Statics, dynamics, and point particle center of mass

Select LEARNING OBJECTIVES:

i. Determine if a system is in an equilibrium or dynamic condition, and specify which one.
ii. Demonstrate the ability to analyze a system in static translational and rotational equilibrium via a torque analysis.
iii. Be able to calculate the center of mass of an object or a system of objects.
iv. Be able to determine if an object or system of objects will be stable based on center of mass location relative to normal forces.

TEXTBOOK CHAPTERS:

- Giancoli (Physics Principles with Applications 7th) :: 9-1, 9-2, 9-3, 9-4
- Knight :: 12.8, 12.2
- BoxSand :: Rotational Mechanics (Statics & Dynamics)

WARM UP: Does the reference axis "o" have to be the actual pivot point that an object rotates about?

Now that we have defined a way to mathematically work with torque, let's revisit Newton's laws of motion as applied to the point particle model (translational motion) and also introduce its form in the rigid body model (rotational motion).

CENTER OF MASS

POINT PARTICLE (COM)

(TRANSLATIONAL MOTION)

$$\sum \vec{F}_{EXT} = m_{sys}\,\vec{a}_{com}$$

RIGID BODIES

(ROTATIONAL MOTION)

$$\sum \vec{\tau}_{EXT,o} = I_o\,\vec{\alpha}$$

"NET EXTERNAL TORQUE"

AXIS "O"

MOMENT OF INERTIA ABOUT AXIS "O"

Statics and dynamics are terms used to describe the different scenarios that fall out of the two versions of Newton's 2nd law above. The scenarios can be summarized by the table below. In this section we will focus mostly on the rotational portion since this is the newer topic being introduced.

	Translational	Rotational
Static Equilibrium	$\vec{v}_{com}=\vec{0}$	$\omega=0$
Dynamic Equilibrium	$\vec{v}_{com}=\text{constant}\neq\vec{0}$ $\vec{a}_{com}=\vec{0}$ $\Sigma\vec{F}=\vec{0}$	$\omega=\text{constant}$ $\alpha=0$ $\Sigma\tau_o=0$
Dynamics	$\Sigma\vec{F}=m\vec{a}_{com}$	$\Sigma\tau_o=I_o\alpha$

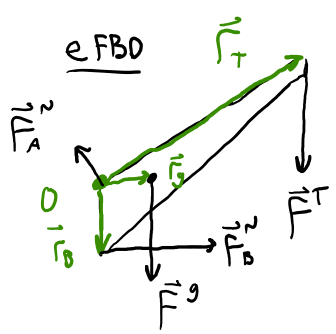
A new quantity shows up when dealing with rotational dynamics, moment of inertia I_o, where the subscribe "o" refers you to the reference axis of rotation. The moment of inertia is analogous to mass in the point particle model. The mass in the point particle model plays the role of inertia, which we can view in the following way: the larger the mass (i.e. the larger the inertia) the harder it is to increase the translation motion of an object. Think about a massive object on a frictionless surface, if you stand on a surface with friction and push the massive object, it is really hard to start to get moving at a high speed. If the object were much less massive, you would have no problem getting the object up to a fast speed. Now we can understand what role the moment of inertia plays in rotational motion. Just like mass, the larger the moment of inertia, the harder it is to increase the rotational motion of an object about an axis. Again, a simplified thought experiment goes something like this: if object that is not rotating has a large moment of inertia, it would require a lot of effort on your part to get it to start rotating at a fast angular speed. If the object has a much smaller moment of inertia, it would be much easier to get it rotating at a fast speed.

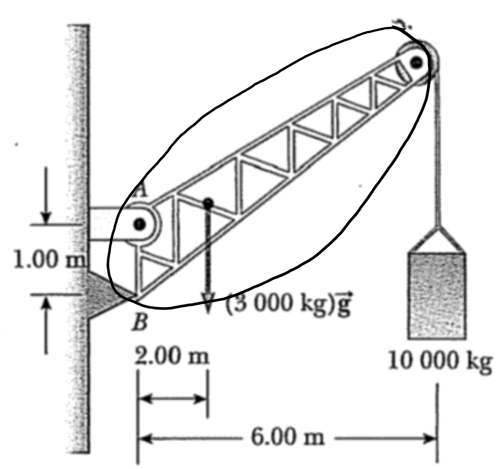
The moment of inertia depends on the reference axis of rotation and the way that the mass is distributed around this reference axis of rotation. For example, if you hold weights in our hands and stretch your arms out you would have a larger moment of inertia about the axis that runs from your feet to your head than you would if you held the weights tight to your body.

PRACTICE: A 3000 kg crane is supporting a 10,000 kg crate. The crane pivots about point A and is at rest pressed up against a support B. Find the torque from the 10,000 kg crate about the point A.

1. 6,000 N
2. 10,000 N m
3. 10,000 N
4. 36,800 N m
5. 588,000 N m

CCW(+)
CW(−)





$$\vec{\tau}^T = \vec{r}_T \times \vec{F}^T$$

? / ?

$$\tau^T = -|\vec{r}_T||\vec{F}^T|\sin\theta$$

$$= -r_{T\perp}|\vec{F}^T|$$

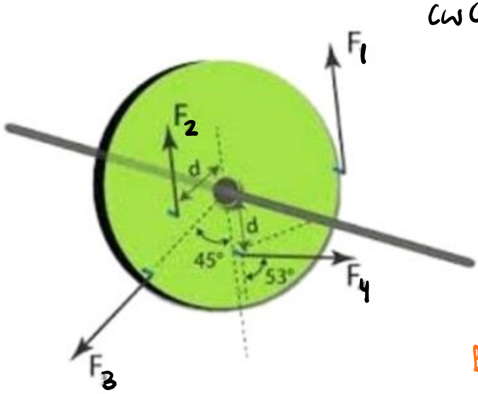
$$= -(6\,m)(10000\,kg)(9.8\,m/s^2)$$

$$\boxed{\tau^T = -588000\ N\,m}$$

PRACTICE: Rank the torques about the center axel from most negative to most positive. All the forces have the same magnitude.

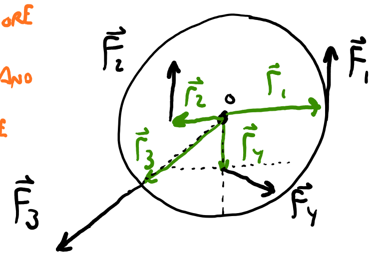
CCW(+)
CW(-)

$\tau_2 < \tau_3 = 0 < \tau_4 < \tau_1$

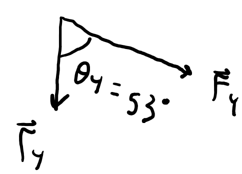


* IGNORE $\vec{F}^g$ AND $\vec{F}^{AXLE}$ B/C @ O

@ FBD



VEC. OP



The light disk has a radius of 1.0 m and is in rotational equilibrium while the following four forces act on it. If **F**[1] = 5 N, **F**[2] = 15 N, **F**[3] is very large, what is the value of **F**[4]?

$$\Sigma\tau_o = I_o \cancel{\alpha}^{0}$$

$$\tau^1 + \tau^2 + \tau^3 + \tau^4 = 0$$

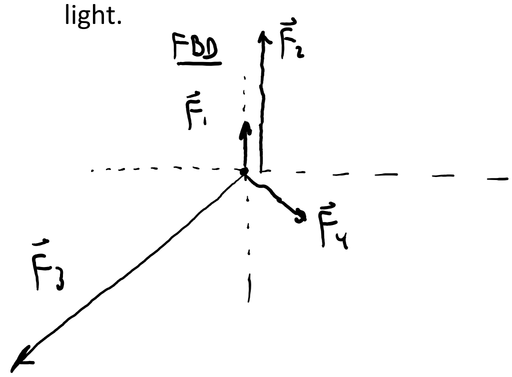
$$r_1F_1\cancel{\sin\theta_1}^{1} - r_2F_2\cancel{\sin\theta_2}^{1} + r_3F_3\cancel{\sin\theta_3}^{0} + r_4\sin\theta_4 = 0$$

$$r_1F_1 - r_2F_2 + r_4F_4\sin\theta_4 = 0$$

$$\vdots$$

$$\boxed{F_4 \approx 6.26\ N}$$

The disk is in rotational equilibrium with the 4 forces in the figure. Would the disk also be in linear equilibrium with only those 4 forces? Ignore the force from the axle for now and assume the disk is very light.



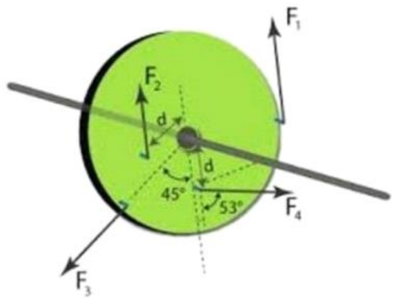
$\Sigma\vec{F} \neq \vec{0}$

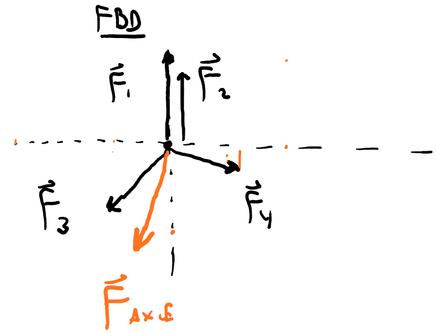
NOT IN LINEAR EQUILIBRIUM.

Sketch a force vector coming from the center of the disk to represent the reaction force, from the axle, required to keep the disk in linear equilibrium. Assuming the force vectors are scaled relative to each other and the disk is very light.

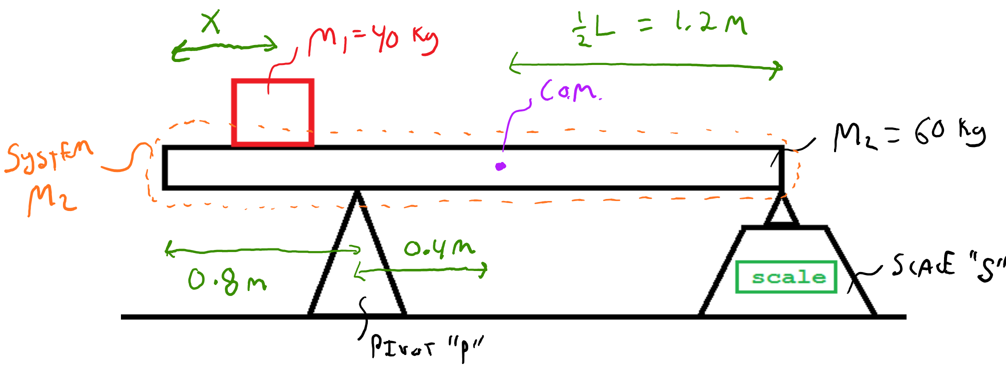
FBD

other and the disk is very light.

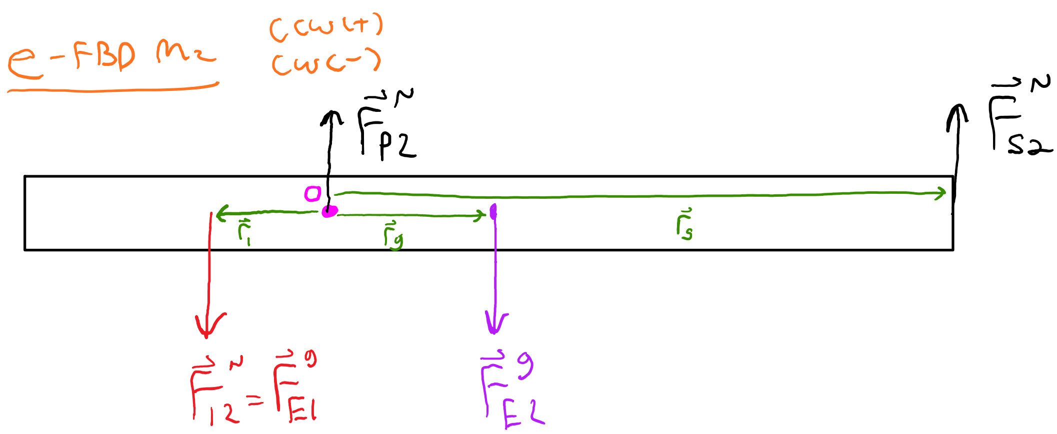




PRACTICE: A table top of mass 60 kg which is uniformly distributed is 2.4 m long and is supported by a pivot 0.8m from the left end, and by a scale at the right end.



1. How far from the left end should a 40 kg box be placed on the table top if the scale is to read 100 N?



$$\Sigma \tau_o = I_o \cancel{\alpha}^{0}$$

$$\Sigma \tau_o = 0$$

$$\tau^1 + \cancel{\tau^p}^{0} + \tau^g + \tau^s = 0$$

$$|\vec{r}_1||\vec{F}^N_{12}| - |\vec{r}_g||\vec{F}^g_{E2}| + |\vec{r}_s||\vec{F}^N_{S2}| = 0$$

$$|\vec{r}_1| m_1 g - |\vec{r}_g| m_2 g + |\vec{r}_s||\vec{F}^N_{S2}| = 0 \quad // \text{ 1 eqn 1 unknown}$$

$$|\vec{r}_1| = \underline{|\vec{r}_g| m_2 g - |\vec{r}_s||\vec{F}^N_{S2}|} = \underline{(0.4)(60)(9.8) - (1.6)(100)} \text{ meters} \approx 0.192 \text{ meters}$$

$$|\vec{r}_1| = \frac{|\vec{r}_g| m_2 g - |\vec{r}_s||\vec{F}^N_{s2}|}{m_1 g} = \frac{(0.4)(60)(9.8) - (1.6)(100)}{(40)(9.8)} \text{ METERS} \approx 0.192 \text{ METERS}$$

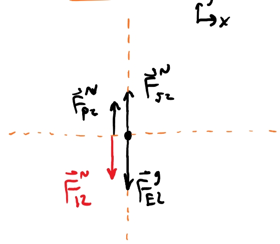
$$X = 0.8 - |\vec{r}_1|$$

$$X \approx 0.608 \text{ METERS}$$

1. With the box at this location, what is the normal force provided by the pivot?

* YOU COULD MOVE THE REFERENCE AXIS TO A DIFFERENT LOCATION SUCH THAT $\tau^P \neq 0$. OR USE REGULAR FBD.

FBD M_2



$$\Sigma F_y = m_2 \cancel{a_y}^{0}$$

$$|\vec{F}^N_{s2}| + |\vec{F}^N_{p2}| - |\vec{F}^N_{12}| - |\vec{F}^g_{E2}| = 0$$

$$|\vec{F}^N_{s2}| + |\vec{F}^N_{p2}| - m_1 g - m_2 g = 0$$

$$|\vec{F}^N_{p2}| = m_1 g + m_2 g - |\vec{F}^N_{s2}|$$

$$|\vec{F}^N_{p2}| = 880 \text{ N}$$

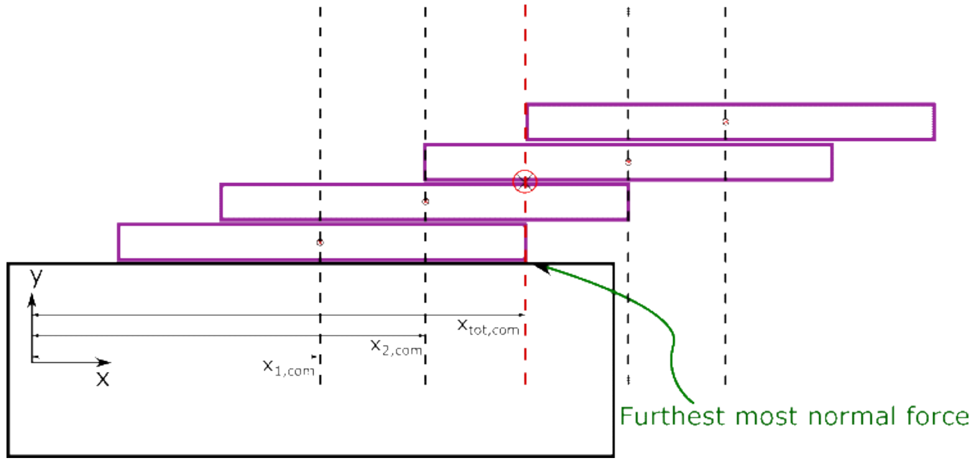
Stability

Often times summing the torque of a system of objects around an axis to determine if the system is in equilibrium or not is tedious. It turns out we also have another method to determine the stability of a system by looking at the total center of mass of the system and its location to the furthest most normal force.

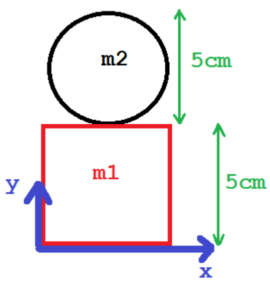
Mass is distributed within a rigid body, however, we can define a "center of mass" (cm) location for an object or system of objects. This is useful because if our object is in a uniform gravitational field (much like the approximate uniform gravitation field near the surface of the Earth), then the force of gravity that acts over the entire body of the object can be simplified and treated as if it acts through only the center of mass of the object. The center of mass will have an x, y, and z-component, but often times we can just consider the x and y-component because the z-component is of no importance to our problem. Mathematically we find the center of mass of a system of objects as follows

$$X_{cm,tot} = \frac{X_{cm,1} \cdot M_1 + X_{cm,2} \cdot M_2 + X_{cm,3} \cdot M_3 + \dots}{M_1 + M_2 + M_3 + \dots} = \frac{\Sigma X_{cm,i} \cdot M_i}{\Sigma M_i}$$

$$Y_{cm,tot} = \frac{y_{cm,1} \cdot M_1 + y_{cm,2} \cdot M_2 + y_{cm,3} \cdot M_3 + \dots}{M_1 + M_2 + M_3 + \dots} = \frac{\Sigma y_{cm,i} \cdot M_i}{\Sigma M_i}$$



PRACTICE: A solid cylinder sits atop a solid cube as shown below. Both objects masses are uniformly distributed. What is the center of mass of the combined system? Let m1 = 0.8 kg and m2 = 0.4 kg.



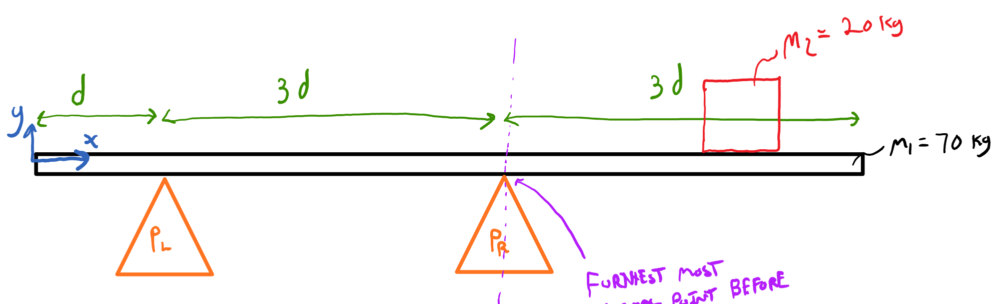
$$X_{cm,TOT} = \frac{m_1 X_{1,cm} + m_2 X_{2,cm}}{m_1 + m_2} \qquad y_{cm,TOT} = \frac{m_1 y_{1,cm} + m_2 y_{2,cm}}{m_1 + m_2}$$

$$X_{cm,TOT} = \frac{(0.8)(2.5) + (0.4)(2.5)}{(.8) + (.4)} \text{ cm} \qquad y_{cm,TOT} = \frac{(0.8)(2.5) + (0.4)(7.5)}{0.8 + 0.4} \text{ cm}$$

$$X_{cm,TOT} = 2.5 \text{ cm} \qquad y_{cm,TOT} = 4.17 \text{ cm}$$

$$\vec{r}_{com}^{\,TOT} = \langle 2.5, 4.17 \rangle \text{ cm}$$

PRACTICE: A 70 kg plank lies on top of two triangular supports. What is the furthest distance, measured from the left side of the plank, that a 20 kg mass could lie without tipping the plank over?



FURTHEST MOST NORMAL POINT BEFORE IT TIPS.

*YOU COULD DRAW AN EFBD FOR m_1, THEN $\Sigma \tau_0 = 0$ WITH $\vec{F}^N_{P_L} \to \vec{0}$ N TO GET ANSWER.

OR USE STABILITY CONDITIONS

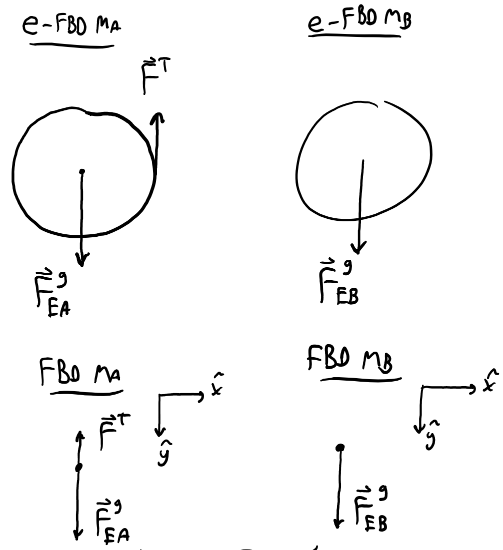
$$X_{cm,TOT} = \frac{m_1 X_{1,cm} + m_2 X_{2,cm}}{m_1 + m_2}$$

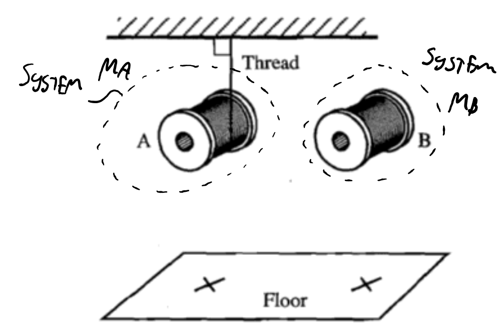
$$4d = \frac{(70)(3.5d) + (40) X_{2,cm}}{70 + 40}$$

$$X_{2,cm} = 5.75\, d$$

PRACTICE: Two identical spools are released from rest. Spool A has it's thread attached to the platform above it and spool B is not attached to the platform. Which spool reaches the floor first?

1. A
2. B
3. The same.





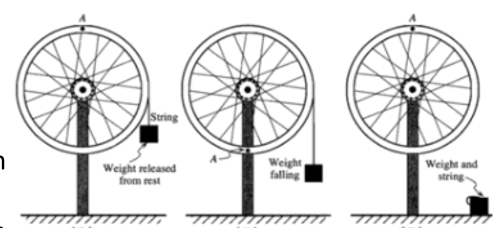
$\Sigma F_{yA} < \Sigma F_{yB}$

Thus $a_{yA} < a_{yB}$

Where does spool A land?

1. To the left of the x
2. To the right of the x
3. On the x $\longrightarrow$ $\Sigma F_{xB} = 0$

PRACTICE: A weight is wrapped around a wheel that is fixed to an axel. The weight is released from rest.



While the weight is falling the wheel is in

1. static translational and dynamic rotational equilibrium
2. dynamic translational equilibrium only.
3. dynamic translational and static rotational equilibrium.
4. static translational equilibrium only.
5. dynamic rotational equilibrium only.

While the string and the weight are no longer attached to the wheel, the wheel is in ASSUME NO FRICTION OR AIR RESISTANCE

1. static translational and dynamic rotational equilibrium.
2. dynamic translational equilibrium only.
3. dynamic translational and static rotational equilibrium.
4. static translational equilibrium only.
5. dynamic rotational equilibrium only.

Questions for discussion:

(1) Discuss the validity of the statement: "torque keeps objects rotating".
(2) Discuss the validity of the statement: "if something is not rotating, it has a net torque of zero".
(3) Can a ruler be in equilibrium at an angle relative to the horizontal?