Fluid dynamics - continuity

Select LEARNING OBJECTIVES:

- Understand the simplifying assumptions leading up the continuity equation.
- Apply the continuity equation to determine velocities or cross sectional areas needed when given certain specifications.
- Demonstrate the ability to determine if necessary and to connect kinematic equations involving fluid motion.

TEXTBOOK CHAPTERS:

- Ginacoli ((Physics Principles with Applications 7th) :: 10-8
- Knight (College Physics : A strategic approach 3rd) :: 13.5
- Boxsand :: Continuity

WARM UP: Questions about buoyancy?

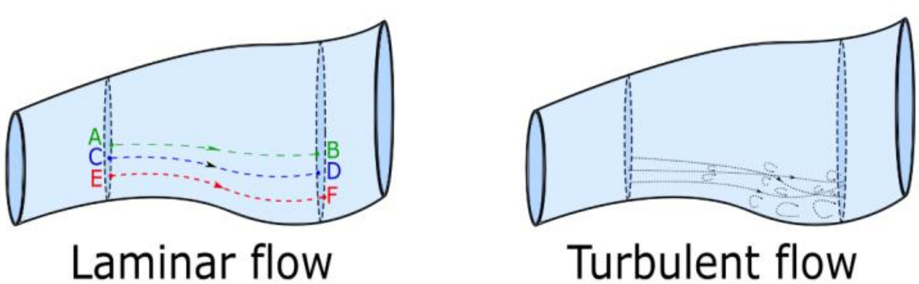
We will now begin to study the properties of moving fluids. In general, this is a very complicated task which would require calculus and perhaps one or more courses that are solely focused on the field fluid dynamics. In this section, we will apply a few approximations with regards to the moving fluid and study the consequences that arise, namely the continuity equation.

How often are weather reports accurate? Sometimes it feels like they can never get the forecast right. Why is this? Well, the dynamics of our atmosphere fall under the realm of fluid dynamics, which means predicting the future (predicting the weather) requires solving fluid dynamic equations. Just like our early studies of kinematics, fluid dynamic equations require initial conditions, however, unlike kinematics, a very small change in the initial conditions can lead to a very different final answer in fluid dynamics. Basically, the study of fluid dynamics is very complicated. Luckily we are able to make a few approximations which will allow us to quantify some aspects of fluids in motion.

Fluid dynamics approximations (model)

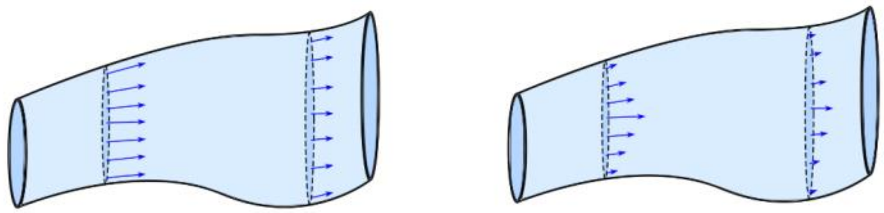
(1) Our first approximation for our model will be to only study fluids that are incompressible, or in other words, the fluid's density is constant.

(2) Our second approximation is restricting ourselves to only study fluids that move in a certain way. Consider two locations A and B within a tube of moving fluid as shown in the left image of the figure below. Our model of the moving fluid will assume that any particle that passes through location A will follow a unique path to get to location B. That unique path is represented by the dashed line between location A and B.



Similarly, if we look at other locations such as C and D, a particle passing through each will also follow unique paths and no paths cross each other. This type of fluid flow is called **laminar flow** (or smooth flow). For comparison, the right image of the figure above shows **turbulent flow** where particles in the fluid that pass through each point follow chaotic paths that change in time, this makes the future position of each particle extremely hard to predict.

(3) The third approximation is to ignore viscous forces (internal friction between "layers" of fluid and external friction between the fluid and the walls of the tube) then the velocity profile (the speed at every location along the cross section of a cylindrical of the tube) is constant. If this were not the case, then kinetic energy of the fluid near the surfaces of the tube would be lost to thermal energy because of the viscous forces (this would mean larger velocities in the center of the tube and velocities approaching zero at the edges). Below is a simplified velocity profile comparison between inviscid flow (without viscous forces) and viscous flow (with viscous forces).



Velocity profile without viscous forces | Velocity profile with viscous forces

When combined, all three of these approximations allow us to make the following statement, "*the mass of fluid that enters a certain region in a given amount of time is equal to the mass of fluid that leaves another region in the same amount of time.*" This is known as the continuity of mass flow rate. Mathematically we write this as…

$$\sum \dot{m}_{IN} = \sum \dot{m}_{OUT}$$

...WHERE...

$$\dot{m} = \rho v A$$

ρ: DENSITY OF FLUID; v: SPEED OF FLUID; A: CROSS SECTIONAL AREA OF ENCLOSURE

*THE DOT ABOVE M INDICATES THAT IT IS A RATE.... $\frac{kg}{m^3} \cdot \frac{m}{s} \cdot m^2 = \frac{kg}{s}$ ✓

What the continuity equation above says is, "*what goes in at some rate, must come out at the same rate*".

If there we restrict our studies to only one type of fluid in a tube at a time, then density is a constant and we use volume flow rate, Q…

$$Q \equiv \frac{\dot{m}}{\rho} = vA$$

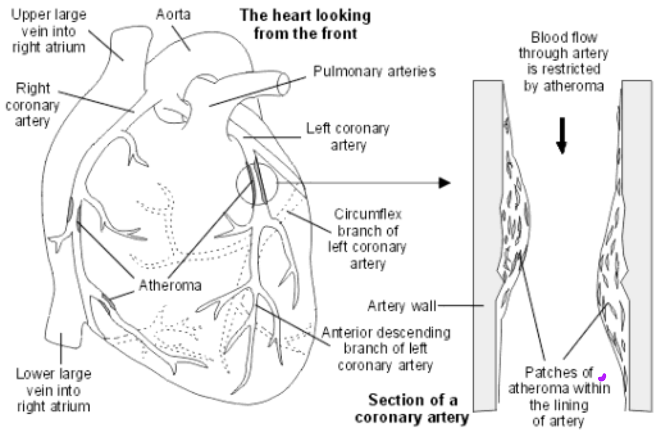
Then our continuity equation becomes…

$$\sum Q_{IN} = \sum Q_{OUT}$$

$$v_1 A_1 = v_2 A_2$$

PRACTICE: Air is compressible. Would the continuity of mass flow rate be a valid expression for air?

PRACTICE: Blood flows through a coronary artery that is partially blocked by deposits along the artery wall. Circle the part of the artery where the flux (volume of blood per unit time) is largest.



Circle the part of the artery where the velocity of the blood is the largest.

PRACTICE: (a) The diameter of a faucet is 9.525 mm, and the speed of the water exiting the faucet is about 1.4 m/s. What is the volume flow rate, in SI units, at the exit location of the faucet?

(b) What is the volume flow rate of the pipe under the sink which has a diameter of 12.7 mm?

PRACTICE: A nozzle of inner radius 4.0 mm is connected to a hose of inner radius 8.0 mm. The nozzle shoots out water moving at 12 m/s. At what speed is the water in the hose moving?

PRACTICE: Why does the diameter of a water stream decrease as the water falls from the faucet?

QUESTIONS FOR DISCUSSION:

(1) Explain why using your finger to cover the end of a garden hose will allow you to water plants that are further away.
(2) In each case below, the tubes are always completely filled with water. In which cases are $v_1=v_2$?