Ray model of light and Snell's law

Select LEARNING OBJECTIVES:

- Understand when the ray model of light is applicable.
- Be able to apply Snell's Law of Refraction to any system.
- Be able to identify all relevant angles in a given system.
- Be able to identify the conditions for Total Internal Refraction.
- Be able to improve use of geometry in analysis.
- Understand the wavelength dependence of index of refraction.

TEXTBOOK CHAPTERS:

- Boxsand :: Snell's law of refraction

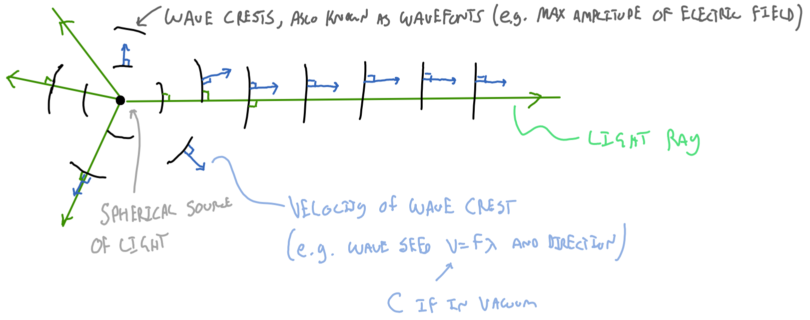
WARM UP: Sound is a traveling wave. If you send sound through two slits will you observe an interference pattern? What might be some restrictions on the apparatus used for sound?

We have explored many ways to model the interference effects of light waves. In fact, the defining characteristic feature of all waves is that they interfere. However, if you recall, the interference patterns obtained in our previous discussions relied on the constraint that the wavelength of the light must be on the order of the size of the object the light encounters. Our normal everyday observations involve light encountering large object (i.e. the size of the objects are much much greater than the wavelength of the light). When light encounters objects with a size much greater than the wavelength of light, the wave nature of light (e.g. interference pattern) is suppressed. We therefore can ignore our mathematical models of light as a wave and invoke some simple geometric principles that light seems to follow under these conditions. This description of light is often referred to as geometric optics, or the ray model of light (ray optics).

The goal of this lecture is to transition from our wave model into the ray model of light, then explore the predictive power of this new ray model.

Ray model of light

Consider a spherical source of light as shown below. You are hopefully familiar with the notion that the light waves are 3-D spherical shells of oscillating electric and magnetic files.

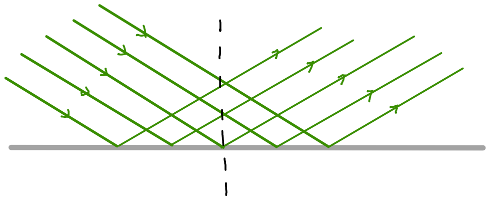


The 2-D mechanical analogy to this is a pebble dropped into water creating circular waves of oscillating particle displacement. Notice that the light waves far from the source are very nearly parallel in any given direction. The wave crests travel at the wave speed, for light in a vacuum this is "c". This should be all review so far. The new concept here is the light ray. We argued in the introduction that the wave nature of light is suppressed when it encounters objects with a size larger than the wavelength of light; so rather than using a wave diagram with crests and troughs we use "light rays" which start form the source and travel perpendicular to the wave crests (i.e. in a straight line). Also notice that a source of light emits an infinite number of light rays but only 4 are shown. You might be curious as to why the light rays must be a straight line? This comes from our observations, for instance the shadow you cast can be predicted based off the assumption that light travels in a straight line. Speaking about assumptions, now is a good time to list our assumptions for our new ray model of light.

Assumptions

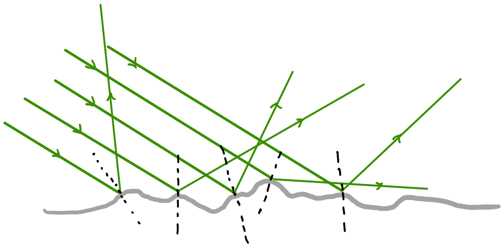
- Light travels in a straight line.
- Rays can cross each other undisturbed.
- Light travels in a straight line until it interacts with matter.
    - Reflection
        - Reflections off surfaces follow the following rule: The angle of incidence is equal to the angle of reflection. Below is two examples of reflections form a rough surface and smooth surface. In both cases each individual ray follows the rule stated above.

SMOOTH (MIRROR-LIKE) SURFACE



SPECULAR REFLECTION

ROUGH SURFACE (EXAMPLE: PAPER)



DIFFUSE REFLECTION

- Transmission
    - When waves encounter a boundary, part of the wave is reflected and part is transmitted. See the Snell's law section for more details.

- Absorption
    - Any time light travels through a medium other than a vacuum, some of the light gets absorbed. In other words, the intensity of light decreases due to some of the light energy being absorbed by the media. This is mathematically modeled by including an imaginary part to the index of refraction as shown below.

$$\tilde{n} = n + iK$$

COMPLEX INDEX OF REFRACTION

THE REAL PART OF THE INDEX OF REFRACTION... THIS IS WHAT WE USE IN THIS CLASS

THE IMAGINARY PART OF THE INDEX OF REFRACTION REFERRED TO AS THE "EXTINCTION COEFFICIENT" BECAUSE IT IS RELATED TO HOW "FAST" THE LIGHT INTENSITY GETS DIMINISHED.

In practice, this complex index of refraction is only needed for conductors with free charges. For most practical purposes, transparent media has an extinction coefficient very near to zero.

- Scattering
    - When light encounters particles it scatters. Below are two main types of scattering.
        - Rayleigh scattering
            - When the suspended particles that the light encounters are on the order of the wavelength of the light, the scattered light can be modeled via Rayleigh scattering. To explain this process we must invoke the wave nature of light again. Recall that light is an oscillating electric and magnetic field. The oscillating electric field causes the molecules, such as oxygen and nitrogen, to polarize and oscillate. These oscillating dipoles then radiate energy via EM waves in many different directions. It is these radiated EM waves that we see as the scattered light. The amount of Rayleigh scattering is proportional to the inverse of the forth power of the wavelength.

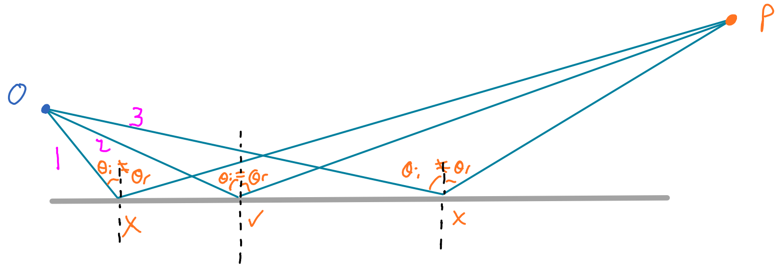
$$\text{SCATTERING} \propto \frac{1}{\lambda^4}$$

This Rayleigh model of scattering correctly predicts why we observe the sky to be blue. Since blue is a shorter wavelength than the other visible colors, it gets scattered more form tiny particles in the atmosphere. Violet has an even shorter wavelength than blue, so why isn't the sky blue? To answer this requires relying on a few different phenomena which I will leave up to you to search for if you are interested.

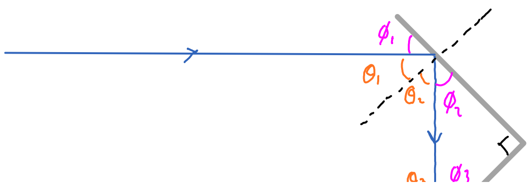
        - Mie scattering
            - When the suspended particles that the light encounters are larger than the wavelength of the light, scattered light is better modeled by the Mie model. This model predicts no strong dependence on the wavelength of the light. Thus all wavelengths are scattered equally strong off of larger particles. Water droplets in clouds are large compared to the wavelength of light. The white color of clouds indicates that all wavelengths are scattered equally which is consistent with Mie scattering.

PRACTICE: Object "O" is a source of light as shown below. Which one of the three paths shown represent light rays that comes from object "O", reflect off the surface and then reach object "P"?

a. Path 1
[✓ b.] Path 2
c. Path 3
d. All paths
e. None of the paths



PRACTICE: Light enters horizontally into the combination of two perpendicular mirrors as shown below. Indicate the direction of the incident light after it reflects off both mirrors.



GEOMETRY

$\theta_1 + \phi_1 = 90$

$\theta_1 = 90 - \phi_1$

$\theta_2 = 90 - \phi_1$

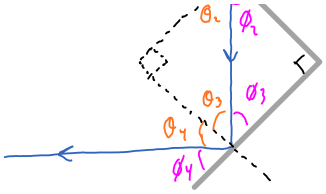
} LAW OF REFLECTION

$\theta_4 + \phi_4 = 90$

$\phi_4 = 90 - \theta_4$

$\phi_4 = 90 - (90 - \theta_2)$

$\phi_4 = \theta_2$



$\theta_2 = 90 - \phi_1$ } LAW OF REFLECTION

$\theta_2 + \theta_3 + 90 = 180$

$\theta_3 = 90 - \theta_2$ } LAW OF REFLECTION

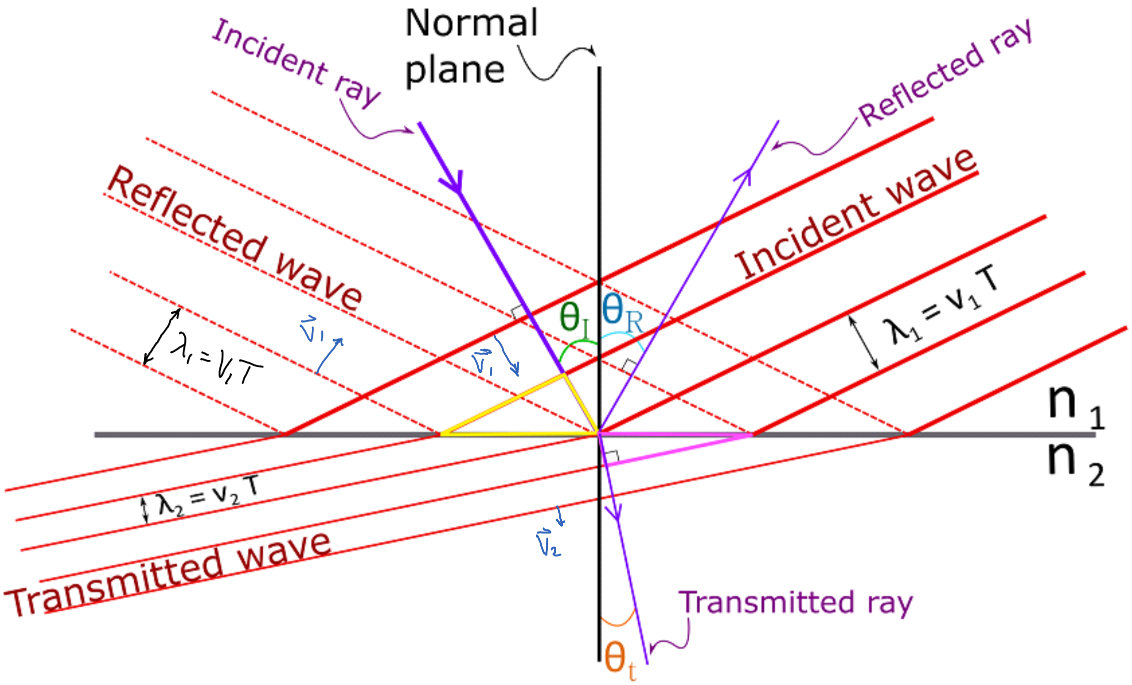
$\theta_4 = 90 - \theta_2$

$\phi_4 = 90 - (90 - \theta_2)$

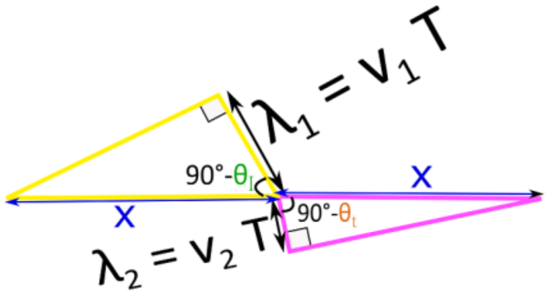
$\phi_4 = \theta_2$

Snell's Law

The first time we encountered light traveling through different media we constrained ourselves to situations where the incident light was always perpendicular to the boundary between the two media that the light was traveling through. As always, some of the light was transmitted, and some was reflected off of the boundary; the transmitted and reflected waves were also perpendicular to the boundary surface. What happens when light is not normal incident on a surface? Well, we already looked at the reflected rays; the reflected angle must be the same as the incident angle. Now what about the transmitted light? To answer this question study the figure below. Light of wavelength λ_1 coming from a medium with an index of refraction n_1 enters a medium with index of refraction n_2. This is represented via the wave model (e.g. the red lines represent wave crests). We alredy studied the effect that differenet media has on light in lecture 35 when we introduced the index of refraction. We know that the wavelength will decrease if n_2 is greater than n_1 as shown in the figure below. By comparing the yellow and pink right triangles we can derive a mathematical expression of how the angle of incidence is related to the angle of transmission (often called angle of refraction). The resulting mathematical model is known as Snell's law.



Snell's Law



Law of reflection

$$\theta_I = \theta_R$$

$$\cos(90^\circ-\theta_I) = \frac{\lambda_1}{X} \qquad \cos(90^\circ-\theta_t) = \frac{\lambda_2}{X}$$

$$\frac{\lambda_1}{\sin(\theta_I)} = \frac{\lambda_2}{\sin(\theta_t)}$$

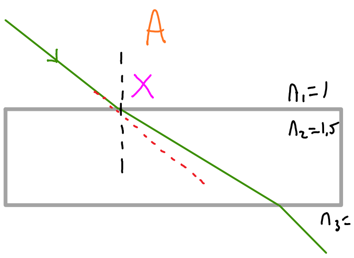
$$n_1\lambda_1 = n_2\lambda_2$$

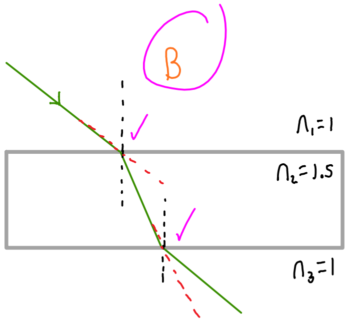
$$\boxed{n_1 \sin(\theta_I) = n_2 \sin(\theta_t)}$$

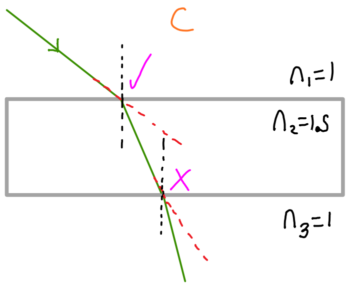
* IF $n_1 < n_2$, Ray REFRACTS TOWARDS NORMAL
IF $n_1 > n_2$, Ray REFRACTS AWAY FROM NORMAL

Notice that the angle of transmission (i.e. angle of refraction) is also measured from the normal. Superimposed on the wave above is the ray model of light (represented in purple). Another important feature is that the incident ray, reflected ray, and transmitted (i.e. refracted) ray all line in the same plane; as shown the plane is the plane of paper you are looking at this image. Basically, the reflected and refracted ray do not bend into our out of the page since the incident is not traveling into or out of the page.

PRACTICE: Which of these ray diagrams are possible?





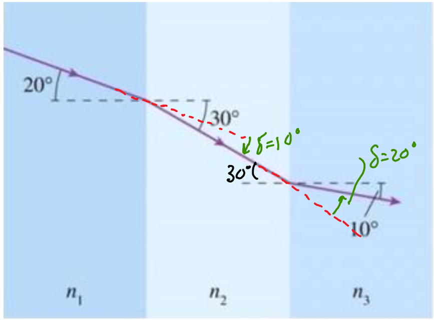


LOOK AT BOUNDARY CONDITIONS ... MUST OBEY SNELL'S LAW

$n_{LOW} \rightarrow n_{HIGH}$ BEND TOWARDS NORMAL

$n_{HIGH} \rightarrow n_{LOW}$ BENT AWAY FROM NORMAL

PRACTICE: Rank the following mediums based on index of refraction.



$n_1 \rightarrow n_2$

BENDS AWAY FROM NORMAL

$n_{HIGH} \rightarrow n_{LOW}$

So

$n_1 > n_2$

$n_2 \rightarrow n_3$

BENDS TOWARDS NORMAL

$n_{LOW} \rightarrow n_{HIGH}$

So

$n_2 < n_3$

$n_1 \overset{?}{\ldots} n_3$

FROM 1→2 or 2→1 $\delta = 10°$ (BENDS "A LITTLE")

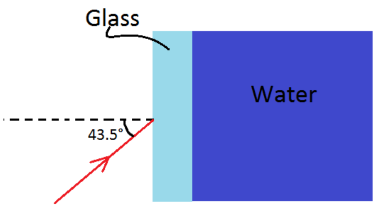
$\boxed{n_3 > n_1 > n_2}$

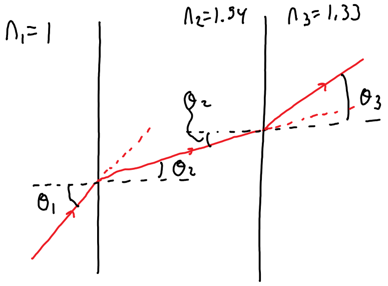
FROM 2→3 OR 3→2 $\delta = 20°$ (BENDS MORE)

SO

$n_3 > n_1$

PRACTICE: An aquarium filled with water has a flat glass sides whose index of refraction is 1.54. A beam of light from outside the aquarium strikes the glass at a 43.5 angle to the perpendicular as shown below. What is the angle of refraction when the light ray enters the water?





LONG

$$\overset{\checkmark}{n_1} \sin \overset{\checkmark}{\theta_1} = \overset{\checkmark}{n_2} \sin \overset{?}{\theta_2}$$

$$\theta_2 = \sin^{-1}\left(\frac{n_1}{n_2} \sin\theta_1\right)$$

$$\theta_2 = \sin^{-1}\left(\frac{1}{1.54} \sin(43.5)\right)$$

$$\theta_2 = 26.55031193$$

$$\overset{\checkmark}{n_2} \sin \overset{\checkmark}{\theta_2} = \overset{\checkmark}{n_3} \sin \overset{?}{\theta_3}$$

$$\theta_3 = \sin^{-1}\left(\frac{n_2}{n_3} \sin\theta_2\right)$$

$$\boxed{\theta_3 \approx 31.2°}$$

SHORT

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 = n_3 \sin\theta_3$$

$$\overset{\checkmark}{n_1} \sin \overset{\checkmark}{\theta_1} = \overset{\checkmark}{n_3} \sin \overset{?}{\theta_3}$$

$$\theta_3 = \sin^{-1}\left(\frac{n_1}{n_3} \sin\theta_1\right)$$

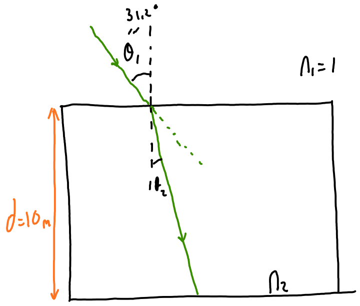
$$\theta_3 \approx \sin^{-1}\left(\frac{1}{1.33} \sin(43.5)\right)$$

$$\boxed{\theta_3 \approx 31.2°}$$

✓

PRACTICE: Light with a wavelength of 569 nm in a vacuum strikes the surface of an unknown liquid at an angle of 31.2° with respect to the normal to the surface. If the light travels at an effective speed of 1.97 x 10^8 m/s through the 10-m-deep liquid, how long does it take for light to travel form the surface to the bottom?

a. 11 ns
b. 29 ns
c. 32 ns
d. 42 ns
e. 54 ns
f. 101 ns



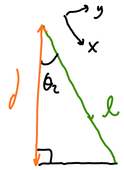
$$\overset{\checkmark}{n_1} \sin \overset{\checkmark}{\theta_1} = \overset{?}{n_2} \sin \overset{?}{\theta_2} \qquad \overset{?}{n_2} = \frac{\overset{\checkmark}{c}}{V_{eff}}\checkmark$$

$$n_1 \sin\theta_1 = \frac{c}{V_{eff}} \sin\theta_2$$

$$\theta_2 = \sin^{-1}\left(\frac{n_1 V_{eff}}{c} \sin\theta_1\right)$$

$$\theta_2 = \sin^{-1}\left(\frac{(1)(1.97\times10^8)}{3\times10^8} \sin(31.2°)\right)$$

$$\theta_2 \approx 19.88724689°$$



$$\cos\theta_2 = \frac{d}{l}$$

$$l = \frac{d}{\cos\theta_2} \approx 10.63418482 \text{ m}$$

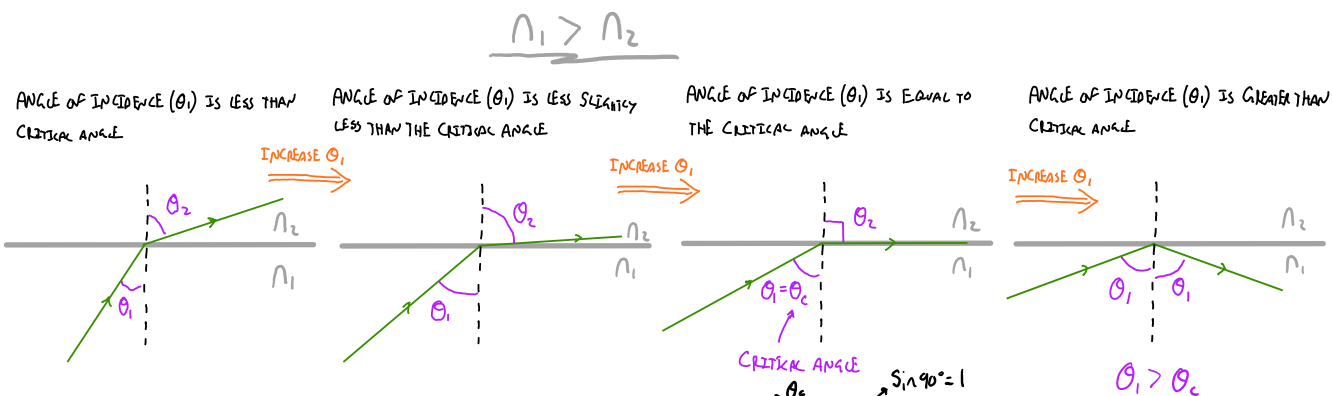
$$\Delta x = V_i \Delta t + \cancel{\tfrac{1}{2} a \Delta t^2}$$

$$l = V_{eff} \Delta t$$

$$\Delta t = \frac{l}{V_{eff}} \approx \boxed{5.4 \times 10^{-8} \text{ sec}} \approx 54 \text{ ns}$$

Total internal reflection

An interesting phenomena occurs under certain geometrical configurations when light travels from a larger index of refraction material to a lower index of refraction material. The phenomena I refer to is known as Total Internal Reflection (TIR). Below are a few sketches to help illustrate TIR.



$$n_1 \sin \overset{\theta_c}{\cancel{\theta_1}} = n_2 \overset{\sin 90^\circ = 1}{\cancel{\sin \theta_2}}$$

$$n_1 \sin \theta_c = n_2$$

$$\boxed{\sin \theta_c = \frac{n_2}{n_1}}$$

* TIR From $n_1 \to n_2$ with $n_1 > n_2$

As you can see from the images above, as you increase the incident angle the transmitted ray gets closer and closer to becoming parallel with the surface. At the critical angle the transmitted ray is no longer "transmitted" into medium 2, rather it is refracted perpendicular to the boundary. Any increase in incident angle from this point will result in the "transmitted" ray being completely reflected back into the medium 1.

It turns out there are many applications that benefit from this total internal reflection phenomena; below are two examples.

Fiber optic cables

Fiber optic cables use TIR to keep light confined inside the glass wires to help transmit information over long distances with little loss in intensity (i.e. signal).

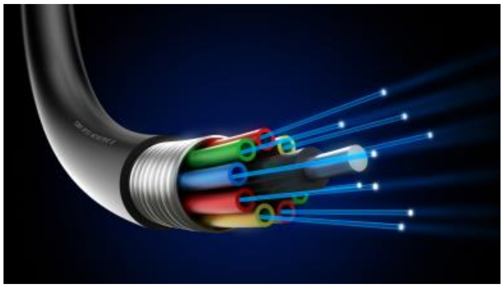
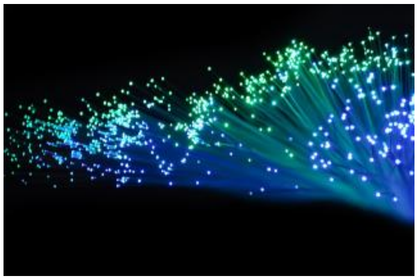
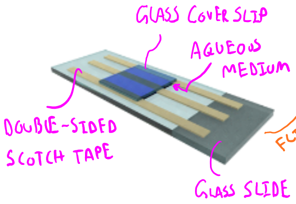


Image credit: http://datatalk.co/communications/fibre-optic-cable-solutions/



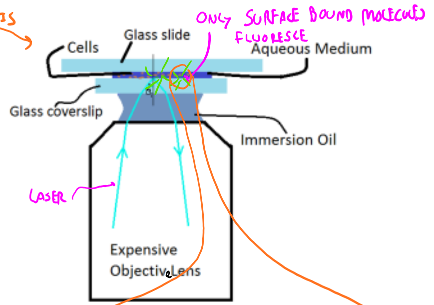
Total Internal Reflection Fluorescence Microscopy (TIRFM)

When light is totally internal reflected, an evanescent wave is created along the surface of the boundary between the two media. This evanescent wave is confined to the boundary surface (i.e. it only penetrates the lower index of refraction material to a depth on the order of the wavelength. Total Internal Reflection Fluorescence Microscopy (TIRFM) uses this surface bound evanescent wave to excite molecules on the boundary surface. When the excited molecules go back to their un-excited state they fluoresce (i.e. they emit light) which is then collected with a microscope and analyzed.



FLIP UPSIDE DOWN AND PLACE ON OBJECTIVE LENS

LOOKS LIKE THIS



VERY CLOSE UP VIEW OF SURFACE BOUNDARY

IMAGE

TURN TIR off

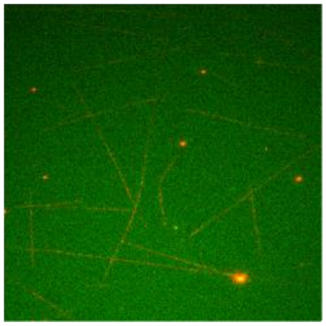


IMAGE WITH $\theta_{INCIDENT} < \theta_c$

NO TIR ... TRANSMITTED LASER EXCITES ALL MOLECULES IN THE AQUEOUS MEDIUM WHICH DROWNS OUT THE IMAGE

"BAD" :(

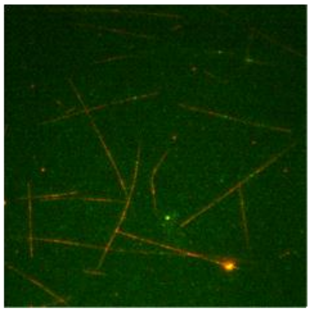
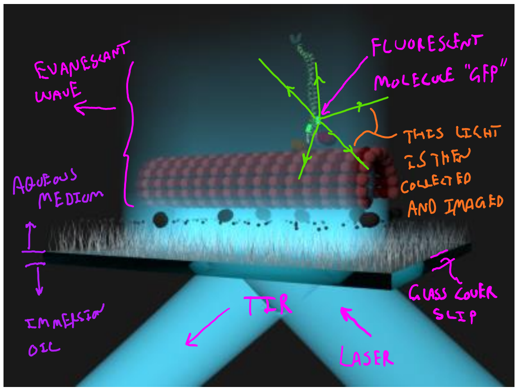


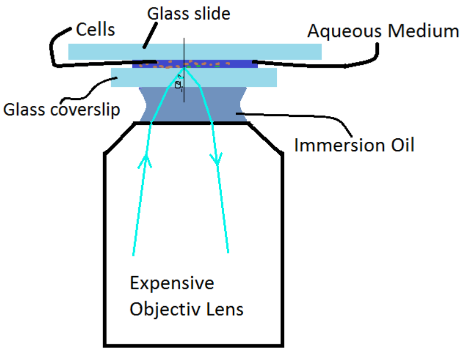
IMAGE WITH $\theta_{INCIDENT} > \theta_c$

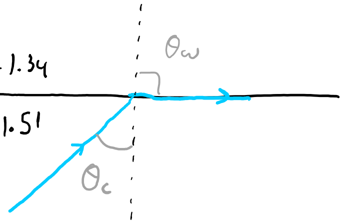
TIR .... ONLY SURFACE BOUND MOLECULES FLUORESCE

"GOOD" :)



PRACTICE: A 488 nm laser is used in a TIRFM microscope like the one shown in the images above. If the aqueous medium has an index of refraction of about 1.34 and the glass coverslip's index of refraction is 1.51, what is the critical angle for light traveling from the glass coverslip to the aqueous solution?





$n_w = 1.34$

$n_g = 1.51$

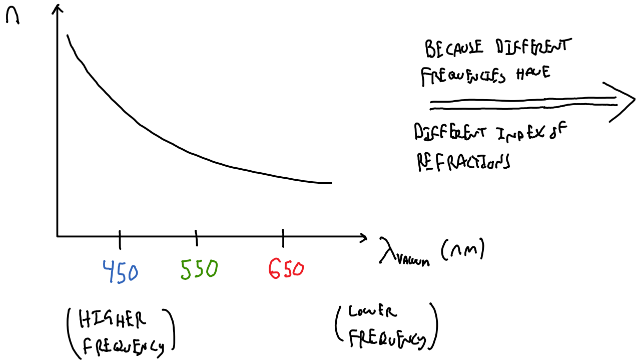
$$n_g \sin\theta_g = n_w \cancel{\sin\theta_w}^{\,1}$$

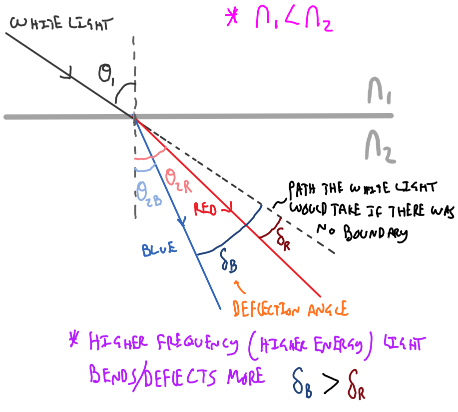
$$n_g \sin\theta_c = n_w$$

$$\boxed{\theta_c = \sin^{-1}\left(\frac{n_w}{n_g}\right)} \approx 62.6°$$

Dispersion

In a vacuum light of all wavelengths travel at the same speed, c. In a medium, the effective speed is slightly different for different wavelengths of light. The dependence of the effective speed of light and the index of refraction on the wavelength is known as dispersion. Below is a plot of the general trend for the index of refractions dependence on wavelength. *Remember, wavelengths change in different medium, thus the wavelength on the axis below is the vacuum wavelength which is a known constant.





There is no nice and easy way to reason out what might causes this frequency dependence (i.e. vacuum wavelength dependence). A not complete but nevertheless interesting model that incorporates what we have learned in this course involves modeling the bound electrons in the transparent medium with spring like bounds to their core atoms which don't move. Then as the light passes through the medium, the oscillating electric field acts as a sinusoidal driving force causing the bound electrons to vibrate back and forth. It turns out accelerating charges (since the electrons are oscillating they are accelerating) emit radiation via EM waves. This radiation of EM waves acts like a damping term because the vibrational energy is being lost to the radiated EM waves. Combine all of these forces together and using Newton's 2nd law to analyze the forces acting on the electrons in the transparent medium results in the expression below.

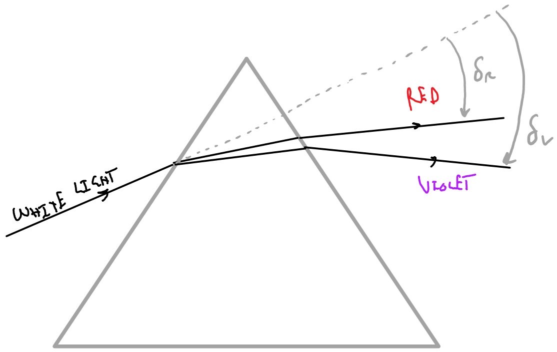
$$\sum \vec{F}_{\substack{\text{ON BOUND} \\ \text{ELECTRON IN} \\ \text{MEDIUM}}} = m_{\text{ELECTRON}}\,\vec{a}$$

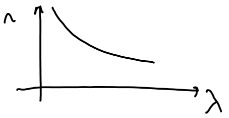
$$\vec{F}_{\text{BINDING}} + \vec{F}_{\text{DAMPING}} + \vec{F}_{\text{DRIVING}} = m\vec{a}$$

$$-kx - b|\vec{v}| + |\vec{F}^{E}|\sin(\omega t) = m a_x$$

At this point you should begin to notice we are just building an equation of motion. This particular equation of motion has forms that we ran into before: a spring like binding force, a velocity dependent damping force, and a sinusoidal driving force.   Now you would hand over the above expression to a mathematician who would then return to you a solution for the position as a function of time x(t). A little more knowledge about electricity and magnetism is required to go further, but that is ok, we are already beyond the scope of this class. Rest assured that following through with this model will lead to the general trend in the frequency dependence of index of refractions. My main goal of going this far is to illustrate just how powerful the physics tools you have in your tool belt are: identifying forces, modeling bonds as spring like forces, recognizing dissipative forces (e.g. damping forces), applying Newton's 2nd law to find an equation of motion, and passing the baton to someone else to do the math part.

PRACTICE: White light enters a glass prism. When the light leaves the prism, the colors have been separated. Match each ray with their color; red or violet.





$\lambda_R > \lambda_V$ .... so $n_R < n_V$

AS n ↑ DEFLECTION ↑

SO VIOLET BENDS MORE

δ

QUESTIONS FOR DISCUSSION:

1. How are rays related to the wave model of light?