Young's Double-Slit Experiment

Select LEARNING OBJECTIVES:

- Understand the two models of light; wave model and particle model.
- Be able to understand the difference between diffraction and interference.
- Be able to understand the condition for far field.
- Understand what the fringe order represents.
- Be able to know when and how to use the small angle approximation.
- Be able to understand the features of an interference pattern (symmetries).

TEXTBOOK CHAPTERS:

- Boxsand :: Single and multi-slit interference

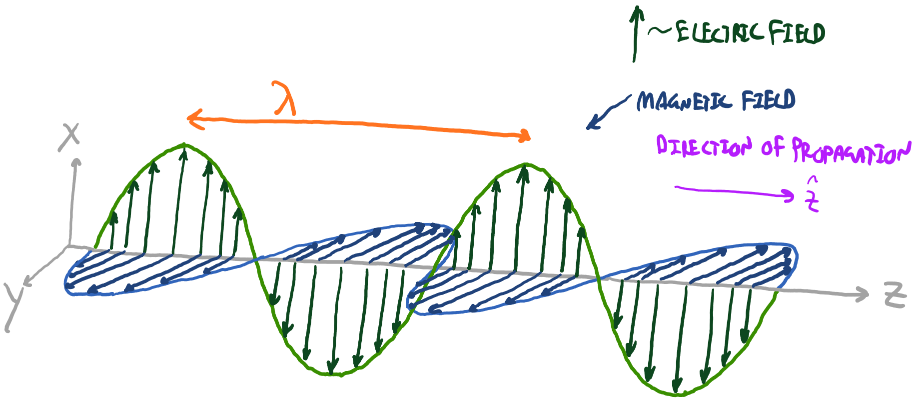
WARM UP: When starting any type of general interference problem, other than standing waves, what is the most important thing to think about? After you have thought about this, what else must you identify before mathematically solving for anything?

In this lecture we will continue to explore traveling waves, however we will shift our focus away from mechanical waves and begin to study electromagnetic waves. Recall that mechanical waves require a medium to propagate through space (e.g. waves on a string need the string to exist to travel, sound waves need air particles, sporting even waves need people…). Electromagnetic waves do not need a medium to travel; they can travel through the vacuum of space. The question is then, what quantity changes with space and time for a light wave? It turns out that the electric field (and magnetic field) that exists in space are the quantities that are oscillating. Therefore light waves are oscillating electric and magnetic fields that propagate through space and time. The electric and magnetic waves are perpendicular to each other and both are perpendicular to the direction of propagation, thus light waves are transverse waves. Below is a sketch of what a light wave can look like.

EXAMPLE OF MATHEMATICAL MODEL

$$\vec{E}(z,t) = E_0 \begin{matrix}\sin \\ \text{or} \\ \cos\end{matrix} (kz - \omega t)\ \hat{x}$$

$$\vec{B}(z,t) = B_0 \begin{matrix}\sin \\ \text{or} \\ \cos\end{matrix} (kz - \omega t)\ \hat{y}$$



Note that electric fields and magnetic fields are vectors, and it is the oscillation of their magnitudes and directions that make up an electromagnetic wave. We will explore the finer details about electromagnetic waves in ph213. For now, it is sufficient to recognize we are still studying traveling waves, and more specifically we are interested in the interference of traveling waves. The wave length of the light wave is shown above as well. We will learn in later lectures that the wave length changes when light travels through different media, thus it is important to remember that unless specified, when given a wavelength for light, that wavelength is measured in a vacuum. Recall that frequency does not change when waves propagate through different media. Below is a table showing the electromagnetic spectrum.

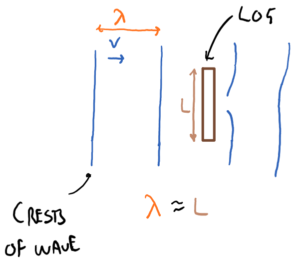
Identification	**Wavelength in vacuum (λ)**	**Frequency *(f)***
Gamma rays	~ 3 pm (picometer 10^{-12})	~ 100 EHz (exahertz 10^{18})
X-rays	~ 3 nm	~ 0.1 Ehz
Ultraviolet	~ 30 nm	~ 10 PHz (petahertz 10^{15})
Violet	~ 400 nm	~ 750 THz (terahertz 10^{12})
Blue	~ 475 nm	~ 632 THz
Green	~ 550 nm	~ 545 THz
Yellow	~ 575 nm	~ 522 THz
Orange	~ 600 nm	~ 500 THz
Red	~ 650 nm	~ 462 THz
Infrared	~ 30 µm	~ 10 THz
Microwave	~ 30 cm	~ 1 GHz
FM radio	~ 3 m	~ 0.1 GHz (100 MHz)
AM radio	~ 300 m	~ 1 MHz (1000 KHz)

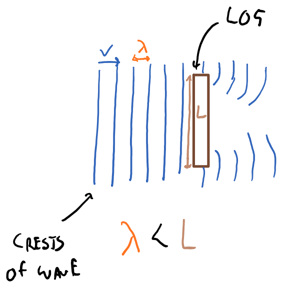
Keep in mind that the identification we use for electromagnetic waves have a range of wavelengths; the wavelengths shown above are very rough estimates for each identification.

*Side note: Are electromagnetic waves really waves? Or particles? Newton's view of light was that it existed as a collection of particles. He even wrote a book called "Opticks" in the early 1700s which was widely accepted for about 100 years as the proper view of light. Around the same time as Newton, Christiaan Huygens proposed a wave theory of light which fell in the shadows of Newton's particle theory of light. Around 100 years after Newton's work, Thomas Young performed experiments supporting the wave nature of light and physicists begin to adopt Huygens' wave theory proposed 100 years earlier. The story doesn't end here. In the early 1900s Max Planck proposed that light waves can be emitted in tiny packets, or quanta, of energy. In 1905 Einstein applied Max Planck's work to explain experimental data from the photoelectric effect, confirming that light can be thought of as packets of energy. Our modern viewpoint is a wave-particle duality interpretation.

The goal of this lecture is to study the wave nature of light via Young's Double slit experiment.

When traveling waves encounter an obstacle, the waves bends around the obstacle by some amount. Particles do not exhibit this bending behavior. Consider shooting a bullet through a small opening; the bullet continues along a straight path. Now imagine a water wave going through the same small opening; the crests of the water wave bend around the corners and fill in the space behind the opening where water did not pass through. This bending nature of waves is known as diffraction. The amount of diffraction depends on the wavelength of the wave and the size of the obstacle. Below is an overview of diffraction as applied to water waves striking a solid object, but remember that this applies to all waves (e.g. sound, light etc..).





Notice that when the wavelength of the wave is on the order of the obstacle length then the wave diffracts more (i.e. it fills in the region directly behind the log sooner) than when the wavelength is much smaller than the length of the log. We will invoke the concept of diffraction when discussing light passing through slits and when encountering obstacles such as single strands of hair.

Let's begin our discussion of the interference of light with some constraints that we must be apply before we jump into Young's double-slit.

Constraints

- Coherent sources.
- Sources have the same frequency.
- The distance (L) from the sources to the viewing screen must be much greater than the distance between the two sources (d). This is often referred to as the "far field approximation" or "Fraunhofer diffraction" . $L >> d$

Our first task is to create coherent sources with the same frequency. One way to do that is to use spherical waves that are very far from the source. The spherical waves far from the source have such a large diameter that they are basically plane waves and since they originated from the same source they have the same frequency (e.g. drop a pebble in water and far from the location of impact, the waves are nearly straight lines when viewed at a close distance). Once these plane waves pass through two open slits, the slits become the new sources; they are coherent sources since the same plane wave passed through both. Another way to get a coherent source is to use a LASER.

Young's Double-Slit Experiment

Attached at the end of this lecture are two full-page sized physical representation of Young's Double-Slit experiment; one for using bright fringes and the other for using dark fringes. The locations of maximum intensity or minimum intensity are referred to as fringes. Study this carefully. Some important features to note:

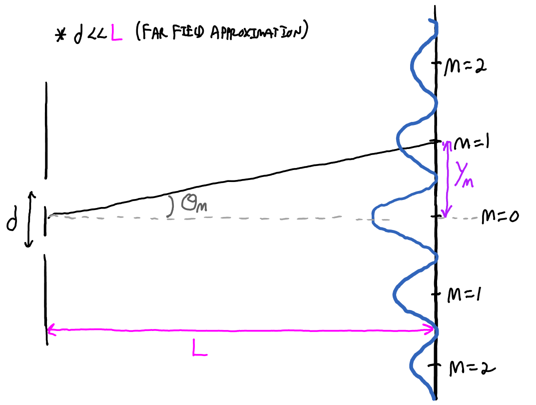
- Far field approximation. The larger the ratio L/d is, the more parallel the path lengths are and the more accurate the $PLD = d \sin(\theta_m)$ is.
- If the angle is small (about less than 10 degrees), then further simplifications can be used. Be careful though, you must confirm that the angle is small before you use the small angle approximation.
- The blue pattern represents intensity seen on a viewing screen. The taller the bumps the brighter the fringe; at the peak is the location of maximum constructive interference. Where the blue meets the black vertical line there is a dark fringe; this is a location of maximum destructive interference.

The last two pages are a detailed way of representing Young's double slit experiment. When solving problems, it would become very tedious to sketch all the information that is included on those two pages. Thus below is a shorthand physical representation that I recommend using when working on problem. Remember that drawing a picture and labeling physical quantities greatly helps identify the underlying physics and it also helps get you started on the right path towards a solution.

BRIGHT FRINGES

CONSTRUCTIVE PHYSICAL REPRESENTATION

* $d \ll L$ (FAR FIELD APPROXIMATION)



PLD = CONSTRUCTIVE) BOTH SLITS IN PHASE

PLD = $m\lambda$

$d\sin(\theta_m) = m\lambda$

$y_m = L\tan(\theta_m)$

SMALL ANGLE APPROXIMATION

IF $\frac{\lambda}{d} \ll 1 \rightarrow \theta_m < 10^\circ$

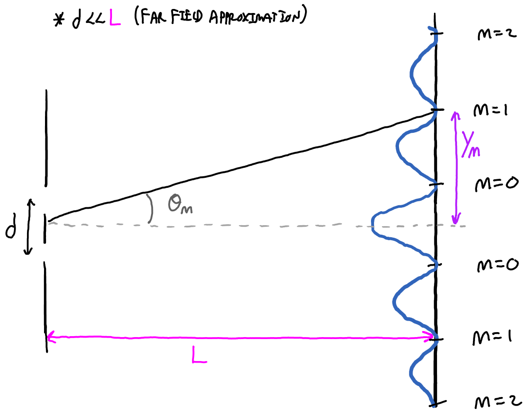
THEN $\sin(\theta_m) \approx \tan(\theta_m) \approx \theta_m$ IN RADIANS

THEREFORE... $y_m \approx \frac{mL\lambda}{d}$

DARK FRINGES

DESTRUCTIVE PHYSICAL REPRESENTATION

* $d \ll L$ (FAR FIELD APPROXIMATION)



PLD = DESTRUCTIVE) BOTH SLITS IN PHASE

PLD = $(m+\frac{1}{2})\lambda$

$d\sin(\theta_m) = (m+\frac{1}{2})\lambda$

$y_m = L\tan(\theta_m)$

SMALL ANGLE APPROXIMATION

IF $\frac{\lambda}{d} \ll 1 \rightarrow \theta_m < 10^\circ$

THEN $\sin(\theta_m) \approx \tan(\theta_m) \approx \theta_m$ IN RADIANS

THEREFORE... $y_m \approx \frac{(m+\frac{1}{2})L\lambda}{d}$

PRACTICE: Which of the following are required to see an interference pattern in a Young's double-slit experiment?

(a) White light
(b) Experimental apparatus in air
[✓ (c)] Single frequency source
[✓ (d)] $\lambda < d$
(e) $\lambda > d$
(f) $\lambda = d$

$d\sin\theta_m = m\lambda$

$\frac{\lambda}{d} = \frac{\sin\theta_m}{m}$

w/ $m = 0, 1, 2, 3, 4 \ldots$

AND $(\sin\theta_m)_{MAX} = 1$

$\therefore \lambda < d$

PRACTICE: Light of wavelength λ_1 illuminates a double slit, and interference fringes are observed on a screen behind the slits. When the wavelength is changed to λ_2, the fringes get closer together. What can you say about the relative wavelengths?

(a) $\lambda_2 > \lambda_1$
[✓ (b)] $\lambda_2 < \lambda_1$
(c) $\lambda_2 = \lambda_1$
(d) Need more information.

$y_m = L\tan(\theta_m)$

IF $y_m \downarrow$

THEN $\tan(\theta_m)\downarrow$

IF $\tan(\theta_m)\downarrow$

THEN $\theta_m \downarrow$

$\rightarrow d\sin\theta_m = m\lambda$

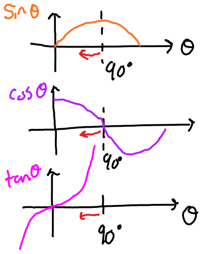
IF $\theta_m \downarrow$

THEN $\sin(\theta_m)\downarrow$

IF $\sin(\theta_m)\downarrow$

THEN $\lambda \downarrow$

$\tan\theta = \frac{\sin\theta}{\cos\theta}$



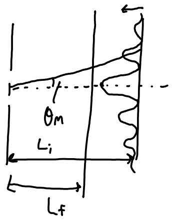
AS $\theta \downarrow$ FROM 90° $\sin\theta$ ALSO $\downarrow$

AS $\theta \downarrow$ FROM 90° $\cos\theta \uparrow$

AS $\theta \downarrow$ FROM 90° $\tan\theta$ ALSO $\downarrow$

PRACTICE: Suppose the viewing screen of a double-slit experiment is moved closer to the double slit. What happens to the interference fringes?

1. They fade out and disappear.
2. They get out of focus.
[✓ 3.] They get brighter and closer together.
4. They get brighter and farther apart.
5. They get brighter but otherwise do not change.



$y_m = L\tan(\theta_m)$

θ_m = CONST

IF $L\downarrow$ $y_m\downarrow$

AMPLITUDE & INTENSITY DECREASE W/ DISTANCE FOR SPHERICAL AND CYLINDRICAL WAVES

PRACTICE: For a double-slit apparatus where λ/d << 1, which of the following can be said about adjacent bright fringes?

(a) The spacing between the mth and (m+1)th fringe increases with increased m.
(b) The spacing between the mth and (m+1)th fringe decreases with increased m.
[✓ (c)] The spacing between the mth and (m+1)th fringe remains constant with increased m.

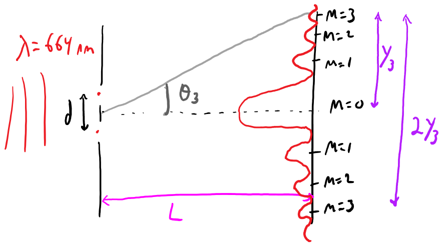
$\frac{\lambda}{d} << 1 \rightarrow$ SMALL θ APPROX.

$$y_m = \frac{mL\lambda}{d} \qquad y_{m+1} = \frac{(m+1)L\lambda}{d}$$

$$y_{m+1} - y_m = \frac{(m+1)L\lambda}{d} - \frac{mL\lambda}{d}$$

$$\Delta y = \frac{L\lambda}{d} \leftarrow \text{CONSTANT}$$

PRACTICE: Red light (λ=664 nm) is used in a double slit experiment with the slits separated by a distance of 1.2 x 10-4 m. The screen is located at a distance of L = 2.75 meters from the slits. Find the distance on the screen between the two third order bright fringes using the small angle approximation and without the small angle approximation.



NO APPROX

$$d\sin\theta_m = m\lambda \qquad y_m = L\tan\theta_m$$

$$\overset{\checkmark}{d}\sin\overset{?}{\theta_3} = 3\overset{\checkmark}{\lambda} \qquad \overset{?}{y_3} = \overset{\checkmark}{L}\tan\overset{?}{\theta_3}$$

2 EQNS 2 UNKNOWNS

$$\theta_3 = \sin^{-1}\left(\frac{3\lambda}{d}\right)$$

$$y_3 = L\tan\left(\sin^{-1}\left(\frac{3\lambda}{d}\right)\right)$$

$y_3 \approx 0.04565629$ METERS

$$\boxed{2y_3 = 0.0913125814 \text{ METERS}}$$

APPROX

$$\frac{\lambda}{d} = 0.0055 << 1 \quad \checkmark$$

$$\overset{\checkmark}{d}\,\overset{?}{\theta_m} = \overset{\checkmark}{m}\overset{\checkmark}{\lambda} \qquad \overset{?}{y_m} = \overset{\checkmark}{L}\,\overset{?}{\theta_m}$$

$$\theta_m = \frac{m\lambda}{d} \qquad y_m = \frac{mL\lambda}{d}$$

$$y_3 = \frac{3L\lambda}{d}$$

$y_3 = 0.04565$ METERS

$$\checkmark \quad \boxed{2y_3 = 0.0913 \text{ METERS}}$$

PRACTICE: What is the difference between diffraction and interference?

(a) Diffraction refers to multi-slit scattering while interference refers to double slit scattering.
(b) Diffraction is relevant only to single slit apparatus. Interference is relevant to all wave phenomena.
(c) Diffraction is the focusing of light to a spot. Interference is the sinusoidal wave patterns of traveling light.
[✓ (d)] Diffraction is a process that changes the direction of light rays. Interference occurs when two light rays (waves) meet at a point.
(e) There is no difference between diffraction and interference.

QUESTIONS FOR DISCUSSION:

1. White light is used in a Young's double-slit experiment. What color do you expect to see at the center maximum? Explain your reasoning.