Diffraction grating and reflection grating

Select LEARNING OBJECTIVES:

- Be able to understand how a wave and interference pattern for a double-slit are similar but different for multi-slit.
- Be able to understand the condition for far field.
- Understand what the fringe order represents.
- Be able to know when and how to use the small angle approximation.
- Be able to understand the features of an interference pattern (symmetries).

TEXTBOOK CHAPTERS:

- Boxsand :: Single and multi-slit interference

WARM UP: Sound is a traveling wave. If you send sound through two slits will you observe an interference pattern? What might be some restrictions on the apparatus used for sound?

In this lecture we will continue to explore light waves interfering after passing through slits. Previously we looked at light passing through two slits. Now we wish to let light pass through n-slits where n is greater than 2. This is often referred to as diffraction grating. Luckily there is no new mathematical forms introduced in this lecture for diffraction grating (i.e. all of the constructive interference conditions for the double-slit apparatus are the same for diffraction grating). There are actually two types of gratings we will discuss, diffraction and reflection gratings.

Before we discuss the two types of grating, let's remind ourselves of the constraints that must be applied for our mathematical models.

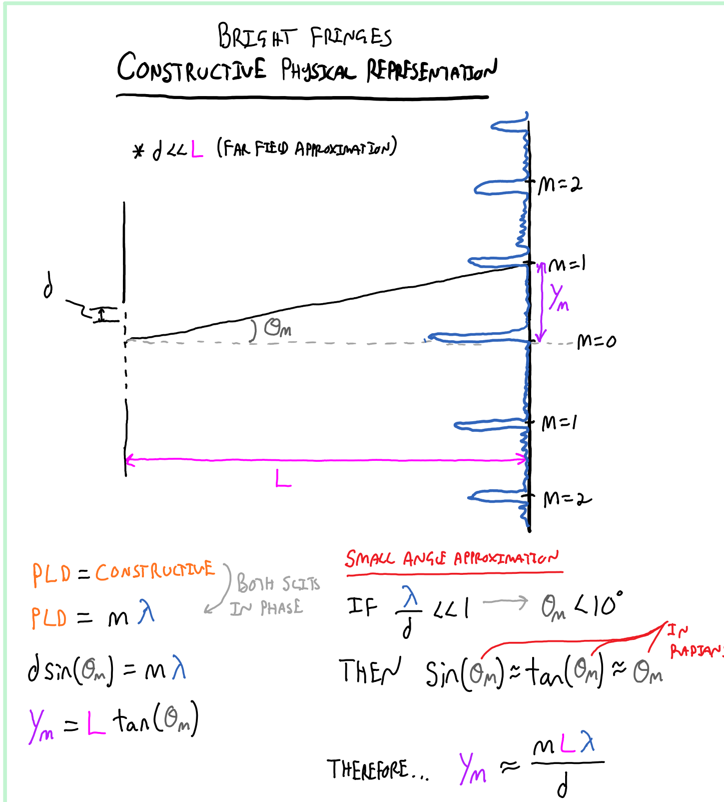
Constraints

- Coherent sources.
- Sources have the same frequency.
- The distance (L) from the sources to the viewing screen must be much greater than the distance between the two sources (d). This is often referred to as the "far field approximation" or "Fraunhofer diffraction" .
  $L >> d$

Diffraction grating

A diffraction grating lets light pass through many slits close together. The light is diffracted as it passes through the slits, hence "diffraction" grating. This is analogous to Young's double-slit experiment, except there are "n" number of slits (often on the order of 500 or more). The last page of this lecture is a detailed physical representation of transmission grating.

The last page is a detailed way of representing diffraction grating. When solving problems, it would become very tedious to sketch all the information that is included on those two pages. Thus below is a shorthand physical representation that I recommend using when working on problem. Remember that drawing a picture and labeling physical quantities greatly helps identify the underlying physics and it also helps get you started on the right path towards a solution.



Some important features to notice is that diffraction grating interference patterns produce much more distinctive bright fringes with large regions of basically dark spaces. For this reason we only consider a mathematical model for bright fringes. The bright fringes are also much more brighter and sharper than the double-slit apparatus. Because of these features, diffraction grating makes for a more precise device for measuring wavelengths.

Since light passes through the slits, diffraction grating is also referred to as transmission grating.

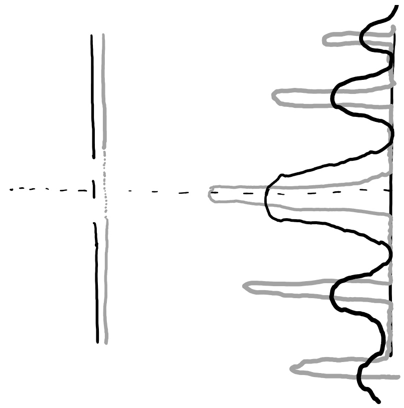
Reflection Grating

A reflection grating does not let light pass through slits, instead it reflects light off of peaks and valleys of a material. The spacing of each peak is analogous to the spacing between slits of diffraction grating (d). Thus, the same mathematical representation as diffraction grating can be used to model the behavior of reflection grating.

PRACTICE: A double slit and a diffraction grating experiment is setup. The slit spacing is the same as the spacing between lines in the grating and the same coherent light is sent through both. If the distance from the scattering target is the same in both cases, which of the following statements are true?

(a) The bright fringes are twice as far apart in the case of the grating.
(b) The bright fringes are sharper in the case of the grating.
(c) The dark fringes are wider in the case of the double slit.
(d) The uncertainty in a measurement is the same in both cases.
(e) The uncertainty in a measurement is greater in the case of the double slit.
(f) The uncertainty in a measurement is greater in the case of the multi-slit.
(g) The spacing between the bright fringes is the same in both cases.

SAME EQUATION FOR BRIGHT SPOT



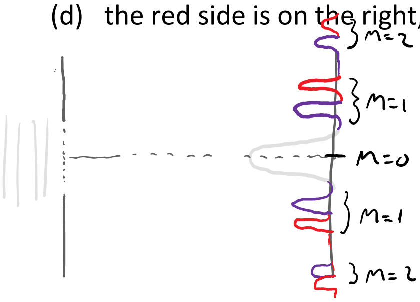
SAME EQUATION FOR BRIGHT SPOT

BUT GRATINGS ARE MUCH SHARPER

AND BRIGHTER

PRACTICE: White light passes through a diffraction grating and forms rainbow patterns on a screen behind the grating. For each rainbow,

(a) the red side is farthest from the center of the screen, the violet side is closest to the center.
(b) the red side is closest to the center of the screen, the violet side is farthest from the center.
(c) the red side is on the left, the violet side on the right.
(d) the red side is on the right, the violet side on the left.



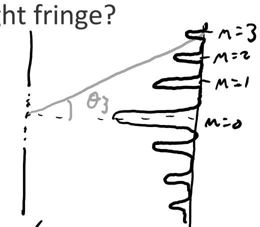
$d\sin\theta_m = m\lambda$

AS $\lambda \uparrow$, $\sin\theta \uparrow$ THUS $\therefore$ $\theta \uparrow$

$\lambda_R > \lambda_V$

PRACTICE: 600-nm-light passes through a diffraction grating with 2500 lines per centimeter. At what angle is the 3rd order bright fringe?

1. 41.1°
2. 12.4 °
3. 8.63 °
4. 26.7 °
5. 17.3 °



$d\sin\theta_3 = 3\lambda$

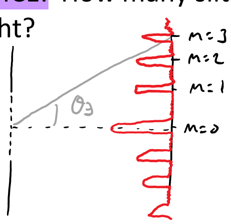
$\theta_3 = 26.7°$

$\frac{\text{SLITS}}{\text{DIST}}$

$d \rightarrow \frac{\text{DIST}}{\text{SLIT}}$

$\frac{1}{25000}\frac{\text{cm}}{\text{SLITS}} \times \frac{1\text{ m}}{100\text{ cm}} = 4\times10^{-6}\ \text{m/SLIT} = d$

PRACTICE: How many slits per centimeter does a grating have if the third order fringe occurs at a 15° angle for 640 nm light?



PLD = CONST

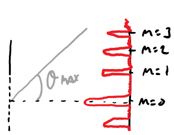
PLD = $m\lambda$

$d\sin\theta_m = m\lambda$

$d = \frac{3\lambda}{\sin\theta_3} \approx 7.186\times10^{-6}\ \text{m}$

$\frac{1}{d} = 139150\ \frac{\text{SLITS}}{\text{m}} \times \frac{1\text{ m}}{100\text{ cm}} \approx 1392\ \frac{\text{SLITS}}{\text{cm}}$

If a screen is placed 115 cm away from the grating, how many total fringes will be observed?



$\theta_{max} = 90°$

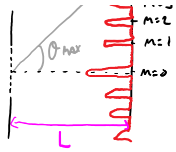
$d\sin\theta_{max} = m_{max}\lambda$

$d = m_{max}\lambda$

$m_{max} = \frac{d}{\lambda} \approx 11.6 \rightarrow$ 11 ON TOP SIDE

+ 11 ON BOTTOM

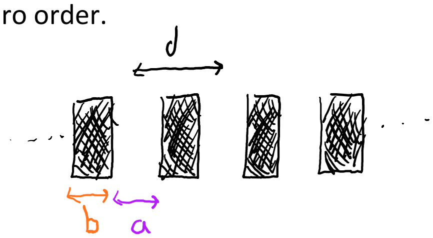
+ CENTRAL MAX



$d = m_{max}\lambda \rightarrow M_{max} = \frac{d}{\lambda} \approx 11.6 \rightarrow$ 11 on top side
+ 11 on bottom
+ central max
23 total

PRACTICE: A diffraction grating is described by its groove spacing **d**. But each groove consists of a clear aperture **a** and an opaque part **b**. For fixed **d** how does changing **a** and **b** (the ratio of **a/b**) affect the diffracted light?

(a) As **b** increases and **a** decreases, more light is transmitted but the diffraction is strong.
(b) As **b** increases and **a** decreases, more light is transmitted and the diffraction is weak.
(c) As **b** increases and **a** decreases, less light is transmitted but the diffraction is strong.
(d) As **b** increases and **a** decreases, less light is transmitted and the diffraction is weak.
(e) As **b** decreases and **a** increases, more light is transmitted, diffraction increases and most of the light goes into higher orders.
(f) As **b** decreases and **a** increases, more light is transmitted, diffraction decreases and most of the light goes into zero order.



SMALLER APERATURE … MORE DIFFRACTION

Applications of diffraction grating

As we have seen, diffraction gratings diffract light more (i.e. the first maximum is bent more) than double slits. Thus diffraction gratings are more widely used in various applications. Below we will look at how physicists use diffraction grating in a few different fields.

Spectroscopy

Spectroscopy relates to the use of interference patterns to determine material composition. As it turns out, every element has a unique fingerprint of light that it emits.

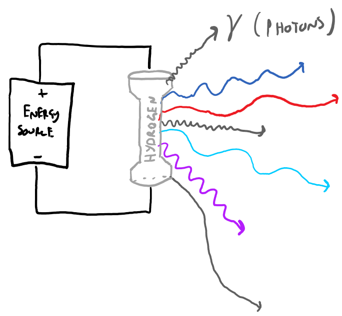
Recall that objects with a temperature above 0 K radiate energy via electromagnetic waves. From a classical stand point, one might argue that the microscopic vibrations of the subatomic particles in atoms have a continuous distribution of energies, thus the EM waves emitted via radiation are also continuous. Unfortunately, this is where the classical view breaks down. Recall from last lecture; in 1905 Einstein confirmed that light behaves like particles called photons. Our best mathematical models of particles (e.g. light, electron, protons etc…) now rely on quantum mechanics which asserts that each particle has an associated wave function that relates to the probability of finding the particle at a certain point in space and time. This interpretation known as the wave-particle duality.

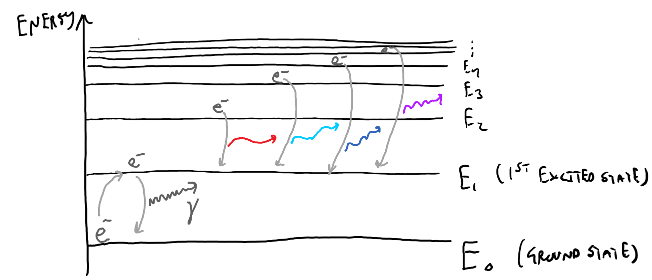
The wave function describes the state of the system (e.g. a systems position and momentum), and this state is related to probabilities of finding a particle with at a particular position or with a particular momentum at a given time. Even though this probabilistic description of particles is far removed from our classical intuitions, we fall back to the same basic fundamental question: How does the state of my system evolve in time? The evolution of this state is governed by the Schrödinger equation which plays the same role as Newton's 2nd law does in classical mechanics. The Schrödinger equation is deeply connected with the energy of the system you are analyzing. Thus, one can use the Schrödinger equation to find the energies of atoms (e.g. hydrogen atom). Since atoms are made up of bound subatomic particles (electrons and protons), the boundary conditions lead to only discrete energies allowed. If you recall our standing wave discussions this sound like a familiar concept. When waved bounced off of boundaries, standing waves were formed only in discrete multiples of integers (m=1,2,3,4…), which also means that the energy of each standing wave was a discrete value for each particular m-value. Because atoms can only have discrete energy states, the energy radiated away in the EM waves atoms emit can also only be discrete energies.

This is some pretty dense concepts so let's recap.

- Atoms with temperature above 0 K radiate EM waves.
- Atoms are only allowed to be in discrete energy levels as per our quantum mechanics model.
- Since atoms radiate EM waves that carry energy away, and atoms can only exist in discrete energy levels, then the EM waves that are emitted are only discrete frequencies (i.e. energy).

At normal temperatures hydrogen does not emit EM waves in the visible spectrum. But we can put energy into hydrogen to rise its temperature, allowing it to emit light in the visible spectrum. Below is a sketch of this process.





At this point in our studies, I do not expect you to commit to memory the discussion above about quantum mechanics. I only seek to help motivate how the light being emitted from each element has its own unique discrete spectrum of wavelengths. Thus, if we send light form an unknown source through a diffraction grating, the grating will diffract the different wavelengths at different angles, allowing us to identify which wavelengths the light was made up of. Once we know the spectrum of wavelengths from the unknown source, we can compare it to the spectrum of known elements to deduce what the source was made of.

PRACTICE: Physicists analyze the electromagnetic spectrum of astrophysical objects to make inferences about which of the following?

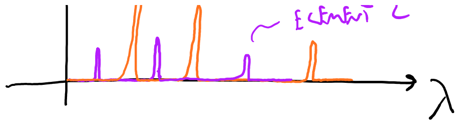
i. Temperature. ~ ALL OBJECTS W/ $T > 0$ K RADIATE EM WAVES
ii. Velocity. ~ DOPPLER SHIFT
iii. Gas pressure. ~ P AND TEMP ARE RELATED .... THERMO!
iv. Overall composition. ~ "FINGER PRINTS" OF ELEMENTS

PRACTICE: The spectral lines of a distant star are shown to match only two elements. What features of the lines can be used to determine the percentage of each element in the star?

i. Frequency.
ii. Wavelength.
iii. Intensity.
iv. Doppler shift

$I_1 > I_2$

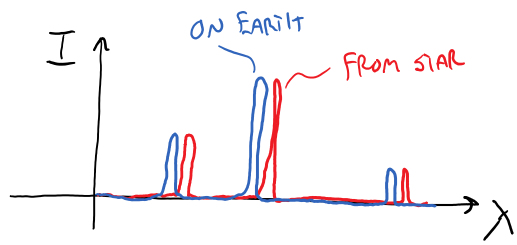
ii. Wavelength.
iii. Intensity.
iv. Doppler shift.



$I_1 > I_2$

THERE IS MORE OF ELEMENT 1

PRACTICE: What feature of the spectral lines could be used to determine the relative motion of the star to Earth?



NOTE: EACH PEAK IS SHIFTED TO GREATER λ

$c = \lambda f$

c = ALWAYS A CONSTANT

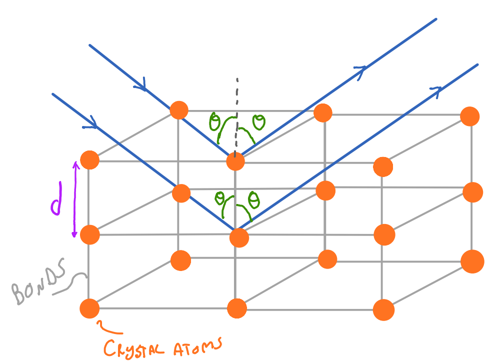
$\lambda \propto \frac{1}{f}$

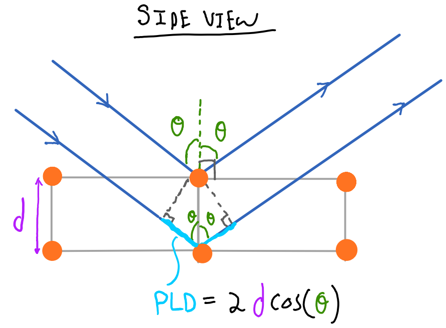
IF $\lambda \uparrow$ THEN $f \downarrow$ } IF $f \downarrow$ THEN $|\Delta \vec{f}_{E\,STAR}| \uparrow$

STAR MOVING AWAY

Crystallography

Crystallography uses EM wave interference patterns to determine the 3-D structure of crystals. Typically X-rays are used to observe the interference patterns when they reflect off crystals; this process is known as X-ray diffraction. Can you predict why X-rays are used? To help illustrate X-ray diffraction, consider a simple cube shaped crystal as shown below. Light is incident on this crystal mostly passes through, but some light is reflected off the atoms in each plane.





As illustrated in the figure above, the PLD between two light waves that reflect off of two adjacent rows is 2 d cos(θ). Thus the condition for constructive interference can be written as follows.

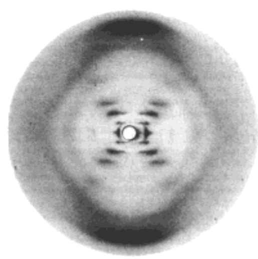
PLD = CONSTRUCTIVE ... AS THE "BRAGG CONDITION" rows is 2 d cos(θ). Thus the condition for constructive interference can be written as follows.

PLD = Constructive

$$2\,d\cos(\theta) = m\lambda$$

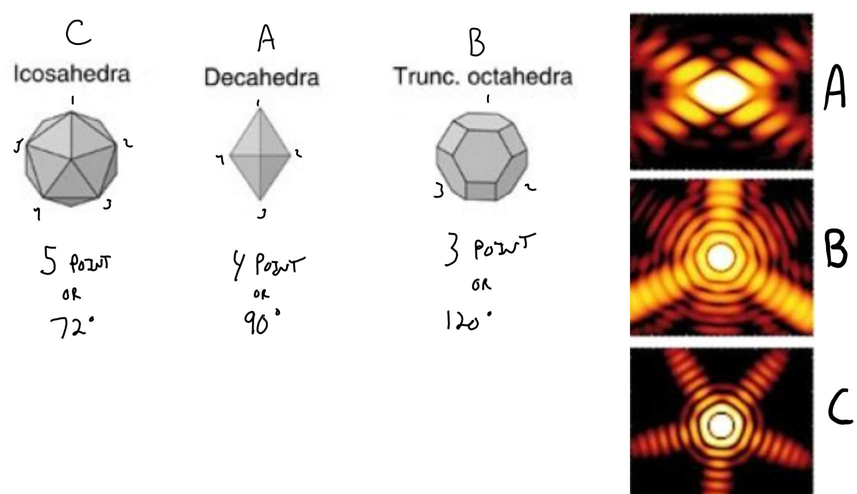
This is known as the "Bragg Condition"

The figure above is known as a cubic structure. Thus this Bragg condition is valid for crystals with cubic structures. We are now at the point where we can ask, how do you determine the structure from the interference pattern? The answer lies in symmetries. The symmetry of the observed interference pattern maintains the same symmetry of the object that the light was scattered from. You are already familiar with this. The double slit experiment with vertical slits produces an interference pattern that spreads out horizontally. If you rotate the slits 90 degrees to the right so that the slits are now horizontal, the interference pattern is also rotated 90 degrees to the right and is now vertical. If you now rotate the slits 90 degrees to the right again, they are vertical and since each slit is identical the interference pattern is identical to the original vertical position.



The image to the left is an X-ray diffraction pattern of DNA first imaged by Raymond Gosling, a graduate student under Rosalind Franklin in 1953. From this interference pattern they were able to determine the double-helix structure of DNA.

PRACTICE: The scattering pattern for 3 different geometries are shown in the figure. The three geometries, which were used to scatter off, are also shown. Match each target with their associated scattering pattern.



Symmetry: 5 point or 72° | 4 point or 90° | 3 point or 120°

Nature Communications

QUESTIONS FOR DISCUSSION:

(1) Explain why you see different colors on the surface of a CD.