

Traveling waves

Select LEARNING OBJECTIVES:

- Be able to take a physical phenomenon and represent it as a history graph or a snapshot graph.
- Be able to identify distinguishing features of mechanical, E&M, and matter waves.
- Be able to mathematically model a traveling wave using a function that has the form of $D(x,t)=D_{\max}\sin(kx-\omega t)$ and be able to manipulate this equation to find relations between D , v_x and a_x .
- Understand the connection between waves and oscillations.
- Understand that the frequency of a wave is dependent only upon the source.
- Understand that the speed of the wave depends on the medium in which it travels.

TEXTBOOK CHAPTERS:

- Ginacoli ((Physics Principles with Applications 7th) :: 11-7, 11-8, 11-15
- Knight (College Physics : A strategic approach 3rd) :: 15.1, 15.2, 15.3, 15.4
- Boxsand :: [Traveling waves](#)

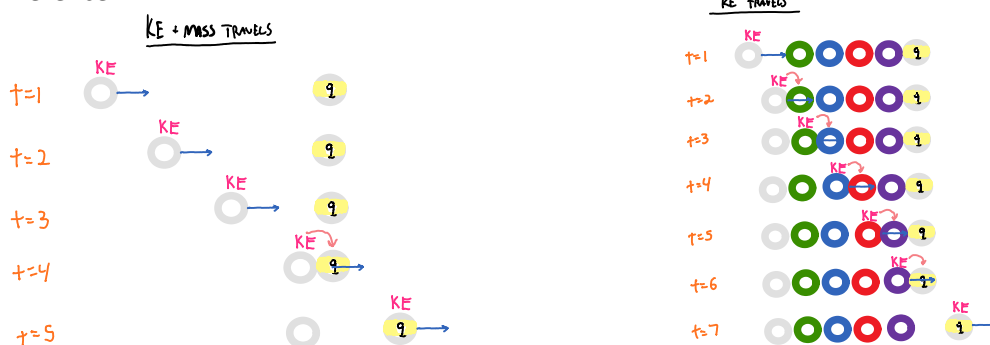
WARM UP: A good rule of thumb is that 10° is considered a small angle. Does this mean $\sin(10^\circ) \approx 10$?

NO MUST BE IN RADIANS ... ex... $\sin(10^\circ) \approx 0.173648 \neq 10$

BUT $10^\circ = \frac{\pi}{18}$ rad ... $\sin\left(\frac{\pi}{18}\right) = 0.173648$ (PRETTY CLOSE :)

$\frac{\pi}{18} = 0.1745$

Think about a game of billiards. You spot the 9 ball near the corner pocket at the far end of the table. In order to get the ball into the pocket you know you must transfer kinetic energy to the 9 ball so it will roll into the pocket. How do you achieve this? You strike the cue-ball with a cue-stick. Once the cue-ball is struck it has a velocity, thus it has kinetic energy. The kinetic energy is associated with the cue-ball, and as it travels across the table it carries with it this kinetic energy until it collides with the 9 ball transferring some of its kinetic energy to the 9 ball along with losing some to thermal energy and sound. In general, particles "carry" kinetic energy with them as they move through space so if we want to transfer energy across space we should send a particle with kinetic energy to that region of space. However the story doesn't end here. Pause for a moment and think; is there another way to transfer energy through space without actually sending a particle to that location? Let's revisit the billiards game to help answer this question. Previously we sent the cue-ball across the table to the corner pocket where the 9 ball was. Is there a way to transfer kinetic energy to the 9 ball but have the cue-ball not actually move across the entire table? There is; line up all the other balls in a row with the cue-ball at one end and the 9 ball at the other. Strike the cue-ball into the edge of the row, and each ball will collide with the previous, then with the next in line until the last ball strikes the 9 ball and sends it into the pocket, all the while the cue-ball is not very far from where it started. What happened there? Energy was transferred originally from the cue-ball to the first ball in the row, then to the next, and next and so on until the kinetic energy finally arrived at the 9 ball sending it forward into the pocket. Below is a sketch of the two scenarios to help illustrate the difference.



So in general, a particle does not have to "carry" energy with it to another region of space. We can construct scenarios where the particles are not displaced very far from their original locations but the energy does travel across space to a different region. In the second billiards example the cue-ball travelled a very short distance and stopped after the collision, boring. What if we connected spring between each billiard ball so that they are all at their equilibrium locations? After the cue-ball collides with the green ball, there will be a restoring force pulling the cue-ball back to the equilibrium location. The cue-ball will thus undergo simple harmonic motion (ignoring friction here) and the kinetic energy transfer down the row will happen over and over again. This is our first example of *wave motion*; energy is propagated from one location to another without bulk propagation of matter. Another way to think about this is that we initially disturbed the cue-ball which in turn disturbed the green ball and so on. Thus an initial disturbance travels down the row. *We can now define a traveling wave as a disturbance that travels with a well-defined speed and carries energy from one location to another without individual particles traveling between the two locations.*

In this lecture we will introduce various types of wave motion and analyze a mathematical model representing the motion of traveling waves.

From our billiards example hopefully you see that a traveling wave is a collection of oscillators, the very same oscillators we studied in the previous lectures. There are many different possible collections of oscillators that can exhibit wave motion, each of which can be identified as one of three types of waves as defined below.

Types of traveling waves *THIS JUST IN (2015)... GRAVITY WAVES CONFIRMED

Mechanical waves

- Waves that require a medium for the disturbance to travel.
 - Examples:
 - Sound waves (*pressure/displacement disturbance*)
 - Waves on a string (*displacement disturbance*)
 - Water waves (*horizontal disturbance in deep water, horizontal and vertical near surface*)

Electromagnetic waves

- Does not require a medium for disturbance to travel.
 - Examples:
 - Visible light (*electromagnetic field disturbance*)
 - Radio waves (*electromagnetic field disturbance*)
 - Microwaves (*electromagnetic field disturbance*)
 - X-rays (*electromagnetic field disturbance*)

Matter waves

- Weird... all matter also exhibits wave-like behaviors
 - Examples of "easily" observable matter waves:
 - Electrons
 - Bucky balls (Spherical C₆₀)

PRACTICE: Match the wave phenomena with the type of wave.

- (a) Crowd at a sporting event. ①
- (b) Diffraction pattern of accelerated electrons through two small slits. ②
- (c) Ham radio waves. ③

1. Mechanical.
2. Matter wave.
3. Electromagnetic.

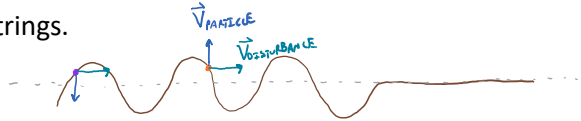
Traveling waves can also be broken down into three modes which describe how the disturbance propagates through space and time. Below is a description for each mode.

Modes of traveling waves

Transverse

- Displacements of oscillators are perpendicular to the direction of disturbance/energy propagation.

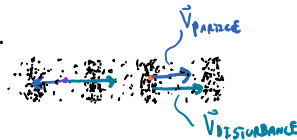
- Example:
 - Waves on strings.



Longitudinal

- Displacement of oscillators are parallel to the direction of disturbance/energy propagation.

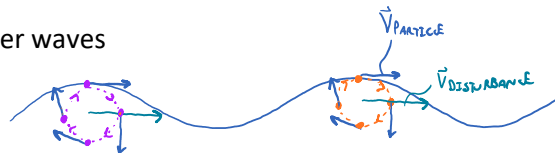
- Example:
 - Sound waves.



Combination of transverse & longitudinal

- Displacements of oscillators are a combination of parallel and perpendicular with respect to the direction of disturbance/energy propagation.

- Example:
 - Surface water waves



PRACTICE: Match the wave phenomena with the type of wave.

- (a) Crowd at a sporting event. (1)
- (b) Earthquake waves. (3) S + P
- (c) Water waves. (3)
- (d) Sound waves. (2)

- 1. Transverse.
- 2. Longitudinal.
- 3. Combination of transverse and longitudinal.

Features of traveling waves

Wavelength

The distance between each successive disturbance is referred to as the *wavelength* (λ). Below is a representation of both a transverse and longitudinal wave showing the wavelength.



Also shown above is the amplitude (A) which represents the same feature as simple harmonic oscillators; the maximum displacement from the oscillator's equilibrium location. Also note that the peaks are referred to as crests and the minima are referred to as troughs; these give us a nice way to identify a fixed location along the disturbance.

Frequency and period

The oscillators that make up traveling waves undergo simple harmonic motion with a fixed frequency (f) and period (T). Thus the frequency is a description for the number of complete crests that pass a given point in space per time. The period then has the interpretation that it is the time it takes between two successive crests passing by the same point in space. The frequency is determined by the source of the wave (e.g. you get to decide at what frequency you wish to oscillate a rope up and down).

Wave speed

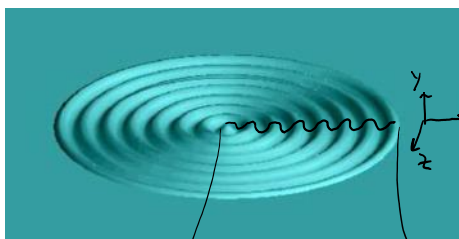
The disturbance that is transmitted across space and time from a traveling wave travels at a well-defined speed. This speed is known as the wave speed (v). Recall the definition of speed ($v = \text{distance}/\text{time}$). Since the crests travel exactly 1 wavelength during the time interval of exactly 1 period the wave speed is defined as

$$v = \frac{\lambda}{T} \quad \text{or} \quad v = \lambda f$$

Refer back to the wavelength section above and carefully study the velocity of the particle and the velocity of the disturbance. The wave speed is the magnitude of the disturbance velocity; note that this wave speed is not the same as the $v(t)$ of the individual oscillators that make up the traveling wave.

The wave speed is a property of the medium which the wave is propagating through. The speed does not depend on the source, amplitude, frequency, etc.. The speed only depends on the medium. For example light has many different frequencies, all of which are determined by the source, but in the vacuum of space all of the frequencies of light travel at the same speed.

PRACTICE: A friend shows you a picture of a series of periodic ripples on the surface of a pond from a stone thrown into the pond. He tells you the picture was taken 2.3 seconds after the source began emitting the wave.



From this picture you can determine:

known
TOTAL TIME = 2.3 s } $T = \frac{\text{TIME}}{\text{# OF CYCLES}} = \frac{2.3 \text{ s}}{7} \approx 0.329 \text{ SEC}$
OF $\lambda = 7$

UNKNOWN
 λ

What if all you knew is that at the instant shown, the outermost ripple is 10.0 m from the source of the ripples. What could you determine then?

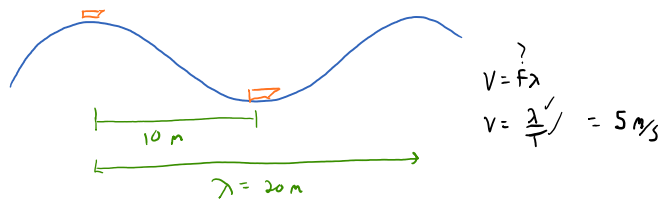
Snapshot IN TIME

known
TOTAL DIST = 10.0 m } $\lambda = \frac{10 \text{ m}}{7} \approx 1.42 \text{ m}$
OF $\lambda = 7$

PRACTICE: If a bigger stone is dropped, do the features of the wave increase, decrease, or remain unchanged.

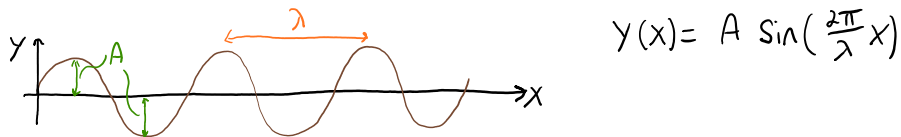
- (a) Speed v UNCHANGED ... PROPERTY OF MEDIUM, NOT SOURCE
- (b) Wavelength λ INCREASED ... (CROSS-SECTIONAL AREA) \uparrow (FORCE FROM WEIGHT)
- (c) Frequency f DECREASED ... $v = \lambda f$ w/ $v = \text{const}$ $f \propto \frac{1}{\lambda}$ AND $\lambda \uparrow$ SO $f \downarrow$
- (d) Amplitude INCREASED ... (CROSS-SECTIONAL AREA) \uparrow (FORCE)

PRACTICE: Two canoes are 10 m apart on a lake. Each bobs up and down with a period of 4.0 seconds. When one canoe is at its highest point, the other canoe is at its lowest point. Both canoes are always within a single cycle of the waves. Determine the speed of the waves.



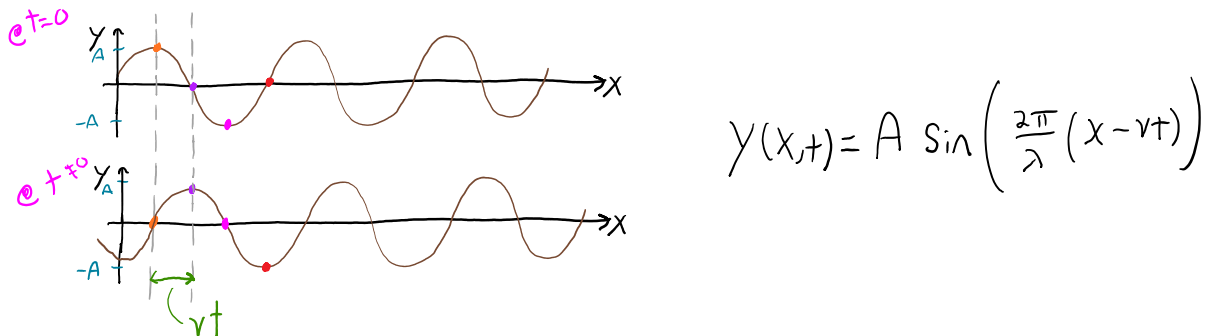
Mathematical model

It is now our goal to introduce the mathematical model for traveling waves. Consider a snapshot of a wave on a string. A snapshot is exactly what it sounds like; imagine using a camera to take a picture of the wave. The picture is then a representation of where the wave is in space (i.e. you could draw an x and y coordinate system on the picture) at the instant you took the picture. Consider a source producing a sine wave along a string, perhaps your snapshot of a wave on a string looks like the image below.



Note that we can overlay a coordinate system where "y" represents the displacement of each segment of the string from its equilibrium location. The amplitude and the wavelength are also labeled. The sine function that mathematically models our snapshot is also shown above.

Now suppose the wave is moving to the right with a speed (v). After an amount of time t , each part of the wave will have moved a distance vt to the right. Below is two snapshots, one at $t=0$ and the other one at time $t \neq 0$.



Study the two snapshot graphs carefully. Remember that the string itself is not moving to the right; this is illustrated with the color dots that always remain at the same x-position. The disturbance is what travels to the right. It is helpful to think about the crest as a particular location of the disturbance, so the crest

moves to the right by a distance vt . The old crest location was at the position x while the new crest location is now located at $(x-vt)$. However the new overall shape of the sine wave is exactly the same as the original. Thus we just replace x with $(x-vt)$ in the argument of the sine function to arrive at the mathematical model shown above. Since we picked initial conditions that correspond to a sine wave and assumed the wave was traveling to the right, this is not the most general form of a traveling wave. Shown below is the most general form where instead of $y(x,t)$ we use $D(x,t)$ to represent the displacement of the individual oscillators from their equilibrium locations (if the displacement happens to be along the y like in our above examples then D can be replaced with y).

$$D(x,t) = \pm D_{\max} \begin{matrix} \text{Sin} \\ \text{or} \\ \text{Cos} \end{matrix} \left(\frac{2\pi}{\lambda} x \pm \frac{2\pi}{T} vt \right)$$

WAVE NUMBER $\equiv k = \frac{2\pi}{\lambda}$
 WITH $v = f\lambda \dots$
 $\frac{v}{\lambda} = f$ AND $2\pi f = \omega$

OTHER USEFUL FORMS

$$D(x,t) = \pm D_{\max} \begin{matrix} \text{Sin} \\ \text{or} \\ \text{Cos} \end{matrix} \left(\frac{2\pi}{\lambda} x \pm \frac{2\pi}{T} t \right)$$

$$D(x,t) = \pm D_{\max} \begin{matrix} \text{Sin} \\ \text{or} \\ \text{Cos} \end{matrix} \left(\frac{\omega}{v} x \pm \omega t \right)$$

$D(x,t) = \pm D_{\max} \begin{matrix} \text{Sin} \\ \text{or} \\ \text{Cos} \end{matrix} (kx \pm \omega t)$

+ or - AND Sin or Cos } BASED OFF OF INITIAL CONDITIONS
 IF kx AND ωt HAVE OPPOSITE SIGNS, WAVE TRAVELS TO THE POSITIVE x DIRECTION...
 IF SAME SIGN THEN TRAVELS TO NEGATIVE x DIRECTION

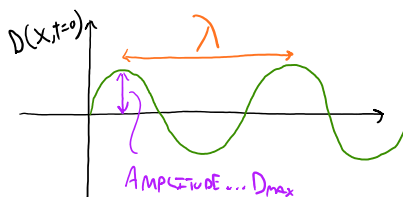
We often define a wave number (k) to simplify the expression. Also note that we made no references to any springs when constructing these equations; the wave number k is not the same thing as the spring constant k (unfortunately we often use the same symbol for multiple quantities, which is why it is very important to always remind yourself what phenomena you are describing and interpret the symbols based off of that context). Although our discussion involved a string, we did not invoke any unique string parameters either. Thus all the variables (wave number, angular frequency, displacement, period, frequency, wavelength) are general features of all traveling waves. The boxed form is often used because it is the least cluttered, however I provided the other useful forms along with our original form we constructed. You should start with the boxed form and convince yourself that all the other forms are equivalent using the general wave relationships for speed, angular frequency, and wave number.

Since the displacement of the wave is a function of both position and time, we must make a choice when plotting to plot a snapshot or a history graph. We have discussed snapshots already. A history plot involves you choosing a single position in space along with direction of wave propagation (in our example this is an x value) and recording the displacements of the individual oscillator at that location as a function of time. Below is a generic example of both a snapshot and history plot.

SNAPSHOT: HOW t CONSTANT

EXAMPLE ... $t = 0$, $D_{\max} = 0$ @ $x = 0$

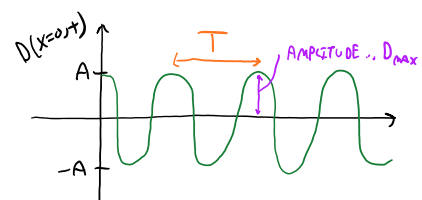
$$D(x, t=0) = D_{\max} \text{Sin}(kx)$$



HISTORY: HOW x CONSTANT

EXAMPLE ... $x = 0$, $D_{\max} = A$ @ $t = 0$

$$D(x=0, t) = D_{\max} \text{Cos}(\omega t)$$



PRACTICE: A wave obeys the equation $D(x,t) = 45 \sin(0.5x - 2t)$, where D and x are in meters and t in seconds.

What is the amplitude of the wave. 45 m

What is the period of the wave? (try without calculator) $\omega = 2$
 $\frac{2\pi}{T} = 2$ } $T = \pi \text{ seconds}$

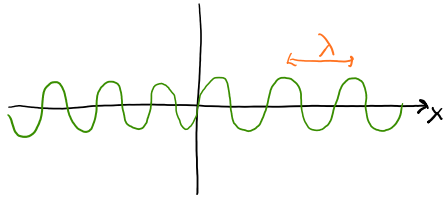
What is the wavelength of the wave? $k = 0.5$
 $\frac{2\pi}{\lambda} = 0.5$ } $\lambda = 4\pi \text{ m}$

Does this wave travel in the positive x direction? $(kx \pm \omega t) \dots$ OPPOSITE SIGNS ... YES

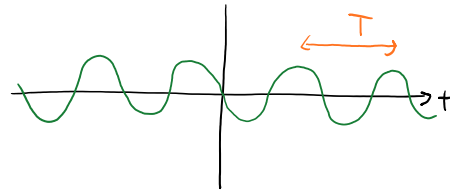
What mode of wave is this? TRANSVERSE OR LONGITUDINAL.

Sketch a snapshot of the wave at $t=0$ seconds as a function of position. Include about two oscillations in both the positive and negative x region.

$$D(x,t=0) = 45 \sin(0.5x)$$

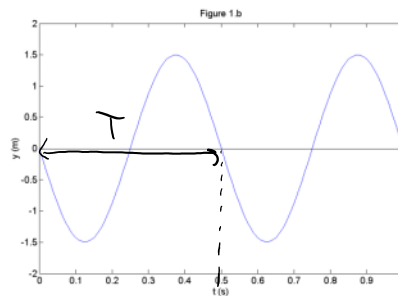
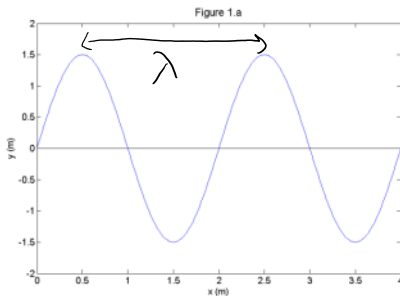


$$D(x=0,t) = 45 \sin(-2t)$$



Sketch the amplitude of the oscillator at $x=0$ as a function of time (history graph). Include about two oscillations in both the positive and negative x region.

PRACTICE: Figure 1.a shows the displacement of particles in a string as a function of position x at time $t = 0$ seconds. Figure 1.b shows the displacement of the particle at $x=0$ as a function of time.



(a) Determine the wavelength, period, wavenumber, frequency, and amplitude.

$$\lambda = 2 \text{ m} //$$

$$k = \frac{2\pi}{\lambda} = \pi \text{ 1/m} //$$

$$A = 1.5 \text{ m} //$$

$$T = 0.5 \text{ sec} //$$

$$f = 2 \text{ Hz} //$$

$$\omega = 2\pi f = 4\pi \text{ rad/sec} //$$

$$D(x,t=0) = 1.5 \text{ m} \sin(\pi x)$$

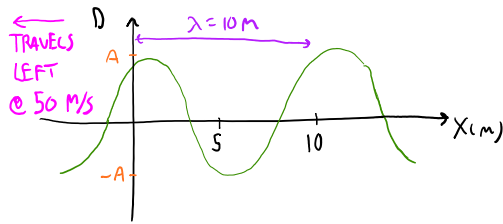
$$D(x=0,t) = 1.5 \sin(-4\pi t)$$

(b) Write the equation for the displacement of this traveling wave as a function of position and time.

$$D(x,t) = \pm D_{\max} \sin(kx \pm \omega t)$$

$$D(x,t) = 1.5 \text{ m} \sin(\pi x - 4\pi t)$$

PRACTICE: What is the frequency of this traveling wave?



$$v = f\lambda$$

$$f = \frac{v}{\lambda} = \frac{50 \text{ m/s}}{10 \text{ m}} = 5 \text{ Hz}$$

QUESTIONS FOR DISCUSSION:

- (1) Explain the difference between the speed of a transverse wave traveling along a string and the speed of a tiny piece of the string.