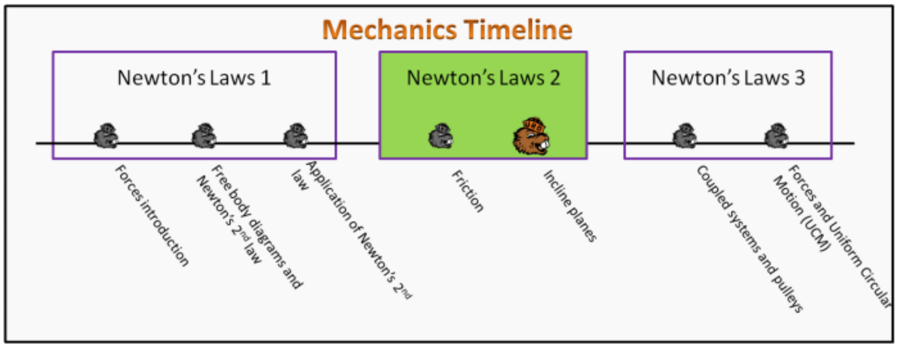
N2.L2.sols: Incline Planes

Monday, January 22, 2018 5:44 PM

Newton's Laws 2

Foundation Stage (N2.2)

lecture 2
Incline planes



Textbook Chapters

- **BoxSand** :: KC videos (Inclined planes)
- **Giancoli** (Physics Principles with Applications 7th) :: 4-8
- **Knight** (College Physics : A strategic approach 3rd) :: 5.4
- **Knight** (Physics for Scientists and Engineers 4th) :: 6.1

Warm up

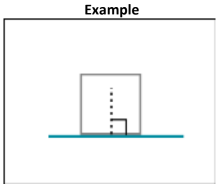
N1.2-1:

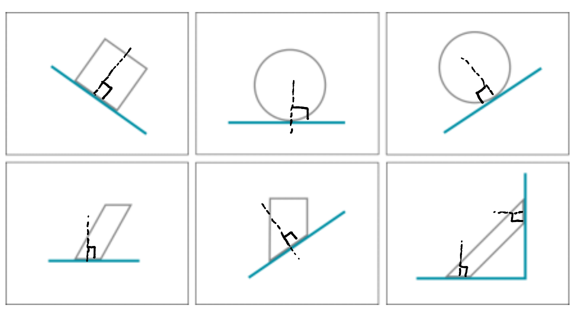
Description: Identify the normal direction between two objects in contact.

Learning Objectives: [?] - Can you identify the objectives from the previous lecture, and this lecture, that this question is relevant to?

Problem Statement: In each of the situations below there is a gray object touching a green object. Normal forces are always perpendicular to the two surfaces in contact. Identify the perpendicular line which the normal force would point along for each scenario. The image on the left is an example.

Example





Selected Learning Objectives

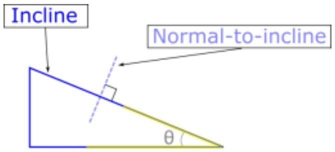
1. Recognize that friction is an interaction between surfaces and therefore depends on both materials.
2. Demonstrate that if two objects are not sliding relative to each other, than the static friction force is as large as it need s to be, up to a maximum value.
3. Demonstrate that if two objects are sliding relative to each other, then the kinetic friction force is constant.
4. Show how the magnitude of the friction force is related to the magnitude of the normal force, via the coefficient of friction ; always true for kinetic friction and only true for static friction when it's a maximum.
5. Identify the mechanisms that go into rolling friction and how the magnitude depends on the normal force.
6. Identify a system where rolling friction is present and not negligible.
7. Show that the friction force is parallel to a surface.
8. (UPMF) Demonstrate that the direction of the friction force is opposite the direction of relative motion (or what relative mo tion would occur without friction) between two surfaces.
9. (UPMF) Demonstrate that if friction is the only force acting in a given direction, then it must be in the same direction as t he acceleration component in that direction.
10. Use multiple approaches to make sense of the direction of friction, e.g. connecting the FBD to the direction of the net force and acceleration.
11. Identify other forces related to friction, including air resistance, viscosity, and cohesion.
12. Recognize when friction can safely be neglected.
13. Identify the direction of acceleration and weigh the value of aligning your coordinate system with that direction as opposed to choosing one that minimizes the number of force components.

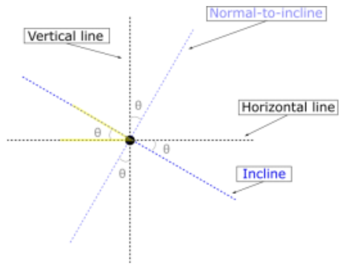
14. Orient the forces in a FBD in the direction they are applied and not rotated to align with a horizontal or vertical direction .
15. Translate a given angle in the physical representation to all like angles in a FBD, which may require geometry.

Key Terms

- Incline plane
- Non-standard coordinate system

Key Equations





Pizza wedge trick

Step 1) Draw horiontal and vertical reference lines on FBD
Step 2) Draw incline line and normal-to-incline lines on FBD
Step 3) Find the angle on the picture and identify the same angle on the FBD
Step 4) Every other "pizza wedge" is the same angle

Key Concepts

- Aligning an axis of your coordinate system along the direction of acceleration can simplify the mathematical representation a nalysis.
- Aligning an axis of your coordinate system so that more vectors are aligned with the axes than those that are not can simplif y the mathematical representation analysis.
- Labeling angles on a FBD can help the process of translating a FBD into a set of Newton's 2nd law equations in the chosen coo rdinates.
- Although not wrong, do not draw the force of gravity vector off at an angle. Rather, draw the FBD exactly as you see the pic ture of the scenario (e.g. gravity points straight downwards).
- The normal force is always perpendicular to the two surfaces in contact with each other.
- Friction is always parallel to the two surfaces in contact with each other.

Act I: Incline planes

Questions

N1.2-2:

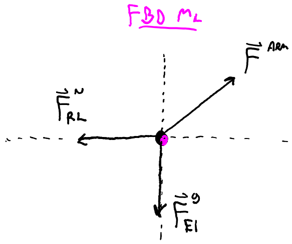
Description: Determine direction of normal force and construct a FBD including the proper directions of all forces. (2 minutes)

Learning Objectives: [14]

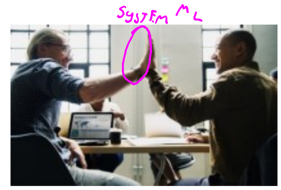
Problem Statement: Give the person next to you a high 5. Now draw a free body diagram for your hand at the instant your hands touch. What direction is the normal force from the right persons hand on the left persons hand?

(1) Left
(2) Right
(3) Upwards
(4) Downwards

FBD M_L



System M_L



N1.2-3:

Description: Match FBDs with snapshots of a roller coaster at various locations along a loop-the-loop. (3 minutes + 1 minute)

Learning Objectives: [13, 14]

Problem Statement: A roller coaster cart goes through a loop-the-loop as shown in the figure below. A group of physics students drew four free body diagrams but didn't label which snapshot each FBD was drawn at. Which set of free body diagrams are matched with the proper snapshot of the cart?

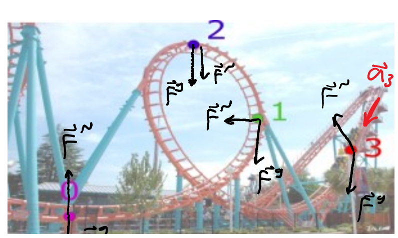
(a)

(1) X

Snapshot	FBD
0	A
1	B
2	C
3	D

(2)

Snapshot	FBD
✓ 0	B
✓ 1	D
✓ 2	A
✓ 3	C



(3) (4)

1	B
2	C
3	D

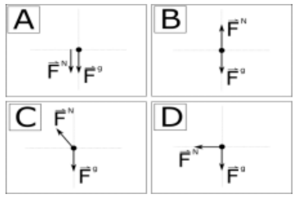
1	D
2	A
3	C

(3)

Snapshot	FBD
0	A
1	B
2	D
3	C

(4)

Snapshot	FBD
0	B
1	C
2	A
3	D

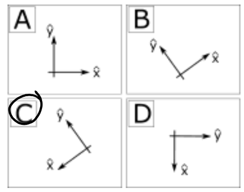


(b) Consider the cart at snapshot 3. Which coordinate system orientation would be best suited for analyzing the forces at this location?

$\vec{a}_3$

WITH

$\vec{a}_3 = \langle a_{3x}, 0 \rangle$



N1.2-4:

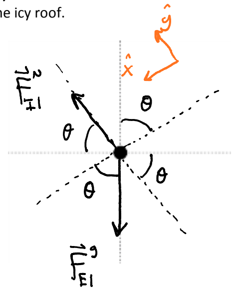
Description: Apply a force analysis on a frictionless incline to determine the acceleration down the incline. (6 minutes)

Learning Objectives: [13, 14, 15]

Problem Statement: ворона (*vorona*) the crow, of mass **m,** is on an icy roof that has an incline of **θ** degrees with respect to the vertical. Friction is negligible.

SYSTEM m_1

(a) Sketch a FBD of the crow while on the icy roof.



(b) Using the wise coordinate system from the previous problem, and Newton's 2nd law, derive an expression for the acceleration of the crow as a function of **θ** and **g** only.

(1) a_x = **g** sin(**θ**)
(2) a_x = **g** cos(**θ**)
(3) a_x = **g** tan(**θ**)
(4) a_x = **g**

$$\sum F_x = m_1 a_{1x}$$

$$|\vec{F}^g_{E1}| \cos\theta = m_1 a_{1x}$$

$$\cancel{m_1} g \cos\theta = \cancel{m_1} a_{1x}$$

$$\boxed{g \cos\theta = a_{1x}}$$

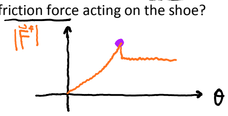
N1.2-5:

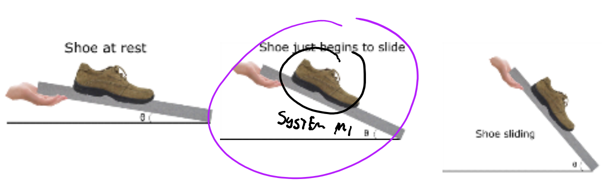
Description: Find the coefficient of static friction for a shoe on an incline. (2 minutes + 3 minutes + 2 minutes + 5 minutes)

Learning Objectives: [1, 2, 4, 7, 10, 13, 14, 15]

Problem Statement: You want to find the coefficient of static friction between your shoe and concrete. You take your shoe and place it on a concrete board that you slowly raise on one end, creating an inclined plane of increasing pitch.

(a) As you raise the board increasing the pitch, which location would you take a snapshot to study the maximum friction force acting on the shoe?





(b) What acceleration would you use to study the features of maximum static friction in this situation?

(1) 0 m/s²

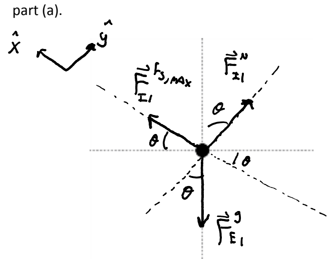
(c) Draw a FBD for the shoe for the snapshot you decided in part (a).

$\hat{x}$ $\hat{y}$

(b) What acceleration would you use to study the features of maximum static friction in this situation?

(1) 0 m/s²
(2) g sin(θ)
(3) g cos(θ)
(4) 9.8 m/s²

(c) Draw a FBD for the shoe for the snapshot you decided in part (a).



(d) Use Newton's 2nd law to determine the coefficient of static friction between your shoe and concrete.

$$\Sigma F_x = m_1 \cancel{a_{1x}}^{0} \qquad \Sigma F_y = m_1 \cancel{a_{1y}}^{0}$$

$$|\vec{F}^{fs,max}_{I1}| - |\vec{F}^{g}_{E1}|\sin\theta = 0 \qquad |\vec{F}^{N}_{I1}| - |\vec{F}^{g}_{E1}|\cos\theta = 0$$

$$\mu_s |\vec{F}^{N}_{I1}| - m_1 g \sin\theta = 0 \qquad |\vec{F}^{N}_{I1}| - m_1 g \cos\theta = 0$$

$$|\vec{F}^{N}_{I1}| = m_1 g \cos\theta$$

$$\mu_s \cancel{m_1 g}\cos\theta - \cancel{m_1 g}\sin\theta = 0$$

$$\mu_s \cos\theta - \sin\theta = 0$$

$$\mu_s = \frac{\sin\theta}{\cos\theta} \implies \boxed{\mu_s = \tan\theta}$$

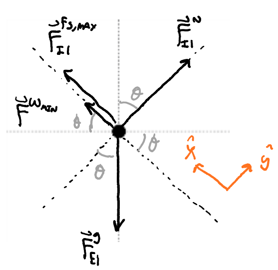
N1.2-6:

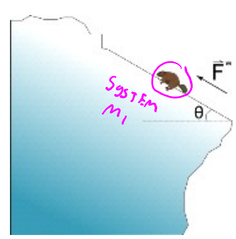
Description: Construct a FBD and apply a force analysis involving a scenario where maximum static friction can point up or down an incline depending on other factors. (6 minutes + 3 minutes)

Learning Objectives: [1, 2, 4, 7, 10, 13, 14, 15]

Problem Statement: A 45 kg Benny slipped during a mountain climbing expedition, and is barely holding on near the edge of a cliff on a 55.0 degree incline relative to the horizontal. In fact, if it wasn't for the wind blowing up the mountain with a constant force, he would begin to slide. The coefficient of static friction between Benny and the snowy surface is 0.15.

(a) What is the minimum wind force to prevent Benny from sliding?





$$\Sigma F_x = m_1 \cancel{a_{1x}}^{0}$$

$$|\vec{F}^{fs,max}_{I1}| + |\vec{F}^{w_{min}}| - |\vec{F}^{g}_{E1}|\sin\theta = 0$$

$$\mu_s |\vec{F}^{N}_{I1}| + |\vec{F}^{w_{min}}| - m_1 g \sin\theta = 0$$

$$\Sigma F_y = m_1 \cancel{a_{1y}}^{0}$$

$$|\vec{F}^{N}_{I1}| - |\vec{F}^{g}_{E1}|\cos\theta = 0$$

$$|\vec{F}^{N}_{I1}| - m_1 g \cos\theta = 0$$

$$|\vec{F}^{N}_{I1}| = m_1 g \cos\theta$$

$$\mu_s m_1 g \cos\theta + |\vec{F}^{w_{min}}| - m_1 g \sin\theta = 0$$

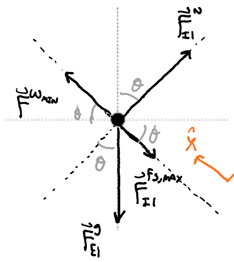
$$|\vec{F}^{w_{min}}| = m_1 g \sin\theta - \mu_s m_1 g \cos\theta$$

$$|\vec{F}^{w_{min}}| = m_1 g \left(\sin\theta - \mu_s \cos\theta\right)$$

$$= (45\ \text{kg})(9.8\ \text{m/s}^2)\left(\sin(55) - 0.15\cos(55)\right)$$

$$\boxed{|\vec{F}^{w_{min}}| \approx 323\ \text{N}}$$

(b) If Benny is to climb up the incline, he will need the wind to pick up. What is the minimum wind force that will begin to push Benny up the hill?



$$\Sigma F_x = m_1 \cancel{a_{1x}}^{\,0}$$

$$-|\vec{F}_{I1}^{f_{s,max}}| + |\vec{F}^{w_{min}}| - |\vec{F}_{E1}^{g}|\sin\theta = 0$$

$$-\mu_s|\vec{F}_{I1}^{N}| + |\vec{F}^{w_{min}}| - m_1 g \sin\theta = 0$$

$$\Sigma F_y = m_1 \cancel{a_{1y}}^{\,0}$$

$$|\vec{F}_{I1}^{N}| - |\vec{F}_{E1}^{g}|\cos\theta = 0$$

$$|\vec{F}_{I1}^{N}| - m_1 g\cos\theta = 0$$

$$|\vec{F}_{I1}^{N}| = m_1 g\cos\theta$$

$$-\mu_s m_1 g\cos\theta + |\vec{F}^{w_{min}}| - m_1 g\sin\theta = 0$$

$$|\vec{F}^{w_{min}}| = m_1 g\sin\theta + \mu_s m_1 g\cos\theta$$

$$\boxed{|\vec{F}^{w_{min}}| = m_1 g\left(\sin\theta + \mu_s\cos\theta\right)}$$

$$= (45\,\text{kg})(9.8\,\text{m/s}^2)\left(\sin(55) + 0.15\cos(55)\right)$$

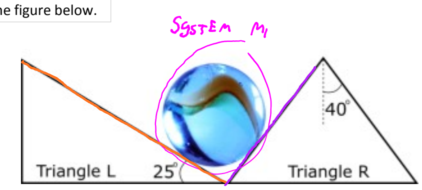
$$\boxed{|\vec{F}^{w_{min}}| \approx 399\,\text{N}}$$

N1.2-7:

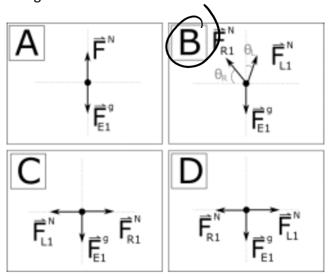
Description: Match the FBD with the picture given, apply a force analysis, and solve simultaneous equations for an object in static equilibrium. (1.5 minutes + 4 minutes + 4 minutes + 6 minutes)

Learning Objectives: [13, 14, 15]

Problem Statement: A 7.50 kg marble is resting in a wedge as shown in the figure below.



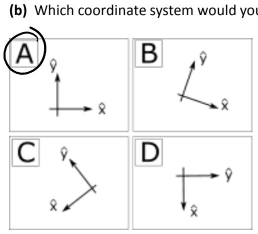
(a) Which FBD correctly represents the forces acting on the marble in the wedge?

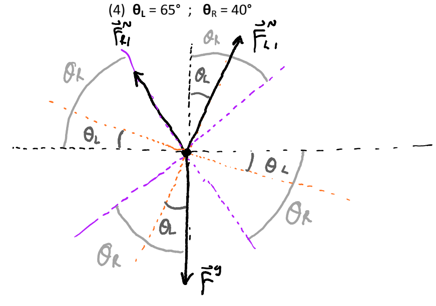


(c) What are the angles θ_L and θ_R?

(1) $\theta_L = 25°$; $\theta_R = 40°$
(2) $\theta_L = 65°$; $\theta_R = 50°$
(3) $\theta_L = 25°$; $\theta_R = 50°$
(4) $\theta_L = 65°$; $\theta_R = 40°$

(b) Which coordinate system would you choose?





(c) Which set of Newton's 2nd law equations applied to this situation correctly describes the forces acting on the marble using a standard coordinate system?

(1) A
(2) B
(3) C
(4) D

[A]

✗ $|\vec{F}_{L1}^{N}| - |\vec{F}_{R1}^{N}| = m\,a_{1x}$

✗ $|\vec{F}_{L1}^{N}| + |\vec{F}_{R1}^{N}| - |\vec{F}_{E1}^{g}| = m\,a_{1y}$

[B]

✓ $|\vec{F}_{L1}^{N}|\sin(\theta_L) - |\vec{F}_{R1}^{N}|\cos(\theta_R) = m\,\cancel{a_y}$

✓ $|\vec{F}_{L1}^{N}|\cos(\theta_L) + |\vec{F}_{R1}^{N}|\sin(\theta_R) - |\vec{F}_{Em}^{g}| = m\,\cancel{a_x}$

[C]

$|\vec{F}_{L1}^{N}|\cancel{\cos}(\theta_L) - |\vec{F}_{R1}^{N}|\cos(\theta_R) = m\,a_x$

$|\vec{F}_{L1}^{N}|\cancel{\sin}(\theta_L) + |\vec{F}_{R1}^{N}|\sin(\theta_R) - |\vec{F}_{Em}^{g}| = m\,a_y$

[D]

$|\vec{F}_{L1}^{N}|\sin(\theta_L) - |\vec{F}_{R1}^{N}|\cancel{\sin}(\theta_R) = m\,a_x$

$|\vec{F}_{L1}^{N}|\cos(\theta_L) + |\vec{F}_{R1}^{N}|\cancel{\cos}(\theta_R) - |\vec{F}_{Em}^{g}| = m\,a_y$

$$|\vec{F}^N_{L1}|\sin(\theta_L) - |\vec{F}^N_{R1}|\cos(\theta_R) = m\,a_x \qquad |\vec{F}^N_{L1}|\sin(\theta_L) - |\vec{F}^N_{R1}|\sin(\theta_R) = m\,a_x$$

$$|\vec{F}^N_{L1}|\cos(\theta_L) + |\vec{F}^N_{R1}|\sin(\theta_R) - |\vec{F}^g_{Em}| = m\,a_y \qquad |\vec{F}^N_{L1}|\cos(\theta_L) + |\vec{F}^N_{R1}|\cos(\theta_R) - |\vec{F}^g_{Em}| = m\,a_y$$

(d) Solve for the magnitude of both normal forces from the wedge on the marble.

$$|\vec{F}^N_{L1}|\sin\theta_L - |\vec{F}^N_{R1}|\cos\theta_R = 0 \qquad |\vec{F}^N_{L1}|\cos\theta_L + |\vec{F}^N_{R1}|\sin\theta_R - m_1 g = 0$$

2 EQNS 2 UNKNOWNS

$$|\vec{F}^N_{R1}|\cos\theta_R = |\vec{F}^N_{L1}|\sin\theta_L$$

$$|\vec{F}^N_{R1}| = \frac{\sin\theta_L}{\cos\theta_R}|\vec{F}^N_{L1}|$$

$$|\vec{F}^N_{L1}|\cos\theta_L + \left(\frac{\sin\theta_L}{\cos\theta_R}|\vec{F}^N_{L1}|\right)\sin\theta_R - m_1 g = 0$$

$$\boxed{|\vec{F}^N_{R1}| \approx 32.2\ N}$$

$$|\vec{F}^N_{L1}|\cos\theta_L + \sin\theta_L \tan\theta_R |\vec{F}^N_{L1}| = m_1 g$$

$$|\vec{F}^N_{L1}|\left(\cos\theta_L + \sin\theta_L \tan\theta_R\right) = m_1 g$$

$$|\vec{F}^N_{L1}| = \frac{m_1 g}{(\cos\theta_L + \sin\theta_L \tan\theta_R)} = \frac{(7.5\ kg)(9.8\ m/s^2)}{(\cos(25) + \sin(25)\tan(40))}$$

$$\boxed{|\vec{F}^N_{L1}| \approx 58.3\ N}$$

Conceptual questions for discussion

1. Consider trying to place a heavy box on top of a tall shelf. You can lift the box directly over your head to place it on the shelf, or you can push the box up an incline to the top of the shelf. Which option would you choose to minimize the amount of force you'd need to exert on the box?
2. Your bathroom scale is placed on an incline plane with sufficient friction so that the scale does not slide down the incline when you step on it. The bathroom scale's reading while on the incline plane will be ______________ the reading of the bathroom scale when on a horizontal surface.
   i. greater than
   ii. less than
   iii. equal to
3. **Case A:** A box that starts from rest and slides down a frictionless incline. **Case B:** The same box that is given an initial velocity up the same incline. For both cases, the only two forces acting on them are the force of gravity and the normal force from the incl ine. The magnitude of acceleration of the box in **Case A** is ___________ the magnitude of acceleration in **Case B**.
   i. greater than
   ii. less than
   iii. equal to

Hints

N1.2-1: No hints.

N1.2-2: Is your FBD in equilibrium?

N1.2-3: No hints.

N1.2-4: Draw your FBD as you see the picture (i.e. gravity is pointing downwards, do not rotate gravity off at an angle).

N1.2-5: The magnitude of the normal force is not equal to mg here. Apply Newton's 2nd law in both the x and the y directions to find an expression for the magnitude of the normal force between the incline and the shoe.

N1.2-6: **Part a**: Draw a FBD and determine which direction friction is pointing. What direction would Benny slide if there was no wind? Also, is this a maximum static friction scenario? If so, what in the problem indicates that it is? **Part b**: Repeat part a hints.

N1.2-7: Use the incline angle pizza wedge trick to help simplify any geometry you may need to do.