Single slit interference

Select LEARNING OBJECTIVES:

- Be able to understand and apply the correct mathematics for single vs double slit which can be misleading.
- Be able to understand the condition for far field.
- Understand what the fringe order represents.
- Be able to know when and how to use the small angle approximation.
- Understand and apply Huygens' principle.

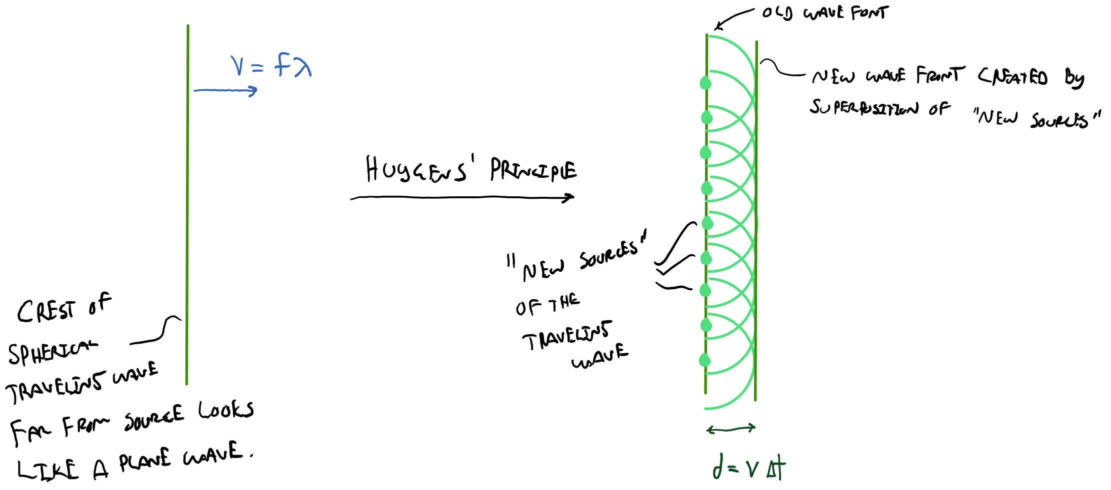
TEXTBOOK CHAPTERS:

- Boxsand :: Single and multi-slit interference

WARM UP: Sound is a traveling wave. If you send sound through two slits will you observe an interference pattern? What might be some restrictions on the apparatus used for sound?

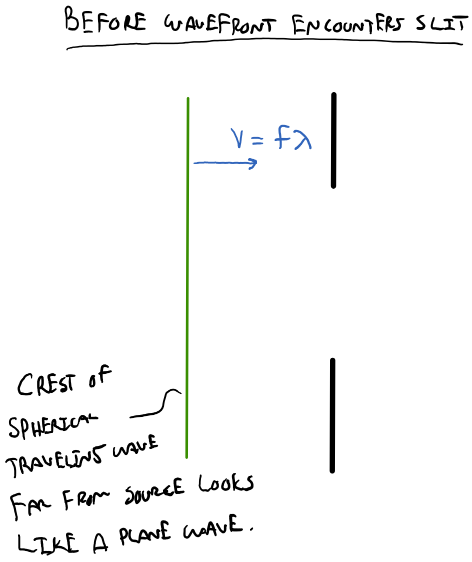
At this point, you hopefully have a solid understanding of the characteristics of waves. With light being a wave, you probably didn't hesitate to buy into the explanation for the observed interference patterns from double or multi slits; that is that each slit acts as a source and light from these sources interfere, as all waves do. But what if I told you, or better you showed you, that light passing through a single slit also produces an interference pattern? Well, light passing through a single slit does produce an interference pattern. We can't use our double and multi slit explanation because there is only one slit, thus only one source, so how does an interference pattern occur? Truly a mind bender. To explain this behavior, a rigorous derivation involves complicated math. However, a fairly simple geometric argument constructed by Huygens provides one explanation for the observed interference pattern. Huygens' principle states that every point on a wavefront (surface on which the disturbance is in the same phase) act as a new spherical source emitting secondary waves at the same frequency as the original traveling wave.

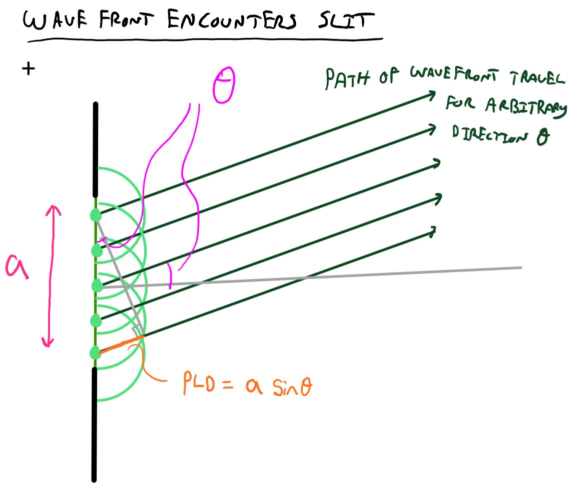
To help visualize this lets consider tracking the motion of a crest from a spherical source of a traveling wave. Far from the source, the radius of curvature of this crest is so large it looks like a plane wave as shown below.



The image on the left is what we typically see far from a source. Although we are talking about spherical sources, it is often helpful to consider water waves (which are confined to 2 dimensions) to help visualize wave fronts. Thus the left image can be thought of as the crest of a water wave far from the source, which you can actually see. Try it, drop a rock in water and follow a crest as it moves away from the source. Huygens' principle states that the disturbance (in this case the crest) propagate because every location in space where the disturbance (crest) exists, the disturbance (crest) creates a new spherical source that emits a traveling wave in the forward direction at the same frequency. This is represented by the image to the right above. Each dot is the "new source" of a spherical wave. As these new waves travel forward, they interfere with each other and we see the net effect, which is another plane wave, but now some distance to the right. This explanation for how waves travel might hold some merit, until you start to think about light waves, what are the "sources" of the new light waves in the vacuum of space? So perhaps this isn't the mechanism for how waves travel, but nevertheless, this model allowed Huygens to correctly predict how waves reflect and refract (transmit through transparent material). Also, this model can also help us argue why interference patterns occur with a single slit.

Shown below is a single slit with a plane wave traveling to the right about to encounter the slit. Once it does, the ends of the wave are blocked, and the empty space where the slit is located becomes a surface of "new sources" as per Huygens' principle. It is the emitted waves from these Huygens' principle sources that are now interfering and creating the interference pattern we observe.



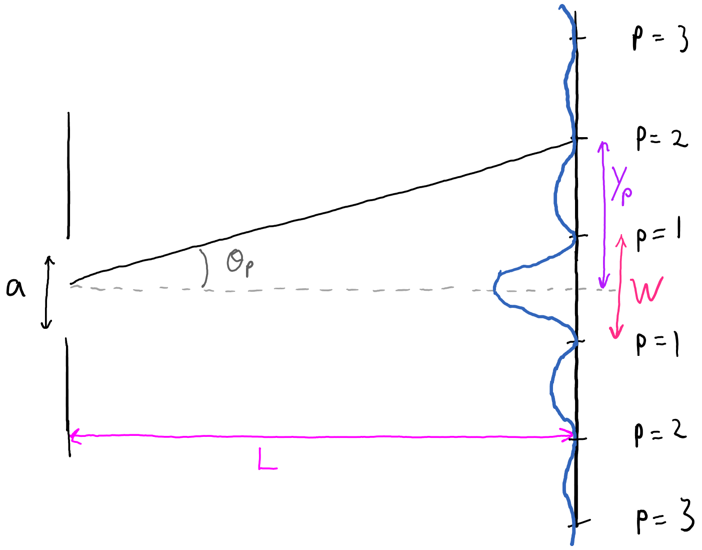


The interference pattern observed from a single slit is similar to double and multi-slit in that there are regions of light and dark locations. Thus a similar mathematical model will be used to predict where locations of fringes will occur. Unlike the seamless transition from double to multi-slit, there is a slight difference with the single slit mathematical model. The functional form remains the same, but the variables represent different locations, thus we use different variables with single slits as shown below.

DARK FRINGES

DESTRUCTIVE PHYSICAL REPRESENTATION

$* \; a \ll L$ (FAR FIELD APPROXIMATION)



PLD = DESTRUCTIVE

$PLD = p\lambda$

$a\sin(\theta_p) = p\lambda$

$y_p = L\tan(\theta_p)$

$W = 2y_1$

SMALL ANGLE APPROXIMATION

IF $\frac{\lambda}{a} \ll 1 \longrightarrow \theta < 10^\circ$

THEN $\sin(\theta_p) \approx \tan(\theta_p) \approx \theta_p$

IN RADIANS

THEREFORE... $y_p \approx \frac{pL\lambda}{a}$

$$W \approx \frac{2L\lambda}{a}$$

Constraints

- Coherent sources.
- Sources have the same frequency.
- The distance (L) from the sources to the viewing screen must be much greater than the distance between the two sources (d). This is often referred to as the "far field approximation" or "Fraunhofer diffraction" . L >> d

PRACTICE: Green light is incident on a very thin slit and illuminates a distant screen. Which of the following statements are true if small angles are assumed.

F 1. If the slit width is doubled than the width of the central maximum will increase by a factor of two.

T [✓ 2.] If the slit width is doubled than the width of the central maximum will decrease by a factor of two.

F 3. If the distance to the screen is doubled than the width of the central maximum will increase by a factor of four.

F 4. If the distance to the screen is doubled than the width of the central maximum will decrease by a factor of four

Small θ_p $\curvearrowright$ $a \sin\theta_p = p\lambda$ $\qquad$ $y_p = L\tan\theta_p$

$a\theta_p = p\lambda$ // $\qquad$ $y_p = L\,\theta_p$ // $\quad \ldots \quad y_p = \frac{p\lambda L}{a}$ // $\qquad w = \frac{2L\lambda}{a}$ //

$w \propto \frac{1}{a}$

$w \propto L$

If $L \to 2L$

$w \to 2w$

If $a \to 2a$

$w \to \frac{1}{2}w$

PRACTICE: A screen is placed 50.0 cm from a single slit, which is illuminated with 690-nm light. If the distance between the first and third minima in the diffraction pattern is 3.00 mm, what is the width of the slit?

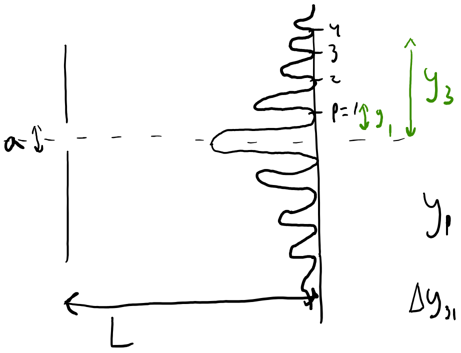
[✓ 1.] 0.23 mm

2. 0.42 mm

3. 0.11 mm

4. 0.37 mm

5. 0.18 mm



$\Delta y_{13} = y_3 - y_1 = 3\times10^{-3}$ m

$y_p = \frac{p\lambda L}{a}$

$\Delta y_{31} = \frac{3\lambda L}{a} - \frac{\lambda L}{a} = \frac{2\lambda L}{a}$

$a = \frac{2\lambda L}{\Delta y_{31}}$ // $= 23\times10^{-5}$ m

PRACTICE: Which one of the following statements best explains why the diffraction of sound is more apparent than the diffraction of light under most circumstances?

1. Sound requires a physical medium for propagation.
2. Sound waves are longitudinal, and light waves are transverse.
3. Light waves can be represented by rays while sound waves cannot.
4. The speed of sound in air is six orders of magnitude smaller than that of light.

[✓ 5] The wavelengths of visible light is considerably smaller than the wavelengths of sound

$a \sin\theta_m = m\lambda$ IF $\lambda \uparrow$ THEN $\theta_m \uparrow$

QUESTIONS FOR DISCUSSION:

(1) A single slit like interference pattern can be observed if coherent light encounters a thin object like hair. Hair is not a slit, rather it blocks the light. Explain this observed phenomena using Huygens' principle.