

Standing waves on strings and standing sound waves in pipes

Select LEARNING OBJECTIVES:

- Be able to differentiate between a general standing wave and a standing wave resonance.
- Be able to determine the boundary conditions (symmetric or antisymmetric).
- Be able to sketch resonant modes.
- Be able to determine the locations of nodes for both pressure and displacement for a sound in a tube.
- Understand what it means to have a series of equations that solve the system - what the m-value represents.
- Be able to solve for the resonances of a system given information about its geometry.
- Have an appreciation for (and ability to identify) all the standing waves around us and in nature.

TEXTBOOK CHAPTERS:

- Boxsand :: [Standing wave resonance](#)

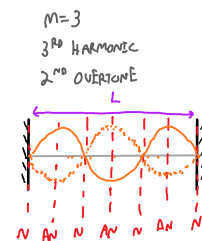
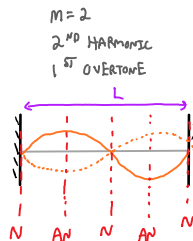
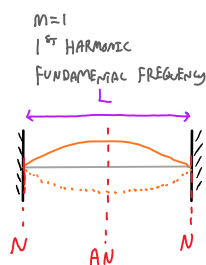
WARM UP: When analyzing standing waves, what feature of the wave should you identify first before applying any mathematical description?

Now that we have a better idea of how standing waves are created and how to describe them with proper terminology, let's look at two case studies: standing waves on a string, and standing sound waves in a tube.

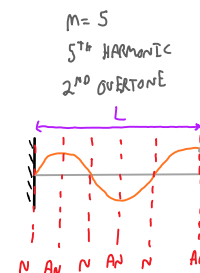
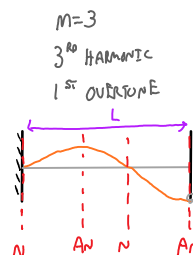
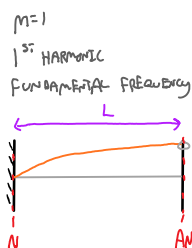
Standing waves on a string

Nodes are labeled N, and antinodes are labeled AN. The orange standing wave represents displacement of the string from the equilibrium position shown in gray.

Symmetric (fixed-fixed)



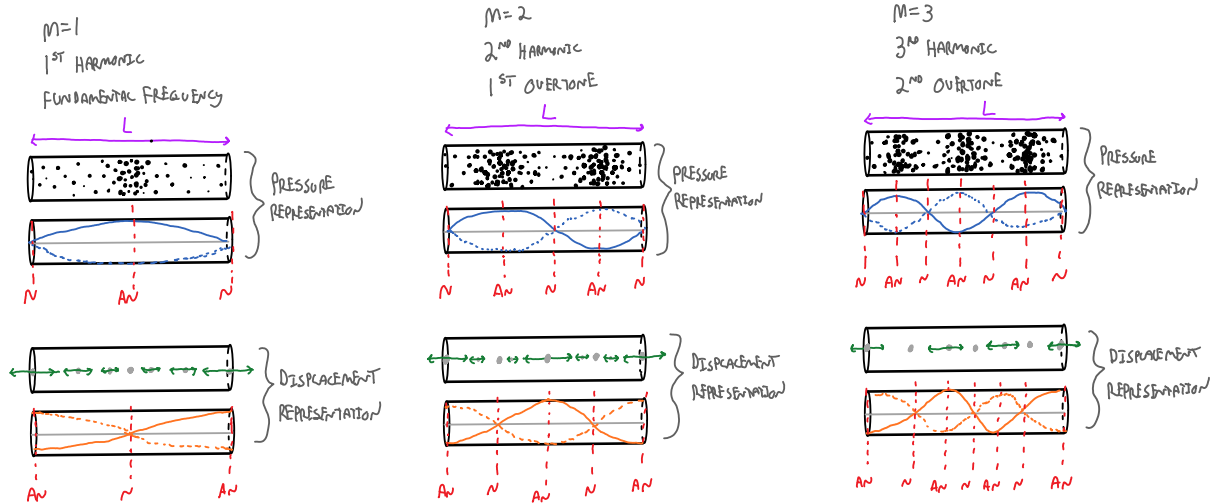
Anti-symmetric (fixed-free)



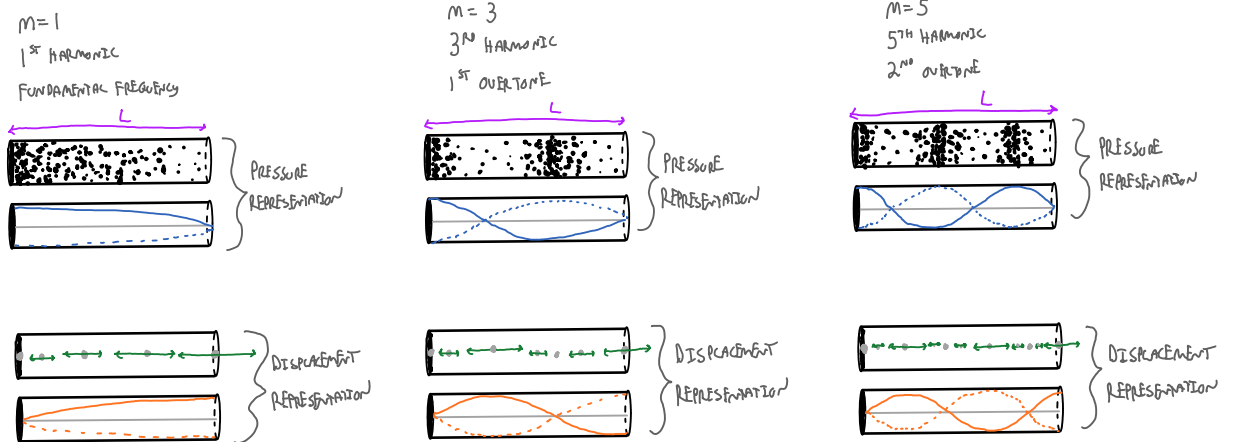
Standing sound waves in pipes

Nodes are labeled N, and antinodes are labeled AN. The orange standing wave represents displacement of the air particles from their equilibrium position. The blue standing wave represents the pressure displacements from its equilibrium pressure (i.e. atmospheric).

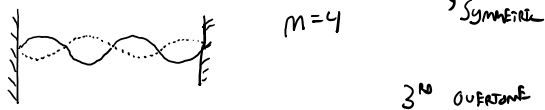
Symmetric (open-open)



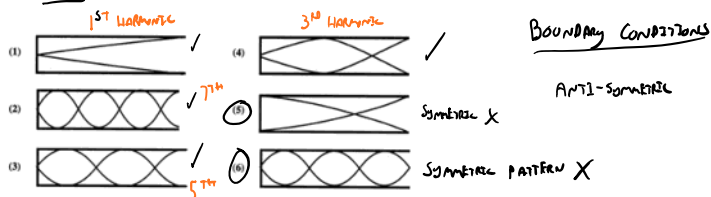
Antisymmetric (closed-open)



PRACTICE: Sketch the 4th harmonic of a fixed-fixed standing wave string. What overtone is this?




PRACTICE: The figure shows representations of standing waves of sound in six identical organ pipes. Which figures do not illustrate a possible resonant situation?



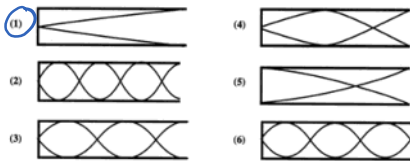
Which of the pipes are resonating in their third harmonic?

Only odd harmonics allowed for Anti-symmetric...
 $M=3$ OR 1st OVERTONE

What must the figures be representing?

- (a) Pressure. *NODES AND ANTI-NODES OP. OF DISP.*
- (b) Displacement. *OPEN END ALLOWS FOR MAXIMUM MOVEMENT* 
- (c) Energy. *CONSTANT*
- (d) Sound intensity level. *GREATEST WHEN CHANGE IN PRESSURE GREATEST (SAME AS PRESSURE REPRESENTATION)*
- (e) Temperature. *X*
- (f) Volume. *X*

PRACTICE: The figure shows representations of standing waves of sound in six identical organ pipes. Which one of the pipes emits sound with the lowest frequency?



$$f_m = \frac{mV}{4L}$$

$v = \text{CONST}$
 $L = \text{CONST}$

$f_m \propto m$ Lowest $f \rightarrow$ Smallest m
 $m=1 \dots$ FUNDAMENTAL FREQ.

PRACTICE: You wish to make an organ for your home but are limited on the length of tubes by the height of your ceiling. Which configuration will give you the lowest possible frequency for the same given length?

- (a) Open-open
- (b) Open-closed
- (c) Closed-closed
- (d) All will have the same fundamental frequency.

ANTI-SYMMETRIC

$$f_m = \frac{mV}{4L}$$

SYMMETRIC

$$f_m = \frac{mV}{2L}$$

$V = \text{CONST}$
 $m = 1$

$$f_1 = \frac{1V}{4L}$$

$$f_1 = \frac{1V}{2L}$$

$L = \text{CONST}$

$$\frac{1}{4}V < \frac{1}{2}V$$

PRACTICE: A guitar string is stretched between two fixed points 0.640 meters apart and that length of string has a mass of 2.86 g. If the tension is set to 88.6 N, what frequency sound will be heard when it is plucked?

- 1. 400 Hz
- 2. 200 Hz
- 3. 330 Hz
- 4. 260 Hz
- 5. 110 Hz

SYMMETRIC

$$f_m = \frac{mV}{2L}$$

STRINGS

$$V = \sqrt{\frac{FT}{\mu}}$$

GENERAL

$$V = f\lambda$$

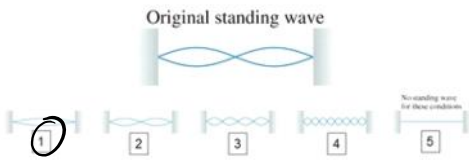
$$f_m = \frac{m}{2L} \sqrt{\frac{FT}{\mu}}$$

$m=1$ FUNDAMENTAL FREQ.

What will the wavelength of the 3rd harmonic be?

SYMMETRIC: $\lambda_m = \frac{2L}{m} \rightarrow m=3$
 $\approx 0.472 \text{ m}$

PRACTICE: Below is a string vibrating in a normal mode. If the tension in the string is quadrupled while the frequency and length of the string are held constant. Which standing wave pattern is produced?



SYMMETRIC
 $f_m = \frac{mV}{2L}$

SWAVE
 $v = f\lambda$

STRINGS
 $v = \sqrt{\frac{FT}{\mu}}$

$\sqrt{\frac{FT}{\mu}} = f\lambda$

w/ $f = \text{CONST}$
 $\mu = \text{CONST}$

$\lambda \propto \sqrt{FT}$

If $FT \rightarrow 4FT$
 $\lambda \rightarrow 2\lambda$

PRACTICE: An open-closed tube of air supports standing waves at frequencies of 225 Hz and 375 Hz, and at no frequencies between these two. Which of the following are also resonances of the same tube.

1. 75 Hz
2. 100 Hz
3. 150 Hz
4. 375 Hz
5. 475 Hz
6. 500 Hz
7. 525 Hz
8. 675 Hz

ANTI SYMMETRIC

$f_m = \frac{mV}{4L}$ $m=1,3,5,7,\dots$

$\Delta f = f_{m+2} - f_m$

$\Delta f = \frac{(m+2)V}{4L} - \frac{mV}{4L}$

$\Delta f = \frac{2V}{4L}$

$\Delta f = 2\left(\frac{V}{4L}\right)$

$\Delta f = 2f_1$

$\Delta f = 375 - 225 = 150 \text{ Hz}$

$150 \text{ Hz} = 2f_1$

$f_1 = 75 \text{ Hz}$

$f_m = mf_1$

$m=1,3,5,7,\dots$

$f_3 = 225 \text{ Hz}$

$f_5 = 375 \text{ Hz}$

$f_7 = 525 \text{ Hz}$

$f_9 = 675 \text{ Hz}$

PRACTICE: An organ pipe open at one end is vibrating in its 5th harmonic with a frequency of 748 Hz. The length of the pipe is 0.602 m. Determine the speed of sound in air in the pipe.

ANTI SYMMETRIC

$f_m = \frac{mV}{4L}$

$f_5 = \frac{5V}{4L}$

$V = \frac{4Lf_5}{5} = 360 \text{ m/s}$

$m=5$

$f_5 = 748 \text{ Hz}$

QUESTIONS FOR DISCUSSION:

- (1) A string/sound wave is vibrating in a normal mode. Question here... something something something....boundary conditions!