General two source interference

Select LEARNING OBJECTIVES:

- Understand what coherence is and what the consequences are if two sources are not coherent.
- Be able to determine path length difference (PLD) for various geometries.
- Apply general two source interference conditions to predict the positions of maximum constructive or destructive interference from two waves.

TEXTBOOK CHAPTERS:

- Boxsand :: Superposition of Waves

WARM UP: Explain how beat frequencies and guitars are related. What might you use beat frequencies for on a guitar?

When we studied standing waves, a wave was generated at one end of the system and reflected off a boundary at the other end. The reflected wave then interfered with any other incoming waves from the source. This system set up the proper conditions to allow us to study standing waves on strings and standing sound waves in tube. Instead of studying wave interference between a single source and the wave reflections off of boundaries, we can study wave interference from multiple sources. In this lecture we will explore the interference resulting from two coherent sources of the same frequency.

Next time you are in a movie theater that is mostly empty, try moving around to different locations and take note of how loud or soft the sound is at each location. You might be able to observe that some locations sound louder than others. This effect is a direct consequence of wave interference. Multiple sources are emitting traveling sound waves which interfere when they meet at your ear. The mathematical model that would describe a movie theater's wave interference pattern is quite complex; sound is being emitted from many sources at different frequencies bouncing around the room off of chairs, walls, people, and the list goes on. The physics behind the model is the same as the physics we will study in this lecture. Interfering waves are modeled mathematically by the superposition principle.

Before we jump right into the types of problems we can tackle, we must first list a few constraints that must be applied in order to observe perfectly constructive and destructive interference patterns.

Constraints

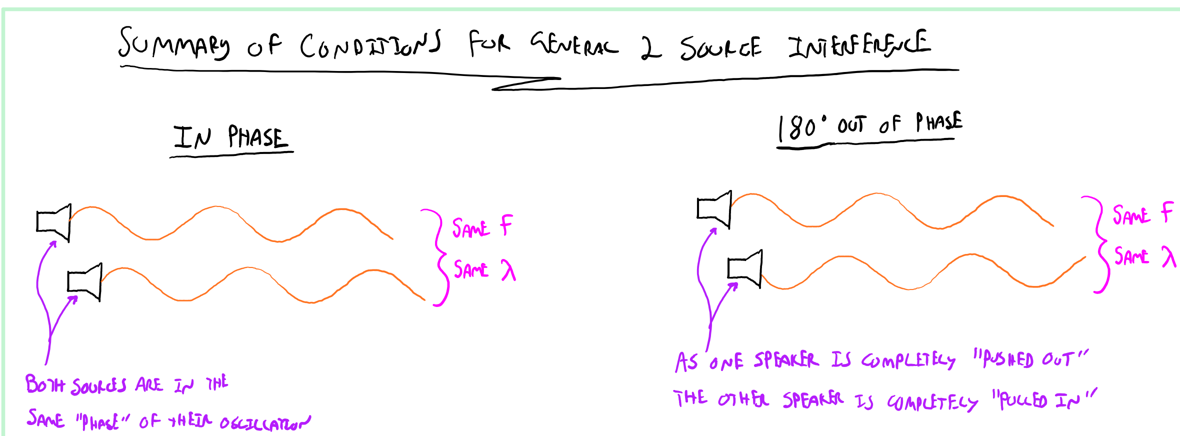
- Coherent sources.
  - The phase relationship between the two sources must remain constant in time.

    Imagine two speakers. They produce sound waves with cones that vibrate back and forth. If the cones of both speakers are in phase and coherent, then the cone from one speaker will be pushed out to its maximum position and pulled back to its minimum position at the same time as the cone form the other speaker. Put both your hands out in front of yourself with your palms facing away from your body. Now move your arms back and forth such that your palms are always side by side. Your two hands are in phase because they reach their maximum and minimum locations at the same time, and coherent because that relationship never changes in time.

    Now imagine two speakers again. This time, let one speaker's cone be pushed out to its maximum position at the same time the other speaker is pulled back to its minimum position. Put both your hands out in front of yourself with your palms facing away from your body. Extend your right hand as far forward as possible and leave it there. Now move your left hand back against your body. Starting from this position, move both your arms so that when your right hand is against your body, your left hand is fully extended. Repeat this process and you are representing two speakers that are 180° out of phase but still coherent because they are at all times 180° of phase.

For this class, we will mathematically model sources that are only in phase or 180° our of phase. But in general they phase that each speaker is in with respect to the other can be anything, so long as that relative phase remains constant in time (coherent sources). Below the next condition is a pictorial representation of two sources that are in phase and 180° out of phase using sound pressure waves to visualize the traveling waves being emitted by each. Note that the speakers do not have to be in the same location in space to be in phase or 180° out of phase.

- Sources have the same frequency and wavelength.
    - We studied what happens when two waves have different frequencies. The result was temporal interference producing a beating type sound with a frequency equal to the magnitude of the difference between the two frequencies. Thus if our two sources have different frequencies, any one location in space will constantly fluctuate between constructive and destructive. We therefore limit our studies to sources with the same frequency, and since they travel through the same media at the same speed, they will also have the same wavelength.



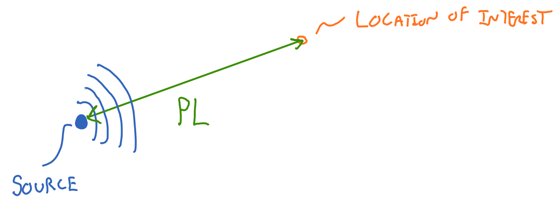
General two source interference mathematical model

We now are able to start the mathematical modeling of general two source interference. The most important line of questioning to ask yourself before you begin any two source interference problem is:

1. What is my Path Length Difference (PLD)?
2. What condition am I looking for (constructive or destructive)?
3. Are my sources in phase or 180° out of phase?

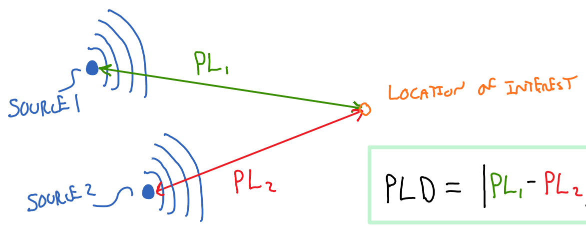
Path Length (PL)

The Path Length (PL) is the distance from one source to your location of interest that you wish to observe an interference effect at. Remember that distance is a scalar. Thus, you would grab a ruler and measure this distance and report back a single value (e.g. 42 meters). There is no direction associated with a PL, it is not a vector. Below is a physical representation of a PL.



Path Length Difference (PLD)

The Path Length Difference (PLD) is the difference between two Path Lengths. Below is a physical representation of how to calculate a PLD.



$$PLD = |PL_1 - PL_2|$$

EXAMPLE: IF $PL_1 = 10\ m$

AND $PL_2 = 8\ m$

THEN $PLD = 2\ m$

Mathematical model

- 2 COHERENT SOURCES W/ SAME f IN PHASE

  CONSTRUCTIVE: $PLD = m\lambda$ WHERE $m = 0, 1, 2, 3, 4, \ldots$

  DESTRUCTIVE: $PLD = (m + \frac{1}{2})\lambda$ WHERE $m = 0, 1, 2, 3, 4, \ldots$

- 2 COHERENT SOURCES W/ SAME f 180° OUT OF PHASE

  CONSTRUCTIVE: $PLD = (m + \frac{1}{2})\lambda$ WHERE $m = 0, 1, 2, 3, 4, \ldots$

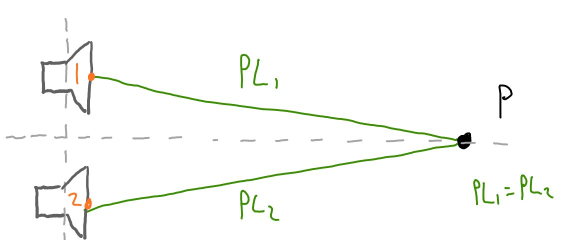
  DESTRUCTIVE: $PLD = m\lambda$ WHERE $m = 0, 1, 2, 3, 4, \ldots$

General 2 source interference is an example of spatial interference, just like standing waves. If the conditions are just right (see the equations above), then there will be locations in space where complete constructive and complete destructive interference occurs. This means that if you were standing at a completely destructive location, you would hear no sound at all coming from the two speakers, and would have to move to another location in order to hear something (hence spatial interference; depends on where you are in space).

PRACTICE: When considering the interference in space between two sources, what is the most important question to ask?

PLD — PATH LENGTH DIFFERENCE

PRACTICE: Two speakers in phase are shown below. If you stand at the location labeled "P", will you be at a maximum constructive, maximum destructive, or somewhere in-between?

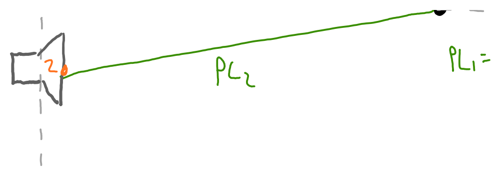


$PLD =$ CONST. (IN PHASE)

$PLD = m\lambda$

$|PL_1 - PL_2| = m\lambda$

$PL_1 = PL_2$: $0 = m\lambda$



$PL_1 = PL_2$ → $|PL_1 - PL_2| = m\lambda$, $0 = m\lambda$

CONSTRUCTIVE

If speaker 2 is now 180 degrees out of phase, what will the interference be at location P?

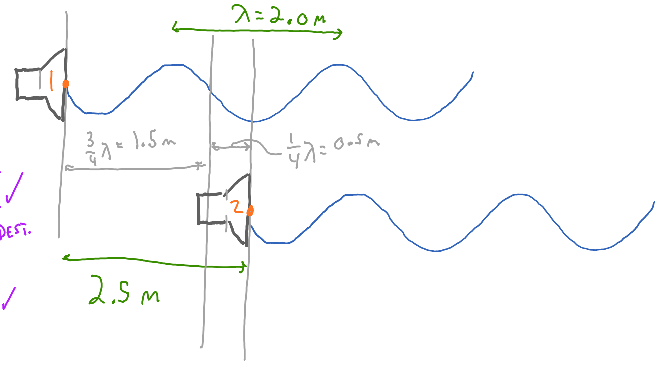
$PLD \stackrel{?}{=} \text{CONST}$) 180° OUT OF PHASE
$PLD \stackrel{?}{=} (m+\frac{1}{2})\lambda$
$0 \neq (m+\frac{1}{2})\lambda$

$PLD = \text{DEST.}$) 180° OUT OF PHASE
$PLD = m\lambda$
$0 = m\lambda$ ✓

INTERFERENCE WILL BE DESTRUCTIVE

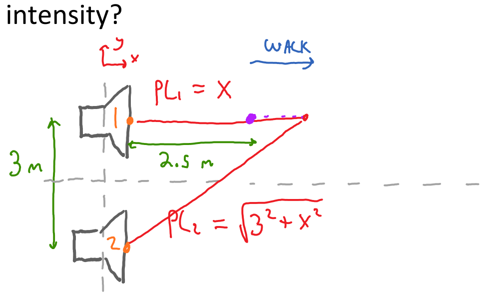
PRACTICE: Two loudspeakers emit (λ=2.0 m) waves are in phase with each other. Speaker 2 is 2.5 meters in front of speaker 1. What, if anything, must be done to cause constructive interference between the two waves?

(a) Move speaker 1 forward (to the right) 0.5 m. 2m ✓
(b) Move speaker 1 backward (to the left) 1.0 m. 3.5 m
(c) Move speaker 1 forward (to the right) 1.0 m. 1.5 m
(d) Move speaker 2 backwards (to the left) 0.5 m. 2m ✓
(e) Move speaker 2 forward (to the right) 0.5 m. 3 m DEST.
(f) Move speaker 2 forward (to the right) 1.0 m. 3.5 m
(g) Move speaker 2 forward (to the right) 1.5 m. 4 m ✓
(h) Nothing. The situation already causes constructive interference.



$PLD = \text{CONST.}$) IN PHASE
$PLD = m\lambda$ $m = 0, 1, 2, 3 \ldots$
$PLD = 0, 1\text{ m}, 2\text{ m}, 3\text{ m}, 4\text{ m}$

PRACTICE: You are standing 2.50 meters directly in front of one of the two loudspeakers shown in the figure below. They are 3.00 meters apart and both are playing a 686 Hz tone in phase. As you begin to walk directly away from the speaker, at what distances from the speaker do you hear a minimum sound intensity?



$PLD = \text{DEST.}$) IN PHASE
$|PL_2 - PL_1| = (m+\frac{1}{2})\lambda$

$V_{sound} = f\lambda$
$\lambda = \frac{V_{sound}}{f}$

$\sqrt{3^2+x^2} - x = (m+\frac{1}{2})\lambda$

$\sqrt{3^2+x^2} = (m+\frac{1}{2})\lambda + x$) SQUARE BOTH SIDES

$3^2+x^2 = \left((m+\frac{1}{2})\lambda + x\right)^2$

$3^2 + x^2 = x^2 + 2x(m+\frac{1}{2})\lambda + (m+\frac{1}{2})^2\lambda^2$

$2x(m+\frac{1}{2})\lambda = 3^2 - (m+\frac{1}{2})^2\lambda^2$

m	x
0	17.88 m

M	X
0	17.88 m
1	5.63 m
2	2.98 m
3	1.70 m
4	0.875 m

@ THESE LOCATIONS SINCE Y/W STARTED AT X= 2.5 m

$$2x(m+\tfrac{1}{2})\lambda = 3^2 - (m+\tfrac{1}{2})^2\lambda^2$$

$$X = \frac{3^2 - (m+\tfrac{1}{2})^2\lambda^2}{2(m+\tfrac{1}{2})\lambda}$$

QUESTIONS FOR DISCUSSION:

(1) With your new knowledge of 2 source interference, describe how active noise canceling headphones work?