Spring-mass oscillator

Select LEARNING OBJECTIVES:

- Apply the sinusoidal equations of motion (position, velocity, acceleration) to SHO - i.e. fit them to the system using the initial conditions.
- Understand the system specific (mass-spring) physical quantities.

TEXTBOOK CHAPTERS:

- Boxsand :: Springs and pendulums

WARM UP: Would the force below give rise to simple harmonic oscillation?

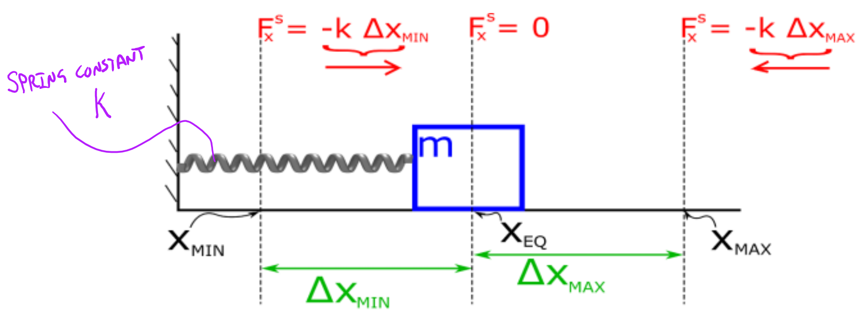
$$F_x = 7.5\,\Delta x$$

In this lecture we will begin to quantify the mass-spring system which is a case study of simple harmonic motion.

Assumptions

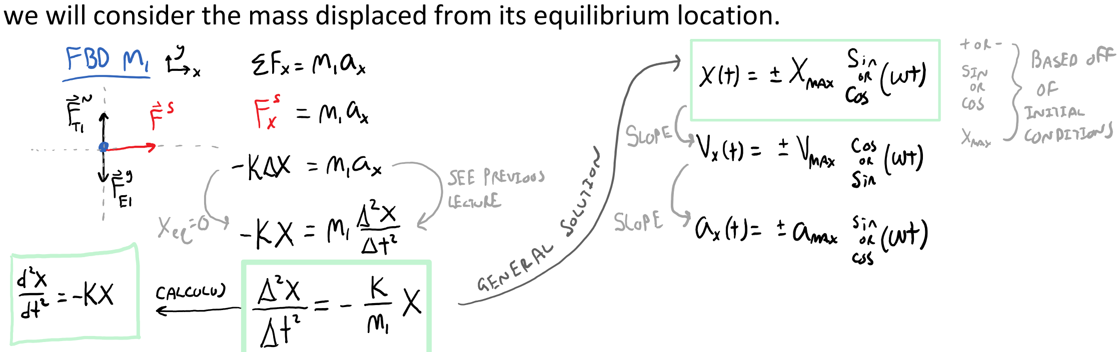
- No friction between surface of mass and table.
- No air resistance.
- The spring is ideal (i.e. it's mass is negligible and it obeys Hooke's law for all Δx).

Below is a basic sketch to help identify relevant quantities for a mass-spring oscillator.



Equation of motion

Recall we can build equations of motion by applying Newton's 2[nd] law. To do this for the spring-mass system, we will consider the mass displaced from its equilibrium location.



- Mass oscillates back and forth between x_{max} and x_{min} with...
  - $|x_{max}| = |x_{min}|$
  - Angular frequency ≡ $\omega = \sqrt{\frac{k}{m}}$
  - Amplitude ≡ $A = |x_{max}|$

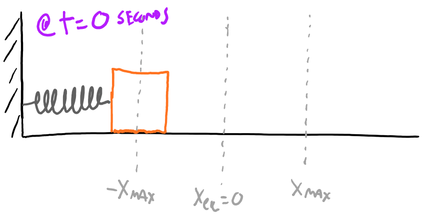
- Period $\equiv T = 2\pi\sqrt{\frac{m}{k}}$
- Frequency $\equiv f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$
- Max velocity $\equiv v_{max} = \omega\, x_{max}$
- Max acceleration $\equiv a_{max} = \omega\, v_{max} = \omega^2 x_{max}$
- Total energy $\equiv E_{tot} = KE + U^s = U^s_{max} = KE_{max}$

$$E_{tot} = \frac{1}{2} m v_1^2 + \frac{1}{2} k \Delta x_1^2 = \frac{1}{2} k A^2 = \frac{1}{2} m v_{max}^2$$

PRACTICE: An object moves with simple harmonic motion. If the amplitude and the period are both doubled, the object's maximum speed is

(a) quartered
(b) halved
(c) unchanged
(d) doubled
(e) quadrupled

PRACTICE: A mass on a frictionless surface is connected to a spring and pulled to the left 15 cm. It is released from rest at t = 0 s and proceeds to make 20 oscillations in 30 seconds. What is the period of oscillation?

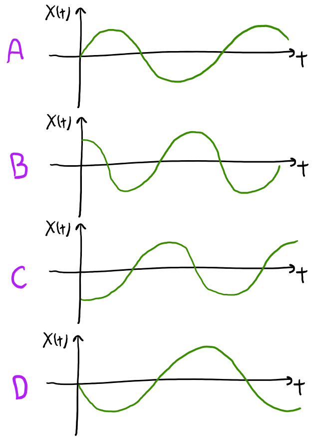


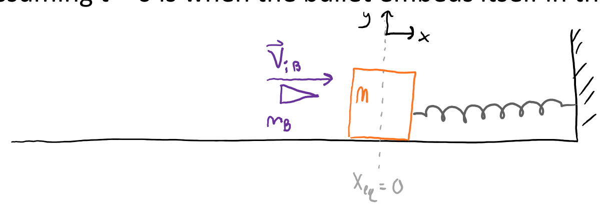
What is the frequency of oscillation?

What is the masses maximum speed?

What are the mass's position and velocity at t = 0.80 s?

PRACTICE: A 16.2 g bullet with an initial speed of 870 m/s embeds itself into a 40.0 kg block, which is attached to a horizontal spring with a force constant of 1010 N/m. Which of the following position as a function of time plots could represent the motion of the bullet + mass system assuming t = 0 is when the bullet embeds itself in the block?





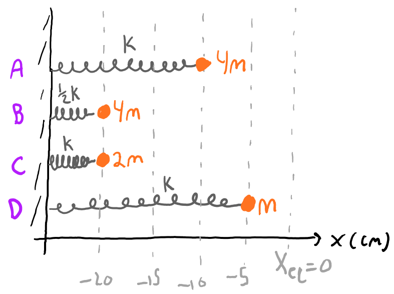
Which physics concepts could be used to determine the maximum speed of the resulting oscillatory motion?

What is the maximum speed of the resulting oscillatory motion.

What is the maximum compression of the spring?

What is the period of the resulting oscillation?

PRACTICE: Four springs have been compressed from their equilibrium position at x = 0 cm. When released, they will start to oscillate. Rank in order the maximum speeds of the oscillations.



QUESTIONS FOR DISCUSSION:

(1) A mass-spring system oscillates in the vertical direction with a frequency *f*. If this system is taken into an elevator which slowly accelerates upwards at a constant rate, what will happen to the frequency of oscillation?