Standing waves resonance

Select LEARNING OBJECTIVES:

- Be able to differentiate between a general standing wave and a standing wave resonance.
- Be able to determine the boundary conditions (symmetric or antisymmetric).
- Be able to sketch resonant modes.
- Be able to determine the locations of nodes for both pressure and displacement for a sound in a tube.
- Understand what it means to have a series of equations that solve the system - what the m-value represents.
- Be able to solve for the resonances of a system given information about its geometry.
- Have an appreciation for (and ability to identify) all the standing waves around us and in nature.

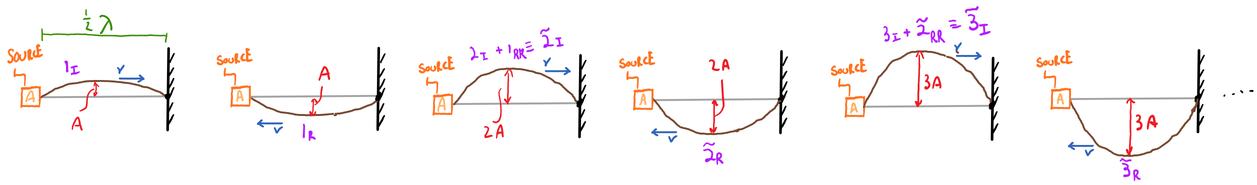
TEXTBOOK CHAPTERS:

- Boxsand :: [Standing wave resonance]

WARM UP: What are some areas of your life influenced by superposition of waves?

Now that we know traveling waves can interfere with each other, let's study a special case of interference with two waves traveling in opposite directions with the same frequency and wavelength in one dimension. As we have seen, we can create two traveling waves moving in opposite directions by letting incident waves reflect off a boundary. The reflected waves then interfere with all other incident waves traveling towards the boundary.

Let's first consider a scenario where you have a vibrating source sending traveling waves down a string which is fixed at the other end. Let the distance between the source and the fixed end be one half of a wavelength as seen below.



The source produces the first incident wave, 1_I, which travels to the right and reflects off a fixed end. The reflected wave, 1_R, picks up a π-phase shift because of the hard boundary. This reflected wave then reflects off the source and again picks up a π-phase shift (source is assumed to be a fixed end since it is a heavy object and its vertical displacements are small). I labeled the reflected wave off of the source as IRR because this is the second time this wave have gone a reflection. As the wave IRR is being reflected by the source, the source also produces another incident wave, 2_I. Now we have two waves in the same place at the same time, so by superpostion, $2_I + 1_{RR}$, the new incident wave traveling to the right from the source is labeled 2_I. This process repeats again and again, each time a wave is created and travels a distance 2 L, energy is added to that wave from the source, increasing the amplitude each time by A. In general, this behavior is true if 2 L is any integer multiplue of the wavelength. Mathematicall this is written as $2\,L = m\,\lambda$, where m is the integer (1, 2, 3, 4, 5…).

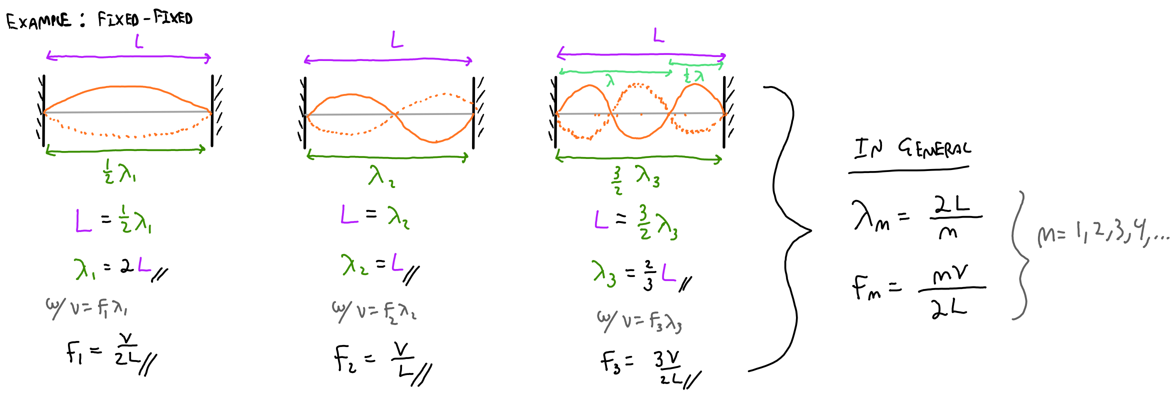
Something should be unsettling about this. When strings vibrate in a fashion as described above, we don't see the amplitude of the string constantly increasing. What is going on here, what did we miss? Well we ignored the energy of the string which is lost to the environment through collisions and thermal energy of the string. Thus, in general, the process described above will occur until the energy supplied to the string by the source matches the energy lost; the system will reach a steady state (i.e. the energy is constant).

Before we look at two specific case studies of standing waves, let's first look at general boundary conditions and how they affect the resulting physical phenomena that occurs.

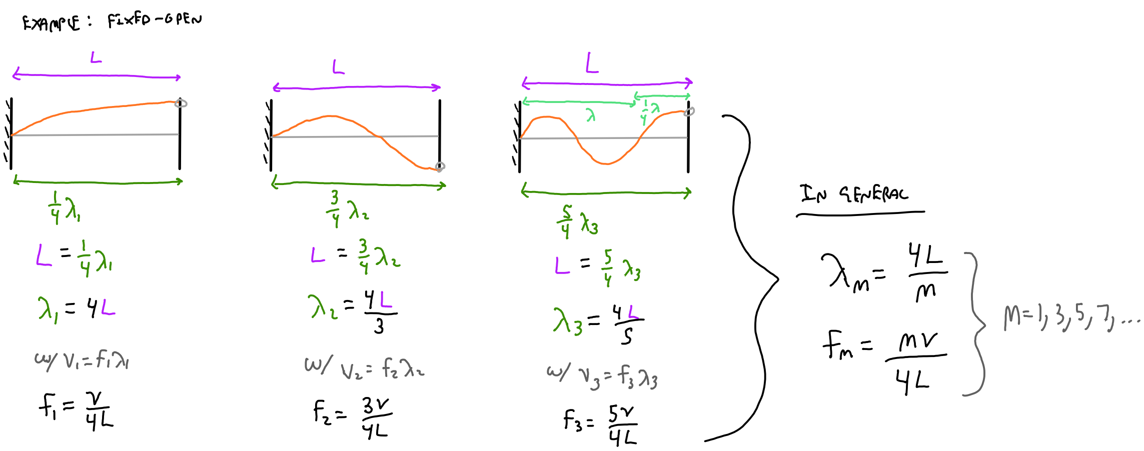
Boundary conditions

The discussions in this section use a general sinusoidal wave shape which represents a quantity of a standing wave. For example, the max values of the sinusoidal wave can correspond to pressure, displacement, electric field strength, etc.. Thus this is discussion is general and applies to all one dimensional standing waves. As discussed in the introduction, if the frequency of your source is just right, a standing wave will form: two traveling waves will interfere to produce a wave which appears to not move at all. Below is a very important physical representation of how to construct general conditions for standing waves based off of boundary conditions.

Symmetric



Anti-symmetric



Study the above carefully. I suggest not remembering the general forms for frequency and wavelength of standing waves, rather remember the constraints that boundary conditions enforce so that you can construct the general forms when needed. With that warning, below is a summary for any symmetric or anti-symmetric systems.

SYMMETRIC

$$\lambda_m = \frac{2L}{m}$$

$$f_m = \frac{mv}{2L} = mf_1 \quad m = 1, 2, 3, 4, \ldots$$

$$\Delta f = f_{m+1} - f_m = f_1$$

ANTI-SYMMETRIC

$$\lambda_m = \frac{4L}{m}$$

$$f_m = \frac{mv}{4L} = mf_1 \quad m = 1, 3, 5, 7, \ldots$$

$$\Delta f = f_{m+2} - f_m = 2f_1$$

Terminology

Often times when we describe standing waves we use new terminology. Below is the working definitions for this class.

Normal modes (m): When a system is vibrating in a standing wave pattern, we often describe this situation by saying that the system is vibrating in a normal mode. The normal mode number is represented by "m", which takes on integer values. Be careful, only odd integers are allowed in anti-symmetric cases.

Fundamental frequency (f_1): The first allowed standing wave in either symmetric or anti-symmetric cases. The normal mode m=1 for the fundamental frequency. Note that all other normal modes of vibration are an integer multiple of this fundamental frequency.

Overtone: In general, the other allowed frequencies of the normal modes of vibration are referred to as overtones.

Harmonic: The allowed integer multiple of the fundamental frequency for standing waves. Basically, this is the normal mode number m. For symmetric cases, the harmonics are all the integers (1,2,3,4…), for anti-symmetric cases only odd harmonics are allowed (1,3,5,7,…). *Note: overtones and harmonics are just "convenient" ways to refer to standing waves, some older text use a different definition of harmonics.

Node: Locations of perfect destructive interference. These show up as the points along the standing wave that are at the equilibrium location.

Antinode: Locations of perfect constructive interference. These show up as the points along the sanding wave that oscillate with maximum amplitude.

Mathematical representation

For fun, let's look at the mathematical way to represent standing waves. We know that standing waves form when two waves with the same frequency and wavelength traveling in opposite directions interfere with each other. So let's begin by writing down the mathematical representation for two individual waves traveling in opposite directions. Remember that this is a general discussion, thus D(x,t) can represent pressure, displacement, etc…

WAVE TRAVELING TO THE RIGHT

$$D_1(x,t) = D_{max}\,\sin(kx - \omega t)$$

WAVE TRAVELING TO LEFT

$$D_2(x,t) = D_{max}\,\sin(kx + \omega t)$$

When the waves meet and interfere, we can invoke superposition to describe the resulting wave that we observe.

$$D_{TOTAL}(x,t) = D_1(x,t) + D_2(x,t)$$

$$D_{TOTAL}(x,t) = D_{max}\,\sin(kx - \omega t) + D_{max}\,\sin(kx + \omega t)$$

$$D_{TOTAL}(x,t) = D_1(x,t) + D_2(x,t)$$

$$D_{TOTAL}(x,t) = D_{max}\sin(kx - \omega t) + D_{max}\sin(kx + \omega t)$$

$$D_{TOTAL}(x,t) = D_{max}\left(\sin(kx-\omega t) + \sin(kx+\omega t)\right)$$

TRIG IDENTITIES

$$D_{TOTAL}(x,t) = D_{max}\left(\sin(kx)\cos(\omega t) - \cos(kx)\sin(\omega t) + \sin(kx)\cos(\omega t) + \cos(kx)\sin(\omega t)\right)$$

$$\boxed{D_{TOTAL}(x,t) = 2D_{max}\sin(kx)\cos(\omega t)}$$

$*\ k = \frac{2\pi}{\lambda} \qquad \omega = \frac{2\pi}{T}$

AMPLITUDE IS FUNCTION OF POSITION $A(x)$

The above mathematical is the general description of two waves traveling in opposite directions interfering with each other. Note that the amplitude is a function of position. A nice way to think about this result is that the amplitude is a wave in space, which is then modulated by the cosine term which contains information about how that amplitude will change in time. At this point, we would now apply our boundary conditions (symmetric and anti-symmetric) to arrive at standing wave conditions. The details of this are beyond the scope of this class so we will conclude the mathematical representation discussion here. Check out this link for a nice visualization of how standing waves from off of different boundaries.

Standing waves are an example of spatial interference. In other words, you can walk around space and observe locations of constructive and deconstructive interference. Once you stop at a certain spot, you will observe the resulting interference condition.

Comparison of standing waves to traveling waves

Below is a short list of some general differences between standing waves and traveling waves.

Traveling Wave	Standing wave
Disturbance travels to different locations in space.	The disturbance oscillates up and down but not to different locations in space. Basically the disturbance is confined to a certain region of space.
The oscillating quantity of traveling waves move through space and time and is never constant. Example: parts of a string on a traveling wave are always moving up and down.	Locations exist where the oscillating quantity of the wave is stationary (nodes). Example: The pressure is a constant value (atmospheric) at nodes.
The oscillators of the traveling wave are never found at their equilibrium locations at the same time.	All oscillators reach their equilibrium locations at the same time.
Energy transported in a transported in a traveling wave form one location in space to another.	The energy in a standing wave is not transported from one location to another; the energy is confined to a certain region in space.

QUESTIONS FOR DISCUSSION:

(1) Can you set up two speakers such that they produce regions of no sound?
(2) What will happen when two surface water waves approach from different directions and run into each other? Will they cancel each other out? Will they bounce off each other? Explain your thoughts.