

Thin lens equation

Select LEARNING OBJECTIVES:

- Be able to apply the thin lens equation for converging and diverging thin lenses, as well as spherical mirrors.
- Use correct sign conventions for the thin lens equation based off of the optical element type and location of objects and images.

TEXTBOOK CHAPTERS:

Boxsand :: [Thin lens equation](#)

WARM UP: An object very far away from a convex mirror is moved toward the mirror at a constant speed. Does the image move faster, slower, or at the same speed?

Ray tracing is often combined under the umbrella term of geometric optics. The term geometric optics suggests that if you are careful enough and construct scaled drawing of the scenario you are analyzing, you can bypass any mathematical models and solve for desired variables via a physical representation (ray tracing diagram). It is great news, you can always default back to drawing a scaled ray diagram to help solve these optical elements problems. However, the mathematical model is often the quick route to an answer so we will spend this lecture discussing the mathematical model for thin lenses and spherical mirrors.

Mathematical model for thin lens (and spherical mirrors)

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$

FOCAL LENGTH OBJECT DISTANCE IMAGE DISTANCE

MAGNIFICATION

$$M = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$$

- IF $|M| > 1$ IMAGE LARGER
- IF $|M| < 1$ IMAGE SMALLER
- IF M IS (+) SAME ORIENTATION AS OBJ
- IF M IS (-) INVERTED ORIENTATION W.R.T. OBJ

* SIGN CONVENTIONS

FOCAL LENGTH

CONVERGING/CONCAVE (+)

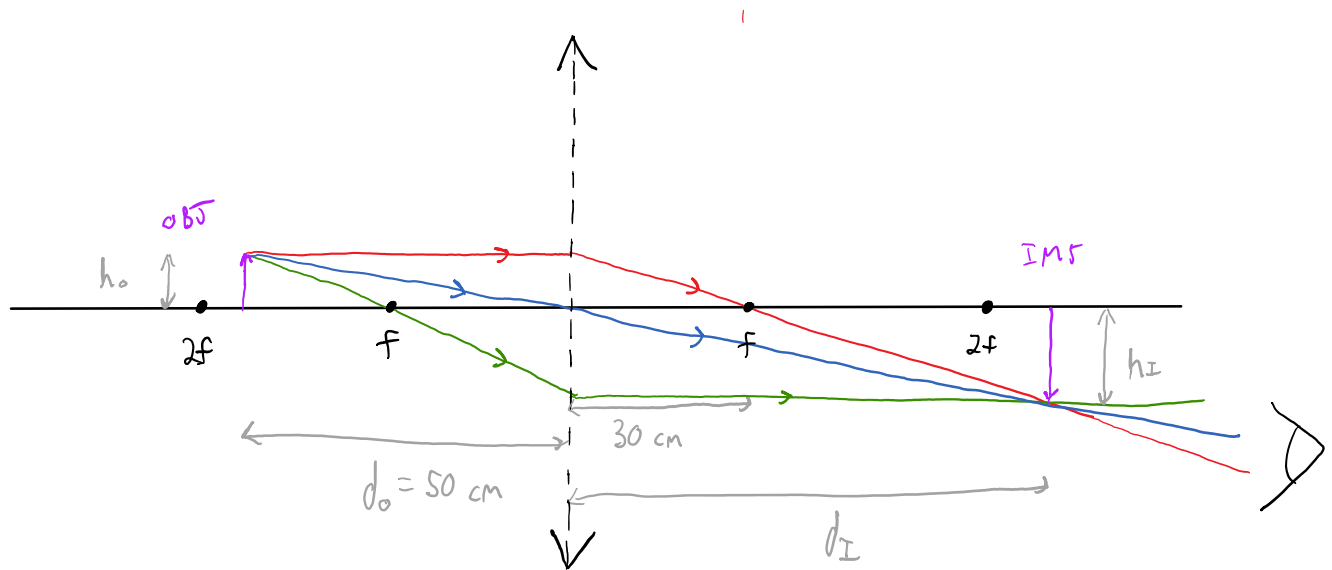
DIVERGING/CONVEX (-)

IMAGE DISTANCE

- IF d_i IS (+), THEN IT IS ON FAR SIDE OF ELEMENT "RIGHT SIDE" REAL IMAGE
- IF d_i IS (-), THEN IT IS ON NEAR SIDE OF ELEMENT "LEFT SIDE" VIRTUAL IMAGE

Good news everybody! The thin lens equation has the same functional form for all 4 optical elements we talked about, thin lenses and spherical mirrors. Bad news everybody! There are some tricky sign conventions you must pay attention to when using the thin lens equation. We will do some practice problems below to help clarify some of the sign conventions.

PRACTICE: A 10-cm-tall object is located 50.0 cm to the left of a converging lens whose focal length is 30.0 cm. Estimate the height of the image by using a carefully constructed ray diagram? Answer in centimeters



Calculate the location of the image.

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$

CONVERGING
 $f(+), d_o(+)$

$$M = -\frac{d_i}{d_o} = -1.5$$

LARGER
 ↑ UPSIDE DOWN

$$\frac{1}{30 \text{ cm}} = \frac{1}{50 \text{ cm}} + \frac{1}{d_i} \rightarrow d_i = 75 \text{ cm}$$

↑ + so on FAR SIDE + REAL

$$h_I = m h_o = -15 \text{ cm}$$

PRACTICE: An object is placed a distance d_o from a converging lens of focal length f . Circle the expression that correctly solves for the distance to the image?

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$

$$\frac{1}{f} - \frac{1}{d_o} = \frac{1}{d_i} \quad \text{INVERT BOTH SIDES}$$

$$\left(\frac{1}{f} - \frac{1}{d_o}\right)^{-1} = d_i$$

$$d_i = f - d_o$$

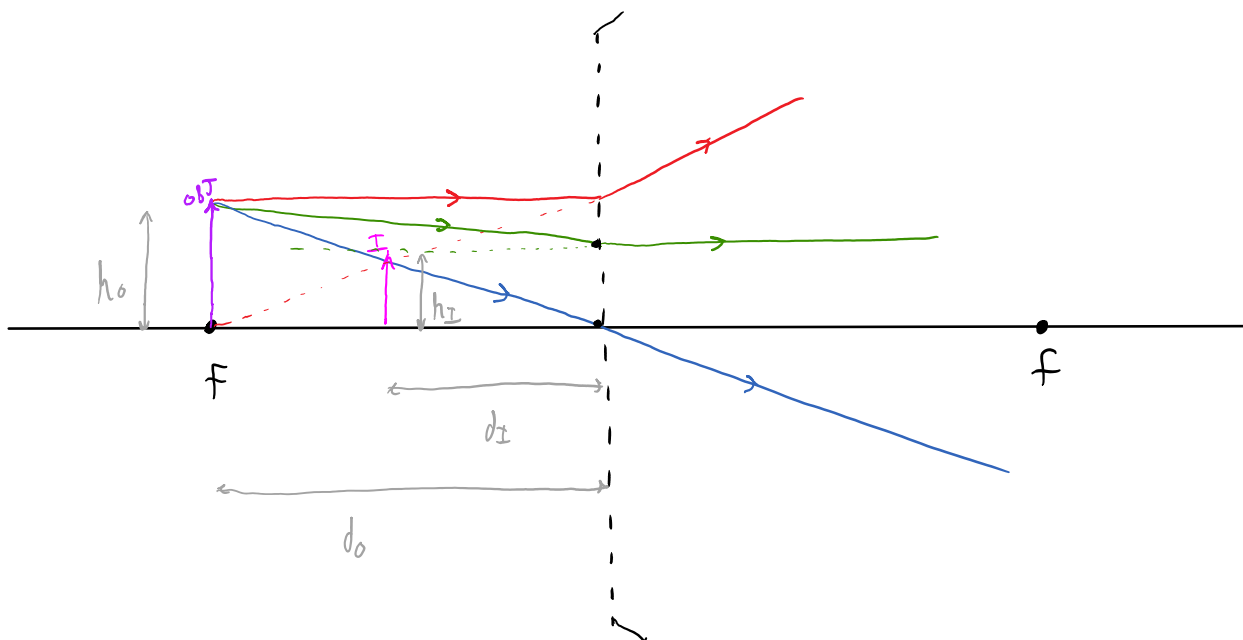
$$d_i = \frac{1-f}{d_o}$$

$$d_i = (f - d_o)^{-1}$$

$$d_i = \frac{1}{\frac{1}{f} - \frac{1}{d_o}}$$

$$d_i = \left(\frac{1}{f} - \frac{1}{d_o}\right)^{-1}$$

PRACTICE: If an object is placed at the focal point of a diverging lens, estimate the magnification of the image using a carefully drawn ray diagram



What is the image distance, in terms of f the focal length?

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$$

$$\frac{1}{-f} = \frac{1}{+f} + \frac{1}{d_i}$$

$$-\frac{2}{f} = \frac{1}{d_i}$$

$$d_i = -\frac{1}{2}f$$

NEAR SIDE + VIRTUAL

DIVERGING
 $f(-), d_o(+)$

$$M = -\frac{d_i}{d_o} = -\frac{-\frac{1}{2}f}{f} = +\frac{1}{2}$$

SMALLER
 UPRIGHT

PRACTICE: A gigantic funhouse mirror has a 20 ft radius of curvature and you are standing 6 ft in front of it. Calculate the image distance and magnification.

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$$

CONCAVE
 $f = +\frac{1}{2}R$

$$R = 20 \text{ ft}$$

$$M = -\frac{d_i}{d_o}$$

$$\frac{1}{6 \text{ ft}} + \frac{1}{d_i} = \frac{1}{10 \text{ ft}}$$

$$M = -\frac{(-15)}{6}$$

$$d_i = \left(\frac{1}{10 \text{ ft}} - \frac{1}{6 \text{ ft}} \right)^{-1}$$

$$M = +2.5$$

$$d_i = -15 \text{ ft}$$

VIRTUAL IMAGE

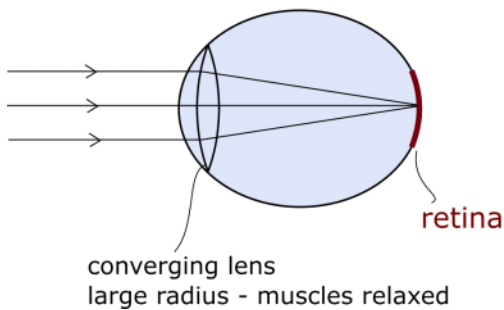
LARGER
 SAME ORIENTATION AS OBJ

The eye as an optical element

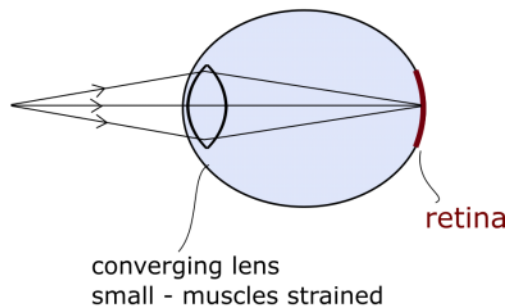
Refractive power $\equiv p = (1 / \text{focal-length})$

- Corrective lens prescribed by power, not focal length
 - Unit is "diopter"

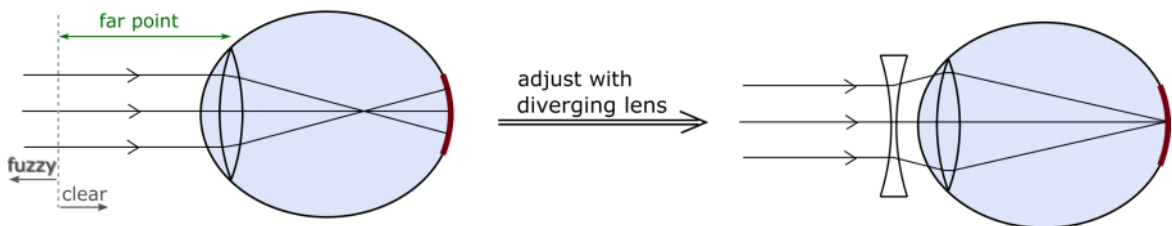
Focus on distant object



Focus on near object

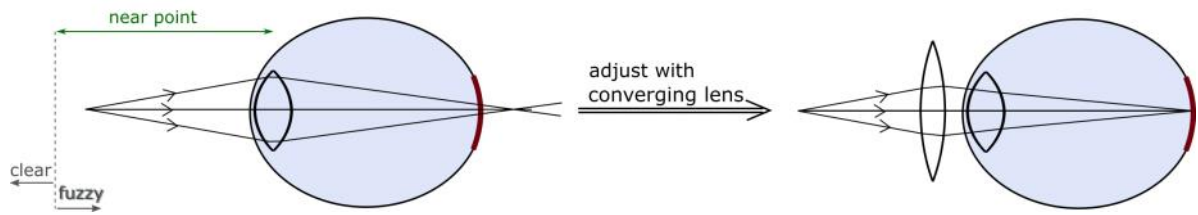


Nearsighted (myopic)



Far point :: furthest distance (measured from eye lens) at which an object can be clearly seen

Farsighted (hyperopic)



near point :: closest distance (measured from eye lens) at which an object can be clearly seen. For the average human eye, the near point is about 25 cm.

QUESTIONS FOR DISCUSSION:

1. When you place a straw in a glass of water at an angle, it looks as though the straw bends. Explain this observation with the concepts covered in this lecture.

