Optical element combinations

Select LEARNING OBJECTIVES:

- Be able to apply the thin lens equation for converging and diverging thin lenses, as well as spherical mirrors when part of a multi element arrangement.
- Use correct sign conventions for the thin lens equation based off of the optical element type and location of objects and images.

TEXTBOOK CHAPTERS:

Boxsand :: [Thin lens equation]

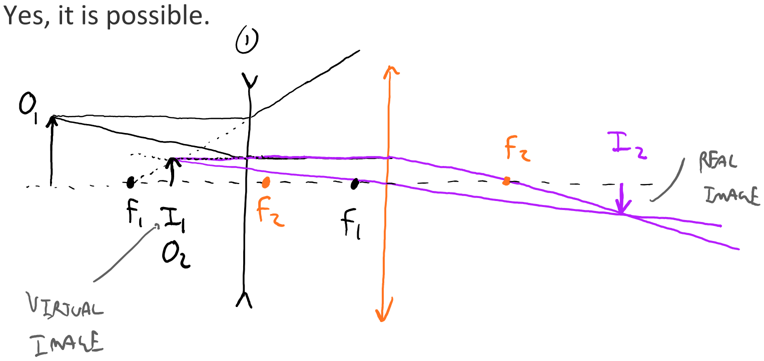
WARM UP: An object very far away from a convex mirror is moved toward the mirror at a constant speed. Does the image move faster, slower, or at the same speed?

We have studied how single thin lenses and spherical mirrors change the behavior of light, namely they bend/reflect the light, the result of which creates images of the original object. There are many applications for these simple optical elements that we have studied so far. And the fun doesn't stop at single optical elements, there are even more applications if you decide to place two or more lenses in line with each other. This lecture will look at how to analyze optical elements in combinations.

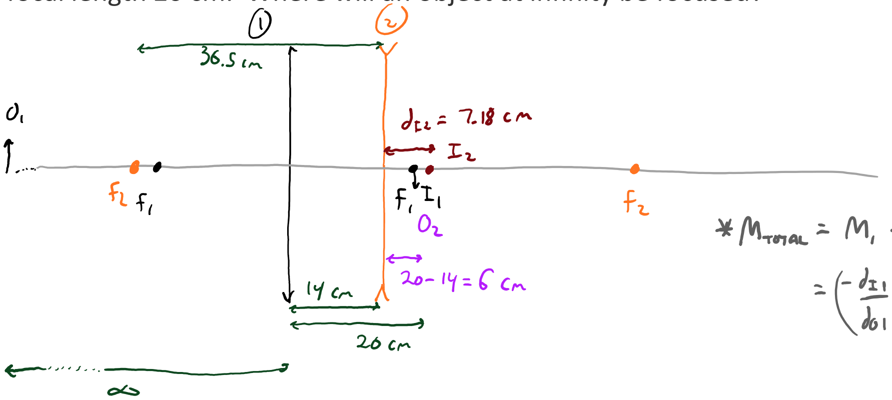
Luckily, we do not have to add to much to our discussion about lenses to be able to analyze lens combinations. The procedure is rather beautiful. When two or more lenses are aligned, you treat each lens as if the other didn't exist, then use the image from the first, as the object for the second, then the image of the second as the object for the third and so on. Basically, if there are 3 lenses in your device, you are doing three single-lens-problems one at a time. Of course, it can't be that easy, there is a new sign convention we must introduce when working with lenses in combination. If the image from the first lens winds up on the far side (right side) of the second lens, we treat it as a virtual object which means the object distance is new negative. Learning the subtleties of these lens combinations is best done through practice problems as given below.

PRACTICE: In a combination of two optics elements, would it be possible to use the second element to make a real image of a virtual image formed by the first lens?

1. No, it is not possible.
2. Yes, it is possible.



PRACTICE: A diverging lens with focal length of 36.5 cm is placed 14 cm behind a converging lens with focal length 20 cm. Where will an object at infinity be focused?



$$* M_{TOTAL} = M_1 \cdot M_2$$
$$= \left(-\frac{d_{I1}}{d_{o1}}\right)\left(-\frac{d_{I2}}{d_{o2}}\right)$$

1ST LENS... IGNORE 2ND

$f_1 (+)$

$d_{o1} (+)$

$$\frac{1}{d_{o1}} + \frac{1}{d_{I1}} = \frac{1}{f_1}$$

$$\cancel{\frac{1}{\infty}}^{\,0} + \frac{1}{d_{I1}} = \frac{1}{20\text{ cm}}$$

$$d_{I1} = 20\text{ cm}$$

2ND LENS IGNORE 1ST

$f_2 (-)$

$d_{o2} (-)$ IMAGE 1 IS OBJECT 2 AND IMAGE 1 IS ON FAR SIDE OF LENS. SO THIS IS A "VIRTUAL OBJ"

$$\frac{1}{d_{o2}} + \frac{1}{d_{I2}} = \frac{1}{f_2}$$

$$\frac{1}{-6} + \frac{1}{d_{I2}} = \frac{1}{-36.5}$$

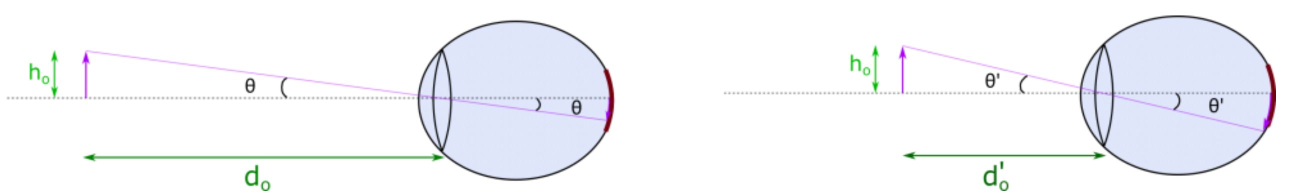
$$\boxed{d_{I2} \approx +7.18\text{ cm}}$$

FAR SIDE OF LENS 2

Applications

Before we list some common application of lenses in combination, we must first talk about angular magnification.

Consider placing an object with height $\mathbf{h_o}$ at some distance $\mathbf{d_o}$ away from your eye. If we look at the special ray from the tip of the object through the center of the lens, we see that this ray makes an angle of $\boldsymbol{\theta}$ with respect to the optical axis. Now move this same object of height $\mathbf{h_o}$ to a distance closer to the eye at a location $\mathbf{d'_o}$. Notice the angle from the same special ray is larger, so we label it as $\boldsymbol{\theta'}$. This thought experiment is shown in the figures below.



Notice that when the object moves closer and the angle increases, the image formed on the retina is larger, thus we perceive the object as being larger when it is closer to our eye. Keep in mind that the actual object has not changed height, the image we see of the object is what has increased from one location to the next. We quantify this apparent increase in size with *angular magnification.* To define angular magnification with a mathematical representation we must first consider the tangent of each angle for each case shown above.

$$\tan\theta = \frac{h_o}{d_o} \qquad\qquad \tan\theta' = \frac{h_o}{d_o'}$$

SMALL ANGLE

$$\tan\theta \approx \theta$$

$$\theta \approx \frac{h_o}{d_o} \qquad\qquad \theta' \approx \frac{h_o}{d_o'}$$

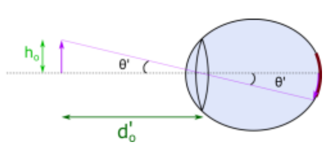
$$\text{ANGULAR MAGNIFICATION} \equiv M = \frac{\theta'}{\theta} \approx \frac{d_o}{d_o'}$$

We can see that as we move the original object from its original location $\mathbf{d_o}$, if $\mathbf{d'_o}$ decreases, then the angular magnification increases. We will use this angular magnification to help analyze some of the applications of lens combinations below.

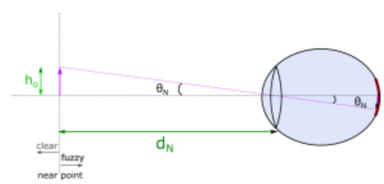
<u>Simple microscope (magnifying glass)</u>

The first application of lens combinations is a simple magnifying glass. Here we will only focus on magnifying glasses that use converging lenses, but know that it is also possible with a diverging lens. You might be thinking, wait a magnifying glass is only one lens, not 2 or more; we consider the eye as the second lens.

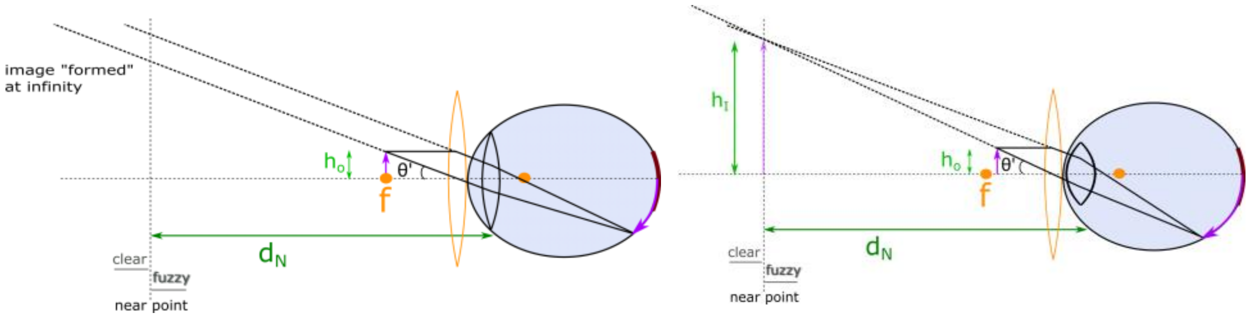
Study the angular magnification images from above carefully; as we move an object closer to our eye, the image formed on our retina gets larger, thus we perceive the object as being larger. Recall that our eyes have a near point which represents the minimum distance an object can be from our eye lens before we cannot clearly focus the image anymore. Thus any closer than the near point the object still looks larger but is out of focus and any fine details are lost to the fuzziness of the image. It is natural to then use the near point distance as the original object distance ($\mathbf{d_o}$) in the angular magnification expression.



$$\boxed{M = \frac{\theta'}{\theta_N} \approx \frac{d_N}{d_o'}}$$



Using this expression for angular magnification is useful because it now references the angular magnification relative to the maximum possible size an eye can see without the aid of any optical elements. The angular magnification as defined above is often referred to as the *magnification power.* Our goal is to get the largest magnification of an object, and we already know an eye by itself can do no better than brining the object to the near point. To do any better we need to use an optical element to bring the object closer than the near point, thus increasing the angular magnification, yet still being allowing our eyes to properly focus on the image. A magnifying glass is one such optical element which is used to help being the object closer than the near point, yet still allows the eye to focus on the image because the image is not closer than the near point. Below are figures showing how a magnifying glass, placed right at the eye, works for two different object locations: one where the object is at the focal point of the magnifying lens, and another where the object is at just the right distance such that the image formed is formed at the near point (**d_N**) of the eye. Note that the angles in the images are exaggerated to emphasize the difference between the two scenarios.



Notice some important features of the images above. When the object is placed at the focal length of the magnifying glass (orange lens) the object is further from the eye, thus a smaller angular magnification. Likewise, the object is closer to the eye lens thus a larger angular magnification is achieved for the scenario on the right. If the object is placed closer to the eye than shown on the right, the image is formed closer than the near point, thus the image will appear larger but will be out of focus. Also note, that when focusing on an image formed at infinity the eye lens is relaxed with a large radius, but when the image is at the near point, the eye lens need to strain to form the image on the retina.

To find the mathematical model for the angular magnification for a magnifying glass, we need only to apply the thin lens equation with our new definition for angular magnification as shown below.

$$\frac{1}{f} = \frac{1}{d_o'} + \frac{1}{d_I} \qquad M = \frac{d_N}{d_o'}$$

$$\frac{1}{f} = \frac{M}{d_N} + \frac{1}{d_I}$$

$$\boxed{M = d_N\left(\frac{1}{f} - \frac{1}{d_I}\right)}$$

IMAGE FORMED BY MAGNIFYING GLASS IS VIRTUAL, THUS d_I IS NEGATIVE, SO LETS PULL THE NEGATIVE OUT SO ALL VARIABLES ARE POSITIVE

$$M = \frac{d_N}{f} + \frac{d_N}{d_I}$$

From the above result, with all variables positive, it is clear that the smallest magnification from the magnifying glass is achieved when **d_I** is infinity, and the largest while still being clear is when **d_I** is at the near point. When **d_I** is less than the near point, the magnification is larger but the eye can no longer focus the image so it is not useful. The value of the angular magnification (a.k.a. magnification power) is often written with an X after the numerical value; for example, if the angular magnification is 5, then it would be written as 5 X, often referred to as "five X power". Finally, with the mathematical model, you should notice that we can also make the angular magnification larger by choosing a smaller focal length lens for the magnifying glass. However, there are limitations as to how small of a focal length lens we can use before other aberrations become an issue, thus we need to look elsewhere for lager magnification devices as discussed in the sections below.

Compound Microscope

Under construction.

Telescope

Under construction.

QUESTIONS FOR DISCUSSION:

1. When you place a straw in a glass of water at an angle, it looks as though the straw bends. Explain this observation with the concepts covered in this lecture.