

Recall that a SHO has a potential energy that is quadratic with respect to the displacement from equilibrium. Thus the total energy of an oscillator is equal to $\frac{1}{2}\ k\ x_{max}^2$. In other words *the energy of an oscillator, and therefore a traveling wave too, is prorportional to the square of the amplitude of the oscillations*. The amplitude is determined by the source as well.

$$\boxed{\text{ENERGY} \propto X_{max}^2}$$

When a speaker plays a constant frequency note for a few seconds or minutes, it is natural to talk about the speaker's power instead of energy. Recall power is energy per time.

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$$\boxed{\text{POWER} \equiv P = \frac{\text{ENERGY}}{\text{TIME}}} \xrightarrow{\text{SI UNITS}} \frac{J}{s} = \text{WATT} \equiv W$$

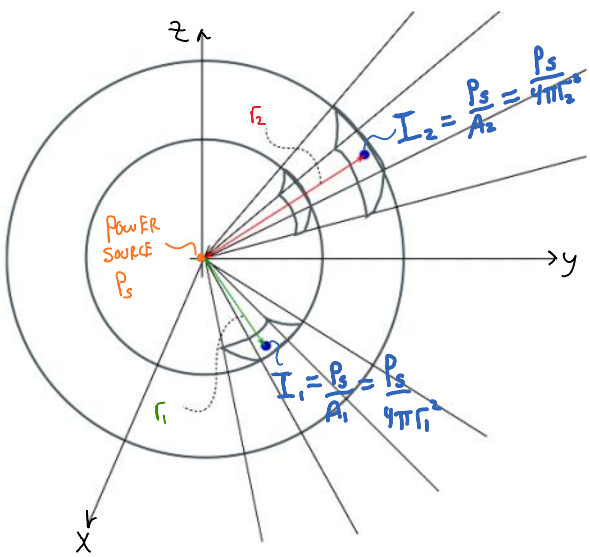
Also note that the energy is proportional to x_{max}^2 and the traveling wave moves through space, the vibrational energy at any given time at a fixed x location also oscillates, so we are really talking about the average power. It should also be clear that power is proportional to the square of the amplitude as well.

Intensity

Imagine dropping a rock into water. The rock displaces the water at the location it entered and sends a disturbance across the surface of the water. The initial crest is located at the entry point, where the energy is a maximum since the amplitude is a maximum. As this wave crest travels across the surface of the water the energy is caries gets spread out over a larger and larger circumference since the circumference of the circular ring increases with the distance from the source (i.e. the radius). Therefore, if you were a bug sitting at some distance far away from the initial location of the dropped rock, the vibrational energy transferred to you is less than the total energy that the rock initially supplied. As the wave travels outwards from the origin, the circumference increases, thus the energy per unit length of the wave must decrease so that the total energy is constant. And since energy is proportional to the square of the amplitude, the amplitude must then decrease as the wave travels outwards. Try this at home! Also remember that energy is proportional to power, so the power also decreases as the wave travels outwards. Since the power is a property of the source, yet it decreases as the wave travels outwards, it is natural to work in terms of power/unit-length. In general, we will be spend most of our time analyzing three dimensional spherical waves where the energy spreads out over the surface of a spherical shell as the wave travels away from its source. Likewise, the natural quantity to think about is power/area which is referred to as intensity. The intensity of a wave is the average power transported across a unit area that is perpendicular to the direction of energy flow. For our class we will always assume unless specified that the area is perpendicular so that we can write…

$$\boxed{\text{INTENSITY} \equiv I = \frac{\text{POWER}}{\text{AREA}}} \xrightarrow{\text{SI UNITS}} \frac{W}{m^2}$$

For a spherical source of power, the energy is spread out over the surface area of a spherical shell, thus the intensity also decreases as the wave travels outwards as shown below.



An important feature of spherical power sources is that the intensity decreases as the inverse square of the distance from the source.

$$I \propto \frac{1}{r^2}$$

This is known as an inverse square law. We have encountered a similar inverse square relationship before when studying the universal law of gravity. An inverse square law will also appear when we study electric and magnetic fields.

It is also interesting to note that since energy is proportional to the square of the amplitude, and energy is proportional to power, and power is proportional to intensity and finally intensity is proportional to the inverse square of the distance of a spherical source, we can determine the amplitude of the oscillations based off of any one of these proportional relationships.

$$\boxed{\left.\begin{array}{l} \left.\begin{array}{l} \left.\begin{array}{l} E \propto X_{max}^2 \\ E \propto P \end{array}\right\} X_{max}^2 \propto P \\ P \propto I \end{array}\right\} X_{max}^2 \propto I \\ I \propto \frac{1}{r^2} \end{array}\right\} X_{max} \propto \frac{1}{r}}$$

For a sinusoidal traveling wave of frequency *f*, the particles of the medium undergo SHM. Again, recall that the vibrational energy is $\frac{1}{2}\,k\,x_{max}^2$. Also recall that the angular frequency ω is equal to 2 π *f*. Modeling the motion of the oscillating particles as a mass-spirng system we can then invoke our definition of angular frequency as also being equal to $\sqrt{\frac{k}{m}}$. Finally, we can do some algebric manipulation to replace the "spring constant" k in terms of mass of the medium parrticle and frequency.

$$\left.\begin{array}{l} E = \frac{1}{2} k X_{max}^2 \\ \left.\begin{array}{l} \omega = 2\pi f \\ \omega = \sqrt{\frac{k}{m}} \end{array}\right\} 2\pi f = \sqrt{\frac{k}{m}} \longrightarrow k = 4\pi^2 f^2 m \end{array}\right\} \boxed{E = 4\pi^2 m f^2 X_{max}^2}$$

Note that the energy transferred in a traveling wave is also proportional to the frequency of the source. The source determines the frequency and the amplitude. We can now do a similar analysis with frequency to see how it is proportional to other quantities like we did with the amplitude. To summarize…

$$\boxed{E \propto f^2 \qquad P \propto f^2 \qquad I \propto f^2}$$

Also note that frequency is a constant, determined by the source. While the amplitude decreases with distance from the source, the frequency does not.

PRACTICE: What are two factors that directly affect sound energy?

a. The wavelength and intensity. X_{max} + f M of oscillators
b. The decibels of the source.
c. The characteristics of the sound source and the substance through which the sound travels.

PRACTICE: Light from a laser forms a 1.31 mm diameter spot on a wall. If the light intensity in the spot is 3.03×10^4 W/m^2, how much energy will the laser output in 10 s?

a. 34.4 J
b. 40.8 mW ?
c. 25.3 J
d. 0.408 N m
e. 34.4 mW ?
f. 25.3 mW ?

$I = \frac{P}{A} = \frac{E/\Delta t}{A}$

$E = I A \Delta t = I \pi r^2 \Delta t = 0.408 \text{ J}$

PRACTICE: The intensity of an earthquake P wave traveling through the Earth and detected at a distance r_1 from the source is I_1. What is the intensity of that wave if a second detector is located at a distance 4 r_1 from the source? Assume a spherical wave.

a. 4 I_1
b. 1/4 I_1
c. 16 I_1
d. 1/16 I_1

$I = \frac{P}{A}$

$I_1 = \frac{P_1}{4\pi r_1^2}$

w/ P_1 CONST

$I_1 \propto \frac{1}{r_1^2}$

IF $r_1 \rightarrow 4r_1$

THEN $I_1 \rightarrow \frac{1}{16} I_1$

PRACTICE: You whisper a word at the same time a dropped book hits the floor. The sound from the book dropping reaches the other side of the room first.

a. True.
b. False.

$V_{SOUND} \propto \sqrt{TEMP}$ w/ TEMP CONST... V_S CONSTANT

Sound level intensity

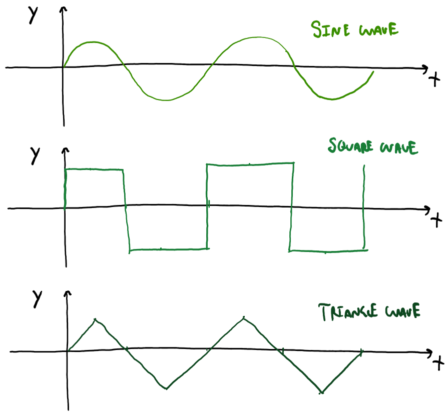
Sound travels as longitudinal pressure waves transporting energy from the source to our ears. The perception of sound to human ears can be characterized by three parameters: pitch, quality, and loudness. All three of these parameters are subjective and vary from person to person. However, they each have connections to actual physical quantities that are not subjective. Pitch is related to the frequency of the source. Quality is related to the shape of the traveling wave. Loudness is related to the intensity of the wave.

Pitch

The pitch of the sound helps us differentiate between the identity of various sources. For example we can tell the difference between a child saying "hi" and an adult saying "hi". The dominate frequency present in the child's voice is a much higher frequency than an adult's dominate frequency.

Quality

The sound produced by a source is almost always a combination of multiple frequencies. The different frequencies present in the sound can have different amplitudes as well. It is then the combined effect of all of these waves that we hear as a sound. Below is an illustration of traveling waves that have a similar periods of amplitude but have different shapes (i.e. different waveforms).



$$\sin(2\pi f_o t)$$

FUNDAMENTAL FREQUENCY

$$\sum_{n=1,3,5,\ldots}^{\infty} \frac{\sin(2\pi n f_o t)}{n}$$

$$\sum_{n=1,3,5,\ldots}^{\infty} (-1)^{\frac{(n-1)}{2}} \frac{\sin(2\pi n f_o t)}{n^2}$$

In general, any complex waveform can be replicated by adding together many sine waves of different frequencies and amplitudes. The quality is how we are able to tell the difference between a clarinet and an oboe playing the same note.

<u>Loudness</u>

The pressure waves that travel to our eardrum fluctuate between a maximum and a minimum value of pressure. Remember pressure differences are associated with some net force, thus our eardrums vibrate back and forth at the same frequency as the source of the pressure waves. So the loudness is related to the pressure amplitude of the wave. Our perception of how loud a sound is also changes with distance from the source. The pressure and distance observation of how we perceive loudness suggests that loudness is proportional to the intensity of the wave. Unfortunately, the range of intensities that humans can hear and tolerate is on the order of 10^{12} W/m^2. It would get really annoying having to always use numbers with orders of magnitude when trying to describe how loud a sound is. We therefore do a clever trick and scale the intensity by logarithmic functions. The resulting loudness scale is referred to as the sound intensity level as shown below..

$$\beta = 10\ \text{dB}\ \log_{10}\left(\frac{I}{I_o}\right)$$

ACTUAL INTENSITY OF SOUND AT A LOCATION FROM THE SOURCE

THRESHOLD OF HEARING

$I_o = 1.0 \times 10^{-12}$ W/m^2

SOUND INTENSITY LEVEL

UNITS "DECIBELS"

Note that the sound intensity level is scaled to our typical human threshold of hearing. We cannot hear any intensity less than the threshold of hearing. Of course this varies between person to person but the value stated above is a good average for humans. Below is the general form if there are multiple sources of sound.

$$\boxed{\beta = 10 \log_{10}\left(\frac{\Sigma I}{I_o}\right)}$$

* IF SPHERICAL SOURCES.. USE $I_1 = \dfrac{P_1}{4\pi r_1^2}$

Logarithmic math review

Before continuing forward it is wise to review some logarithmic math.

Extracting argument of log

$$A = B \log_x\left(\frac{Y}{Z}\right)$$

DIVIDE BY B

$$\frac{A}{B} = \log_x\left(\frac{Y}{Z}\right)$$

RAISE BOTH SIDES TO THE BASE "X"

$$X^{\left(\frac{A}{B}\right)} = X^{\left(\log_x\left(\frac{Y}{Z}\right)\right)}$$

$$X^{\left(\frac{A}{B}\right)} = \frac{Y}{Z}$$

Log subtraction

$$\log_x(Z) - \log_x(Y) = \log_x\left(\frac{Z}{Y}\right)$$

Log addition

$$\log_x(Z) + \log_x(Y) = \log_x(ZY)$$

Expand powers of log argument

$$\log_x(Y^Z) = Z\log_x(Y)$$

PRACTICE: Testing a sound system with several speakers set up so as to simulate a single point source, a consumer noted that she could get as close as 1.2 m with the volume full on before sound hurt her ears (120 dB). What is the intensity of the sound at her location?

a. 1.0 dB
b. 1.0 W/m^2
c. 1.2 W/m^2
d. 120 dB
e. 120 W/m^2

$$\beta = 10\text{ dB}\log_{10}\left(\frac{\Sigma I}{I_o}\right)$$

$$\frac{\beta}{10\text{ dB}} = \log_{10}\left(\frac{\Sigma I}{I_o}\right)$$

$$10^{\beta/10\text{ dB}} = \frac{\Sigma I}{I_o}$$

$$\Sigma I = I_o 10^{\beta/10\text{ dB}}$$

$$I = \left(1\times10^{-12}\frac{W}{m^2}\right)10^{\left(\frac{120}{10}\right)}$$

PRACTICE: A sound is found to have a sound intensity level difference between 10 dB from one moment to the next. What is the ratio of the intensity of the sound at the observer, final over initial (I_f/I_i).

a. 2.0
b. 100
c. 0.01
d. 10
e. 0.1

$$\left.\begin{aligned}\beta_i &= 10\text{ dB}\log_{10}\left(\frac{I_i}{I_o}\right)\\ \beta_f &= 10\text{ dB}\log_{10}\left(\frac{I_f}{I_o}\right)\end{aligned}\right\}\Delta\beta = \beta_f - \beta_i$$

$$\Delta\beta = 10\text{ dB}\log_{10}\left(\frac{I_f}{I_o}\right) - 10\text{ dB}\log_{10}\left(\frac{I_i}{I_o}\right)$$

$$\Delta\beta = 10\text{ dB}\log_{10}\left(\frac{I_f}{I_o}\cdot\frac{I_o}{I_i}\right)$$

$$\Delta\beta = 10\text{ dB}\log_{10}\left(\frac{I_f}{I_o}\right)$$

$$10^{\left(\frac{\Delta\beta}{10}\right)} = \frac{I_f}{I_o}$$

PRACTICE: Listening to a speaker you recorded the sound intensity level to be 97 dB from a 6.3 W source. The source is then amplified, and the new sound intensity level is 107 dB. What is the new power coming from the source?

$$\beta = 10\,dB \log_{10}\left(\frac{I}{I_o}\right)$$

$$\Delta\beta = 10\,dB \log_{10}\left(\frac{I_f}{I_i}\right)$$

$$\Delta\beta = 10\,dB \log_{10}\left(\frac{P_f}{A_f}\cdot\frac{A_i}{P_i}\right)$$

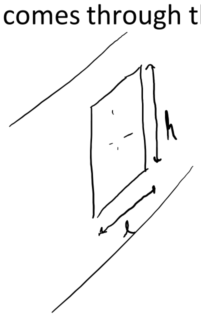
$$\Delta\beta = 10\,dB \log_{10}\left(\frac{P_f}{P_i}\right)$$

SAME LOCATION $A_f = A_i$

$$P_f = P_i\,10^{\Delta\beta/10\,dB} \rightarrow P_i\,10^{(10/10)}$$

$$P_f = 10\,P_i = 63\,W$$

PRACTICE: Sound is coming through an open window whose dimensions are 1.1 m x 0.75 m. The sound intensity level is 95 dB above the pain threshold of human hearing. How much sound energy comes through the window in one hour? Use 120 dB for the pain threshold of human hearing.



$$\beta = 120\,dB + 95\,dB$$

$$\beta = 10\,dB \log_{10}\left(\frac{I}{I_o}\right)$$

$$I = I_o\,10^{(\beta/10\,dB)}$$

$$\frac{P}{A} = I_o\,10^{(\beta/10\,dB)}$$

$$\frac{E}{\Delta t A} = I_o\,10^{(\beta/10\,dB)}$$

$$E = I_o A \Delta t\,10^{(\beta/10\,dB)} \approx 9.4\times10^{12}\,J$$

QUESTIONS FOR DISCUSSION:

(1) If the intensity is doubled at the same location in space, is the sound intensity also doubled?
(2) Your digital sound intensity level reader is showing 0 dB in the room you are in. Does this mean there is no sound? Explain your answer.