<u>Kinetic theory of gases</u>

Select LEARNING OBJECTIVES:

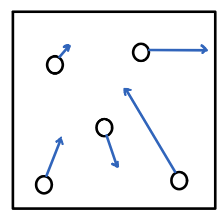
- Develop a microscopic model of a gas where each particle has a different speed but the distribution of speeds is predictable at a given temperature.
- Make the connection between the microscopic particle speeds and the macroscopic temperature.
- Connect the average kinetic energy per particle to the overall total thermal energy - realizing that thermal energy is simply the summation of all of the microscopic KE of each particle.
- Understand the concept of equilibrium as a state where the energy gradient is zero.
- Using the concept of energy equilibrium to manifest as constant temperature.
- Define the Equipartition Theorem - specifically for gasses

TEXTBOOK CHAPTERS:

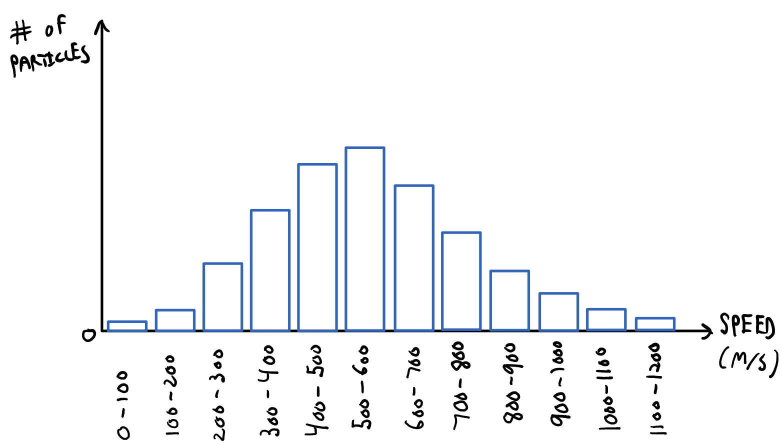
- Giancoli (Physics Principles with Applications 7^{th}) :: 13-9, 13-10
- Knight (College Physics : A strategic approach 3^{rd}) :: 12.2
- BoxSand :: Kinetic Theory of Gases

WARM UP: Questions about matter?

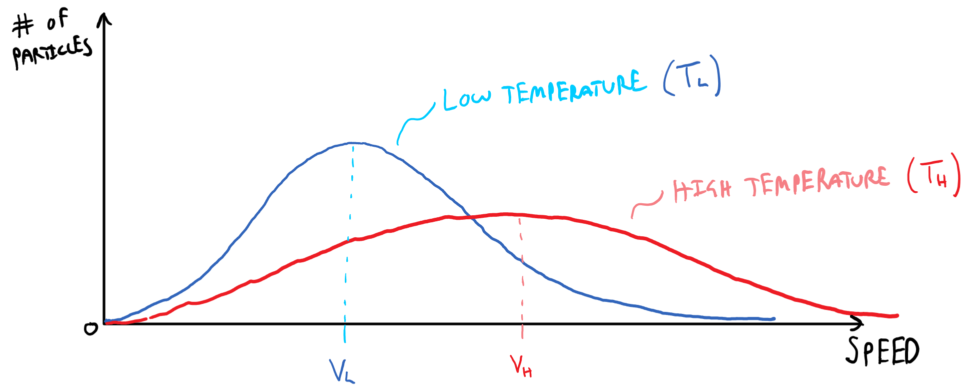
In this lecture we are going to connect our newly refined microscopic model of gases to the macroscopic measurements we can make with a sample of gas. The connection between the microscopic world and macroscopic quantities such as temperature and thermal energy is often referred to as the kinetic theory of gases.



Let's begin this lecture by taking a closer look at our microscopic model of a gas. This picture in your mind of this microscopic model hopefully looks something like the image to the right. Here we see a gas that consists of particles (e.g. molecules) with random motion (i.e. random direction and random speeds). Consider for a moment that we let this container of gas sit in a room for a long period of time. If we were able to isolate each particle and measure its speed we could create a histogram that shows that number of particles that we observed at a specific speed. This histogram might look something like the image below.



The histogram above shows that the gas particles have a distribution of speeds within the container. The shape of this distribution suggests that if you were to pick at random a particle within the container, then you will most likely pick a particle with a speed between 500 m/s to 600 m/s since that range contains the largest number of particles compared to the other ranges. Typically, gases have very large numbers of particles (think on the order of Avogadro's number) so that instead of looking at a histogram we can create a continuous distribution function as seen below. This type of distribution is known as a Maxwell-Boltzmann distribution.

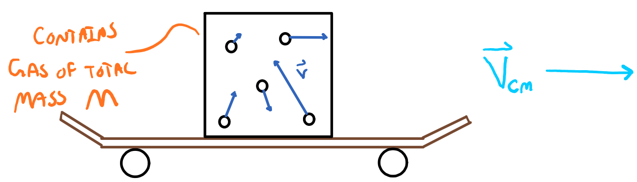


If you look at the blue curve above, it follows the same shape at this histogram that was seen before. Now consider turning the temperature of the room up to a higher temperature, wait a long time again, then come back and create a similar histogram as we did before. If you carry out that process you would notice that the shape of the distribution shifted which is represented by the red curve above. Some important features to recognize are that the number of particles between our low temperature and high temperature experiment didn't change, thus the area under the blue curve must equal the area under the red curve. Also note that a higher temperature suggest that more particles are moving at a faster speed than the low temperature experiment. In fact, the most likely speed is larger for higher temperatures, similarly the average speed is larger for higher temperatures.

Now that we have seen that gases at different temperatures have a distribution of speeds, and each temperature for a given gas has a different average speed let's move onto defining thermal energy.

Thermal energy (Eth)

Thermal energy is a measure of the microscopic kinetic energy of a gas. To get a better understanding of what this statement means, let's consider a container of a monatomic gas that sits on top of a skateboard moving at a constant speed to the right at shown in the figure below.



First let's consider the center of mass of this system. The center of mass reduces to a single point with a velocity that points to the right. This represents the collective motion of the object as a whole. We can then determine the translational kinetic energy of this collective motion which is the familiar 1/2 m v^2. However, the v that goes into this form of kinetic energy is the center of mass velocity of the entire setup. On top of this collective motion represented by the center of mass, there is also the motion of each individual gas particles within the box that sits on top of the skateboard. Thus we must also include this microscopic translational kinetic energy due to the motion of gas particles. So our system, which includes the skateboard and the gas has a kinetic energy that is both the macroscopic translational kinetic energy from the center of mass motion plus the microscopic translational kinetic energy from the motion of each individual gas particle. There may even be rotational and vibrational kinetic energy of the gas molecules if they are diatomic. It is this microscopic kinetic energy of each individual particle that we refer to as the thermal energy of the gas. (If the particles interact with each other via attractive forces then the thermal energy also includes the energies associated with those interactions. For our class we will not consider gases that interact like that so the thermal energy is just the microscopic kinetic energy).

$$KE_{system} = \underbrace{KE_{cm}}_{\frac{1}{2}M_{tot}|\vec{V}_{cm}|} + \underbrace{KE_{micro}}_{E^{TH}}$$

$$KE_{micro} = \underbrace{KE_{TR} + KE_{ROT} + KE_{VIB}}$$

TOTAL TRANSLATIONAL + ROTATIONAL + VIBRATIONAL KINETIC ENERGY

Since we have covered the macroscopic kinetic energy in detail before, let's just take a look at the microscopic kinetic energy. Let's find the average microscopic translational kinetic energy per particle; add all the individual translational kinetic energies of each particle and divide by the total number of particles (N). If each gas particle has a mass of m, this would look like the following...

$$\overline{ke}_{TR} = \frac{\frac{1}{2}m_1V_1^2 + \frac{1}{2}m_2V_2^2 + \frac{1}{2}m_3V_3^2 + \cdots}{N}$$

TRANSLATIONAL KINETIC ENERGY PER PARTICLE

THE VELOCITIES ARE THE CENTER OF MASS VELOCITY

$$\overline{ke}_{TR} = \frac{\sum_i^N \frac{1}{2}m_iV_i^2}{N}$$

At this point a bit more statistics would allow us to define some useful average quantities. The statistical analysis is not of importance so I will just provide the end result and briefly discuss its meaning.

$$\overline{ke}_{TR} = \frac{\sum_i^N \frac{1}{2}m_iV_i^2}{N} \xrightarrow{\text{STATISTICS}} \overline{ke}_{TR} = \frac{1}{2}\bar{m}V_{RMS}^2$$

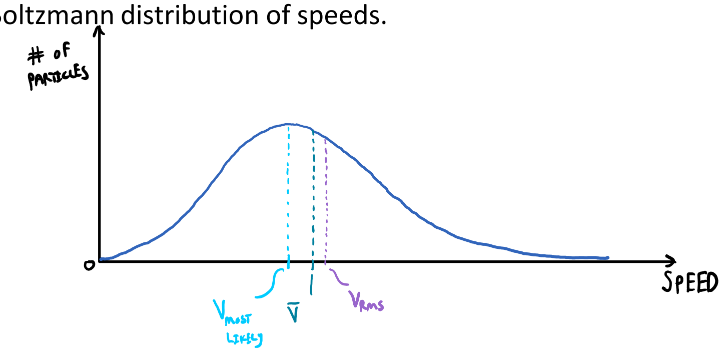
AVERAGE MASS OF A PARTICLE

ROOT MEAN SQUARE SPEED

$$V_{RMS} \equiv \sqrt{\overline{V^2}}$$

MEAN SPEED OR AVERAGE SPEED

What shows up after the statistical analysis is an average mass of each particle (which is just equal to the mass of the molecule if the gas is made up of one type of molecule) and a speed which is called the root mean squared speed. You might cover this more thoroughly in a proper thermodynamics class in your future. For our class all you should take away is that this root mean squared speed is just an average quantity derived from some statistics which allows us to eventually write a nice functional form relating average translational kinetic energy to temperature. Below is a rough sketch if you are curious as to where this root mean square speed would show up on our Maxwell-Boltzmann distribution of speeds.



Back on track, let's find the total translational kinetic energy due to the microscopic motion of the gas particles. To do this, add up all the individual translational kinetic energies of each particle which we found above. This results in the following...

gas particles. To do this, add up all the individual translational kinetic energies of each particle which we found above. This results in the following…

$$\overline{KE}_{TR} = \overline{ke}_{TR1} + \overline{ke}_{TR2} + \overline{ke}_{TR3} + \ldots$$

$$\overline{KE}_{TR} = \sum_{i}^{N} \overline{ke}_{TR_i}$$

$$\boxed{\overline{KE}_{TR} = N\,\overline{ke}_{TR} = N \tfrac{1}{2} \overline{m} v_{RMS}^2}$$

A similar analysis can be done for rotational and vibrational microscopic kinetic energy, however we will not quantify any diatomic molecules so there is no functional form provided for their contributions. Now we can finally construct the thermal energy of the gas which is defined as the total kinetic energy of the gas (plus any interaction energy which in this class we won't cover).

$$E^{TH} = N\,\overline{ke}_{TR} + N\,\overline{ke}_{ROT} + N\,\overline{ke}_{VIB} + NU_{OTHER}$$

TOTAL KINETIC ENERGY

IF GAS IS NOT MONATOMIC THEN THERE CAN ALSO BE ROTATIONAL AND VIBRATIONAL KINETIC ENERGY

ANY INTERACTIONS BETWEEN THE PARTICLES

The above thermal energy relationship is a general form for any type of molecule, but for this class let's go back to monatomic gases that do not have any attractive interactions…

$$\boxed{E^{TH} = N\,\overline{ke}_{TR} = N \tfrac{1}{2} \overline{m} v_{RMS}^2}$$

*FOR MONATOMIC GASES

PRACTICE: If you double the speed of the molecules in a gas, by what factor does the thermal energy of that gas change by?

To relate thermal energy to temperature we rely on what Boltzmann postulated, "each degree of freedom per particle contributes a total of 1/2 k_B T to the total contribution of thermal energy". Here temperature must be in kelvin and the k_B term is known as Boltzmann's constant. To summarize…

BOLTZMANN'S POSTULATE

$$\boxed{E^{TH} = N D \tfrac{1}{2} k_B T}$$

THERMAL ENERGY OF A GAS OF 1 MOLECULE TYPE

PARTICLES IN THE GAS

THE DEGREE OF FREEDOM OF THE GAS MOLECULE

BOLTZMANN'S CONSTANT

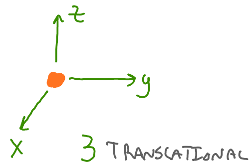
TEMPERATURE

$$*\, k_B \approx 1.38 \times 10^{-23} \, \frac{kg\,m^2}{s^2\,K}$$

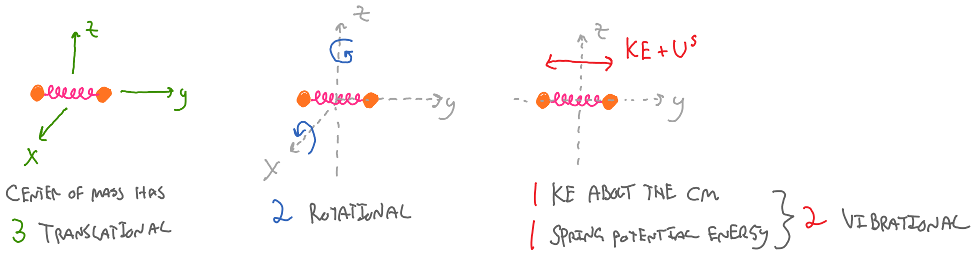
The degree of freedom is a unitless integer which captures type of motion the gas molecule might be able to undergo. For this class, we will only deal with monatomic gases, but the figure below shows the possible degrees of freedom for both monatomic and diatomic gases.

Degree of freedom (D)

Monatomic particle: D = 3



Diatomic particle: D = 7



Since translational motion has 3 degrees of freedom, we can now combine our kinetic theory of gases result with Boltzmann's postulate to get the following relationship…

$$\overline{KE}_{TR} = \frac{3}{2} k_B T$$

By combining these two ideas, we now see that temperature is a macroscopic quantity that is a measure of the average microscopic translational kinetic energy.

PRACTICE: What is the root mean square speed of helium atoms at 2 K? Helium is a monatomic gas with an atomic mass of 4 u.

PRACTICE: If the temperature of a gas doubles, by what factor does the average translational kinetic energy of the particles change by?

Questions for discussion:

(1) We initially introduced a container filled with gas on a skateboard moving at some speed to illustrate the difference between the center of mass velocity and the velocity of the individual particles. If the container was initially on a skateboard at rest at with a temperature of T, would the temperature of the gas increase, decrease, or stay the same if you set the skateboard in motion?
(2) Can you define a temperature of a vacuum?
(3) Can you define a temperature of a single particle?