Spring-mass oscillator

Select LEARNING OBJECTIVES:

- Apply the sinusoidal equations of motion (position, velocity, acceleration) to SHO - i.e. fit them to the system using the initial conditions.
- Understand the system specific (mass-spring) physical quantities.

TEXTBOOK CHAPTERS:

- Boxsand :: [Springs and pendulums]()

WARM UP: Would the force below give rise to simple harmonic oscillation?

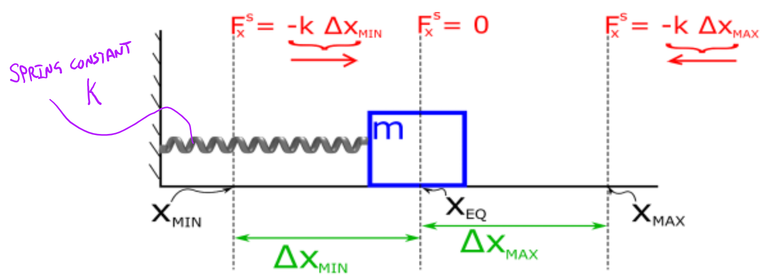
$$F_x = -7.5\,\Delta x$$

In this lecture we will begin to quantify the mass-spring system which is a case study of simple harmonic motion.

Assumptions

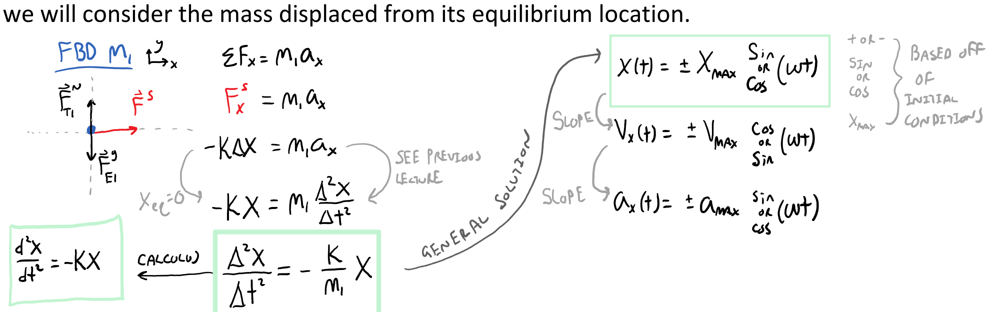
- No friction between surface of mass and table.
- No air resistance.
- The spring is ideal (i.e. it's mass is negligible and it obeys Hooke's law for all Δx).

Below is a basic sketch to help identify relevant quantities for a mass-spring oscillator.



Equation of motion

Recall we can build equations of motion by applying Newton's 2nd law. To do this for the spring-mass system, we will consider the mass displaced from its equilibrium location.



- Mass oscillates back and forth between x_{max} and x_{min} with...
  - $|x_{max}| = |x_{min}|$
  - Angular frequency $\equiv \omega = \sqrt{\frac{k}{m}}$ $\qquad \omega = 2\pi f$
  - Amplitude $\equiv A = |x_{max}|$
  - Period $\equiv T = 2\pi\sqrt{\frac{m}{k}}$
  - Frequency $\equiv f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$
  - Max velocity $\equiv v_{max} = \omega\, x_{max}$
  - Max acceleration $\equiv a_{max} = \omega\, v_{max} = \omega^2 x_{max}$
  - Total energy $\equiv E_{tot} = KE + U^s = U^s_{max} = KE_{max}$

$$E_{tot} = \frac{1}{2} m v_1^2 + \frac{1}{2} k \Delta x_1^2 = \frac{1}{2} k A^2 = \frac{1}{2} m v_{max}^2$$

PRACTICE: An object moves with simple harmonic motion. If the amplitude and the period are both doubled, the object's maximum speed is

(a) quartered
(b) halved
(c) unchanged
(d) doubled
(e) quadrupled

$V_{MAX} = \omega X_{MAX}$

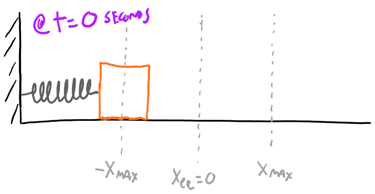
$V_{MAX} = \frac{2\pi}{T} X_{MAX}$

$V_{MAX} \propto \frac{A}{T}$

IF $A \to 2A$
AND $T \to 2T$
THEN $V_{MAX} \to \frac{2}{2} \frac{A}{T}$

SO V_{MAX} IS UNCHANGED.

PRACTICE: A mass on a frictionless surface is connected to a spring and pulled to the left 15 cm. It is released from rest at t = 0 s and proceeds to make 20 oscillations in 30 seconds. What is the period of oscillation?



$T = \frac{\text{TIME}}{\text{ONE OSCILLATION}} = \frac{30\ s}{20} = \frac{3}{2}\ s$

What is the frequency of oscillation?

$F = \frac{1}{T} = \frac{1}{3/2\ s} = \frac{2}{3} \frac{1}{s}$ OR $\frac{2}{3}$ Hz

What is the masses maximum speed?

$V_{MAX} = \omega X_{MAX}$

$= 2\pi f X_{MAX}$

$= 2\pi \left(\frac{2}{3}\right)(0.15) \frac{m}{s} \approx 0.628\ m/s$

What are the mass's position and velocity at t = 0.80 s?

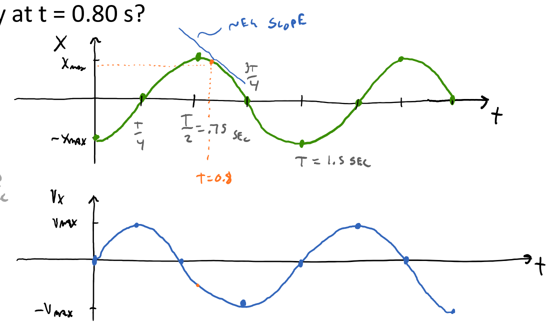
MODEL: $X(t) = \pm X_{MAX} \ \begin{matrix} \sin \\ \text{or} \\ \cos \end{matrix} (\omega t)$

$X(t) = -X_{MAX} \cos(\omega t)$ $\quad \omega = 2\pi f$

$X(t) = -0.15 \cos(4.189\, t)$ METERS $\quad \omega \approx 4.189\ \frac{RAD}{SEC}$

$X(t = 0.8) = -0.15\ m \cos\left(4.189\ \frac{RAD}{SEC}(0.8\ SEC)\right)$

$X(t = 0.8) \approx 0.147\ m$



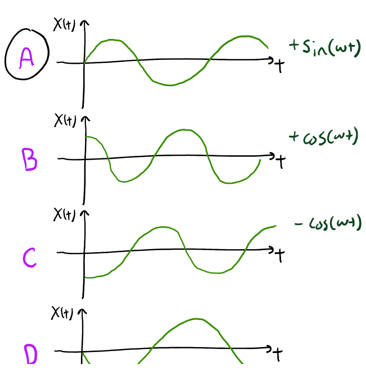
$V_x(t) = \omega X_{MAX} \sin(\omega t)$

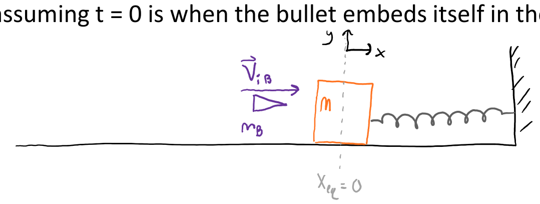
$V_x(t) = V_{MAX} \sin(4.189\, t)$

$V_x(t) = 0.628\ \frac{m}{s} \sin\left(4.189\ \frac{RAD}{SEC}\, t\right)$

$V_x(t = 0.8) \approx -0.131\ m/s$

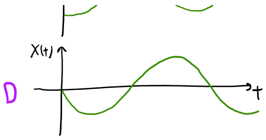
PRACTICE: A 16.2 g bullet with an initial speed of 870 m/s embeds itself into a 40.0 kg block, which is attached to a horizontal spring with a force constant of 1010 N/m. Which of the following position as a function of time plots could represent the motion of the bullet + mass system assuming t = 0 is when the bullet embeds itself in the block?





Which physics concepts could be used to determine the maximum speed of the resulting oscillatory motion?

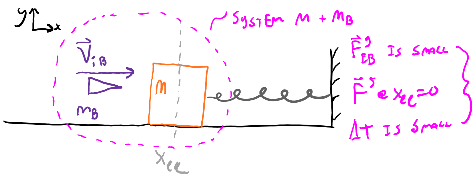
COLLISION W/ $\Sigma \vec{F}_{EXT} \approx \vec{0}$ SO ... CONSERVATION OF MOMENTUM



motion?

COLLISION w/ $\sum \vec{F}_{EXT} \approx \vec{0}$ So ... CONSERVATION OF MOMENTUM

What is the maximum speed of the resulting oscillatory motion.



$$\vec{P}_{sys\,i} = \vec{P}_{sys\,f}$$

$$P_{ixB} + P_{ixm} = P_{F\,B+m}$$

$$m_B V_{iBx} + 0 = (M_B + M) V_f$$

$$V_f = \frac{m_B}{(M_B + M)} V_{iBx} \approx 0.352 \text{ m/s}$$

THIS IS SPEED OF $M_B + M$ RIGHT AFTER COLLISION @ $X_{eq} = 0$ SO THIS IS ALSO V_{MAX}

What is the maximum compression of the spring?

SYSTEM: BULLET + BLOCK + SPRING

$$KE_i + \cancel{U^g_i} + U^s_i + \cancel{E^{TH}_i} + \cancel{W^{NC}_{INT}} + \cancel{W_{EXT}} = KE_f + \cancel{U^g_f} + U^s_f + \cancel{E^{TH}_f}$$

$$KE_i + \cancel{U^s_i}^{\,0} = \cancel{KE_f}^{\,0} + U^s_f$$

$$\frac{1}{2}(M_B + M) V_i^2 = \frac{1}{2} K \Delta X_{max}^2$$

$$\Delta X_{max}^2 = \frac{(M_B + M) V_i^2}{K}$$

$$\Delta X_{max} = \sqrt{\frac{(M_B + M)}{K}\left(\frac{M_B}{M_B + M} V_{iBx}\right)^2} \approx 0.070 \text{ m}$$

OR 7 cm

What is the period of the resulting oscillation?

GENERAL

$$\omega = \frac{2\pi}{T}$$

$\therefore T$?

MASS - SPRING SYSTEM

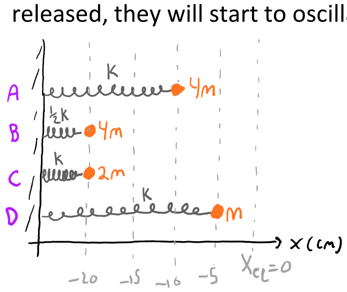
$$\omega = \sqrt{\frac{K}{m}}$$

w/ $m = M_B + M$

$$\omega = \sqrt{\frac{K}{M_B + M}} = \frac{2\pi}{T}$$

$$T = 2\pi \sqrt{\frac{M_B + M}{K}} \approx 1.25 \text{ SECONDS}$$

PRACTICE: Four springs have been compressed from their equilibrium position at x = 0 cm. When released, they will start to oscillate. Rank in order the maximum speeds of the oscillations.



$V_{max} = \omega X_{max}$ $\omega = \sqrt{\frac{K}{m}}$

$V_{max} = \sqrt{\frac{K}{m}} X_{max}$ FROM WHERE?

$U^s_{max} \rightarrow KE_{max}$

$\frac{1}{2} K X_{max}^2 = \frac{1}{2} m V_{max}^2$

$V_{max}^2 = \frac{K}{m} X_{max}^2$

$V_{max} = \sqrt{\frac{K}{m}} X_{max}$

$V_{max} = \omega X_{max}$

a) $V_A = \sqrt{\frac{K}{4m}}\, 10 = \frac{10}{2}\sqrt{\frac{K}{m}} = 5\sqrt{\frac{K}{m}}$

b) $V_B = \sqrt{\frac{K}{2(4)m}}\, 20 = \frac{20}{\sqrt{8}}\sqrt{\frac{K}{m}} \approx 7.07\sqrt{\frac{K}{m}}$

c) $V_C = \sqrt{\frac{K}{2m}}\, 20 = \frac{20}{\sqrt{2}}\sqrt{\frac{K}{m}} \approx 14.1\sqrt{\frac{K}{m}}$

D) $V_D = \sqrt{\frac{K}{m}}\, 5 = 5\sqrt{\frac{K}{m}}$

...SO...

c) $v_C = \sqrt{\frac{k}{2m}}\,20 = \frac{20}{\sqrt{2}}\sqrt{\frac{k}{m}} \approx 14.1\sqrt{\frac{k}{m}}$

D) $v_D = \sqrt{\frac{k}{m}}\,5 = 5\sqrt{\frac{k}{m}}$

...so...

$v_A = v_D < v_B < v_C$

QUESTIONS FOR DISCUSSION:

(1) A mass-spring system oscillates in the vertical direction with a frequency *f*. If this system is taken into an elevator which slowly accelerates upwards at a constant rate, what will happen to the frequency of oscillation?