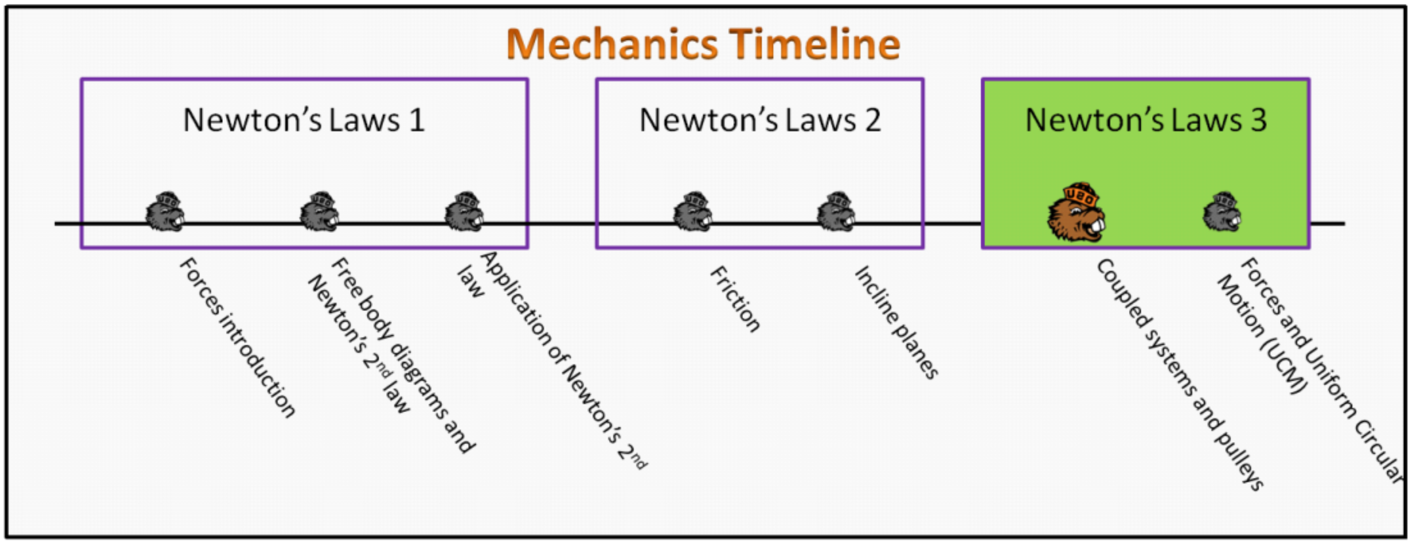
N3.2.L1.sols: Coupled Systems and Pulleys

Monday, January 22, 2018 5:44 PM

Newton's Laws 3
Foundation Stage (N3.2)

lecture 1
Coupled systems and pulleys



Textbook Chapters

- **BoxSand** :: KC videos (coupled systems ; mechanical advantage)
- **Giancoli** (Physics Principles with Applications 7th) :: 4-5 ; 4-7
- **Knight** (College Physics : A strategic approach 3rd) :: 5.7 ; 5.8
- **Knight** (Physics for Scientists and Engineers 4th) :: 7.1 ; 7.2 ; 7.3 ; 7.4 ; 7.5

Warm up

N3.2-1:

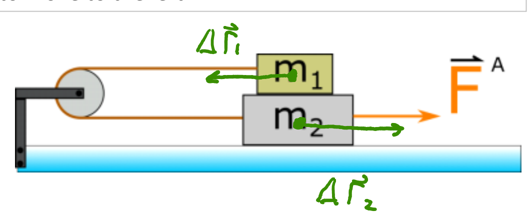
Description: Determine how the magnitude of acceleration for two objects connected via a pulley are related to each other.

Learning Objectives: [?] - Can you identify the objectives from the previous lecture, and this lecture, that this question is relevant to?

Problem Statement: Consider two boxes on top of each other as shown below. Initially the system is at rest. If you apply a force on the bottom box as shown, the bottom box will move to the right and the top box will begin to move to the left.

(a) For every meter that m_2 moves to the right, how many meters does m_1 move to the left?

(1) 1 meter
(2) 2 meters



(b) If it takes m_2 1 second to travel 1 meter to the right, the speed of m_2 is 1 m/s. Using your answer from part (a), what is the speed of m_1 during this same 1 second interval?

$|\Delta\vec{r}_1| = |\Delta\vec{r}_2|$

Using your answer from part (a), what is the speed of $\mathbf{m_1}$ during this same 1 second interval?

$|\Delta \vec{r}_1| = |\Delta \vec{r}_2|$

$\frac{|\Delta \vec{r}_1|}{\Delta t} = \frac{|\Delta \vec{r}_2|}{\Delta t}$

(1) 1 m/s
(2) 2 m/s

(c) If the magnitude of acceleration of $\mathbf{m_2}$ is 1m/s^2, what can you conclude about the magnitude of the acceleration of $\mathbf{m_1}$ using your answers from (a) and (b)?

$|\Delta \vec{v}_1| = |\Delta \vec{v}_2|$

$\frac{|\Delta \vec{v}_1|}{\Delta t} = \frac{|\Delta \vec{v}_2|}{\Delta t}$

(1) 1 m/s^2
(2) 2 m/s^2

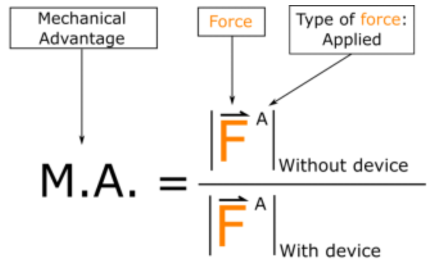
Selected Learning Objectives

1. Analyze situations involving multiple objects and choose appropriate systems.
2. Differentiate between external and internal forces.
3. Use 3rd law force pairs to analyze coupled systems.
4. Use constraints to analyze coupled systems.
5. Choose coordinate systems relative to each other that simplify the analysis.
6. Choose different systems (e.g. the system 1 and 2, or the combined 1 2 system) for the same physical situation and use the analysis from both to arrive at a solution.
7. Identify the features of an ideal pulley.
8. Demonstrate the ability to use a FBD and Newton's 2nd law to determine the mechanical advantage of a pulley system.
9. Identify systems that exhibit uniform circular motion.
10. Draw physical representations from multiple directions and choose the perspective that helps the analysis the most.
11. Draw a FBD that only includes real forces.
12. Identify which forces are responsible for keeping the object traveling in a circle.
13. Define centripetal force.
14. Show that the direction of the net force, and thus acceleration, are perpendicular to the direction of velocity during UCM.
15. Show that the direction of the net force and acceleration point towards the center of the circle during UCM.
16. Identify the radially inward direction and align the coordinate system with it.
17. Identify the tangential direction.
18. Realize that UCM is a situation where the speed of the object affects the magnitude of the acceleration (v^2/r) and thus net force.
19. (UPMF) Explain the nature of fictitious forces that arise in UCM and inertia's role in the effect.
20. Show how period, frequency, speed, and distance are related.

Key Terms

- Newton's 3rd law force pairs
- Coupled system
- Ideal pulley
- Mechanical advantage

Key Equations



In words: The mechanical advantage of a device is equal to the magnitude of the applied force without the device divided by the magnitude of the applied force with the device.

Key Concepts

- Coupled systems involve multiple objects that are connected to each other via some geometric constraint. For example, if two boxes are touching each other and you push on one end of one of the boxes, then both boxes move the same distance in the same amount of time. This constraint leads to both boxes having the same magnitude acceleration as well.
- Force pairs will never show up on the same FBD because each force acts on a different object. If you choose a system that includes both objects, then the force pairs are internal and thus are not drawn on a FBD which shows all the external forces acting on the system.
- An ideal rope (i.e. massless rope) is one that has negligible mass compared to the objects connected to it. This feature of an ideal rope leads to the magnitude of tension in the rope being equal at all locations along the rope.
- An ideal pulley (i.e. massless-frictionless pulley) is one that has negligible mass compared to the objects connected to it, and also has extremely low friction in its bearings. These two features of an ideal pulley lead to the magnitude of tension in the rope around the pulley being equal on each side.

Act I: Coupled systems

Questions

N3.2-2:

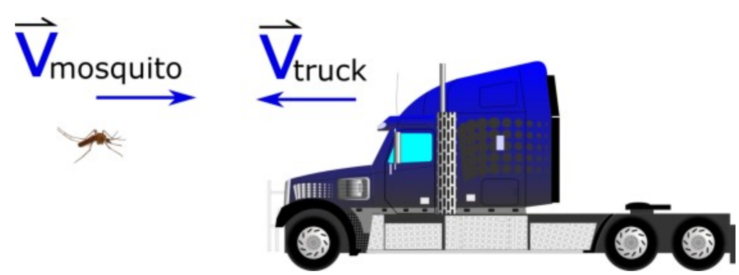
Description: Conceptual questions exploring Newton's 3rd law. (2 minutes)

Learning Objectives: [3]

Problem Statement: A mosquito and truck are traveling at different constant speeds and are about to collide. The mosquito and truck are heading in opposite directions. During the collision, the magnitude of the from the truck on the mosquito is ____________ than the magnitude of force from the mosquito on the truck.

(1) greater than
(2) less than
(3) equal to
(4) not enough information

3^{RD} LAW



$$|\vec{F}^N_{mT}| = |\vec{F}^N_{Tm}|$$

.... BUT....

$$\Sigma\vec{F}_{on\,m} = m_m\vec{a}_m \qquad \Sigma\vec{F}_{on\,T} = m_T\vec{a}_T$$

$$\vec{F}^N_{Tm} = m\,\vec{a}_m \qquad \vec{F}^N_{mT} = M_T\,\vec{a}_T$$

ACCELERATION

SAME $|\vec{F}^N|$ BUT SINCE MASS DIFFERENT THE EFFECT IS ...

SAME $|\vec{F}^N|$ BUT SINCE MASS DIFFERENT THE EFFECT IS MUCH DIFFERENT

N3.2-3:

Description: Conceptual questions exploring Newton's 3rd law. (3 minutes)

Learning Objectives: [3]

Problem Statement: Consider the free body diagram below for a system. Which pairs of forces in this FBD shown could be interaction force pairs (i.e. Newton's 3rd law pairs)?

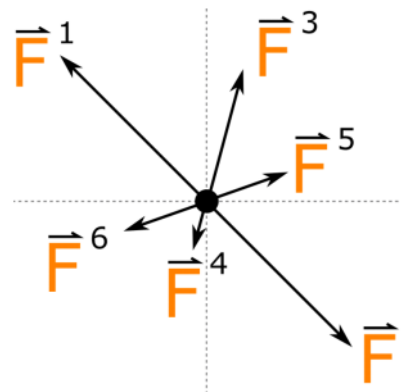
(1) $\vec{F}^1$ and $\vec{F}^2$
(2) $\vec{F}^3$ and $\vec{F}^4$
(3) $\vec{F}^5$ and $\vec{F}^6$
(4) None of these forces are force pairs

$$\Sigma \vec{F}_{EXT} = m\vec{a}$$

ONLY EXTERNAL FORCES GO ON FBD.

$$|\vec{F}^A_{12}| = |\vec{F}^A_{21}|$$

FORCE PAIRS ON DIFFERENT OBJECTS

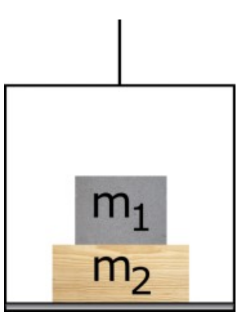


N3.2-4:

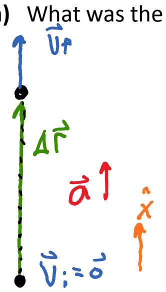
Description: Use multiple systems to analyze two coupled objects to find information about normal forces between the two. (4 minutes + 2 minutes + 4 minutes + 4 minutes)

Learning Objectives: [1, 2, 3, 4, 6]

Problem Statement: A 10 kg box is atop a 5 kg box and both are in an elevator. The elevator starts from rest and accelerates upwards to a speed of 2.00 m/s in 1.50 seconds.



(a) What was the magnitude of the acceleration of the top block in m/s2?



$$V_{fx} = \cancel{V_{ix}}^{0} + a_x \Delta t$$

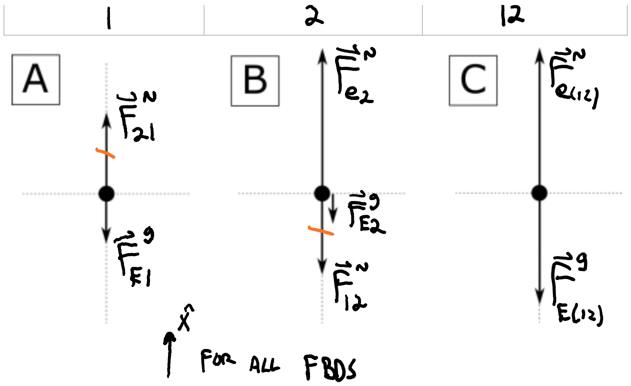
$$2\ m/s = a_x (1.5\ s)$$

$$\boxed{a_x = \frac{4}{3}\ m/s^2} = |\vec{a}|$$

Δx v_{ix} v_{fx} a_x Δt	
K	UK
$V_{ix} = 0$	Δx
$V_{fx} = 2\ m/s$	a_x
$\Delta t = 1.5$	

1	2	12
A	B $\vec{F}^N_{e2}$	C $\vec{F}^N_{e(12)}$

(b) Match the FBD with the appropriate system: top box, bottom box, or top+bottom box.



(b) Match the FBD with the appropriate system: top box, bottom box, or top+bottom box. $a_{1x} = a_{2x} = a_{12x} = a_x$

(c) What is the magnitude of the normal force from the bottom box on the top box? USE SYSTEM 1

$\sum F_x = m_1 a_{1x}$

$F^N_{21} - m_1 g = m_1 a_x$

$F^N_{21} = m_1 a_x + m_1 g$

$\boxed{F^N_{21} \approx 111\ N}$

(d) What is the magnitude of the normal force from the elevator on the bottom box?

USE SYSTEM (12) $\longleftarrow$ OR $\longrightarrow$ USE SYSTEM 2

$\sum F_x = M_{12} a_{12x}$

$F^N_{e(12)} - M_{12} g = M_{12} a_x$

$F^N_{e2} - (m_1+m_2)g = (m_1+m_2)a_x$

$F^N_{e2} = (m_1+m_2)g + (m_1+m_2)a_x$

$\sum F_x = m_2 a_{2x}$

$F^N_{e2} - m_2 g - F^N_{12} = m_2 a_x$

$F^N_{e2} = m_2 a_x + m_2 g + F^N_{12}$

$= m_2 a_x + m_2 g + m_1 a_x + m_1 g$

$F^N_{e2} = (m_1+m_2)a_x + (m_1+m_2)g$

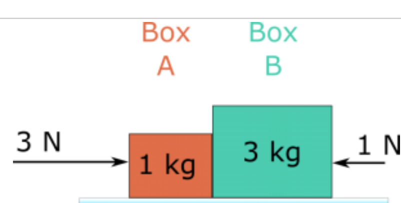
$\boxed{F^N_{e2} \approx 167\ N}$

N3.2-5:

Description: Compare net force magnitude and direction on a coupled system of 2 objects. (2 minutes + 3 minutes)

Learning Objectives: [2, 4, 6]

Problem Statement: The diagram below shows two blocks, on a frictionless horizontal surface, with two external forces acting, one on each block as shown.



(a) Compared to the net force on the smaller block, the net force on the larger block is ________________

(a) Compared to the net force on the smaller block, the net force on the larger block is ________________

(1) in the same direction.
(2) in opposite directions.
(3) The net force vanishes so there is no direction.

$\vec{a}_A = \vec{a}_B = \vec{a}_{AB} \equiv \vec{a}$

$\Sigma \vec{F} = m\vec{a}$

SAME DIRECTION

(b) Compared to the net force on the smaller block, the net force on the larger block is ________________

(1) equal in magnitude.
(2) larger in magnitude.
(3) smaller in magnitude.

$|\Sigma \vec{F}| = m|\vec{a}|$ w/ $|\vec{a}|$ CONSTANT

$|\Sigma \vec{F}| \propto m$

$m_A < m_B \therefore |\Sigma \vec{F}_A| < |\Sigma \vec{F}_B|$

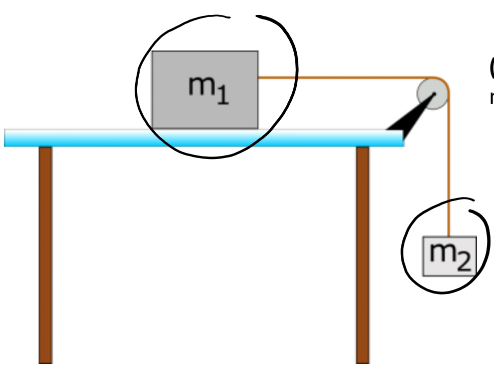
Act II: Coupled systems with pulleys

N3.2-6:

Description: Explore the features of two objects coupled via a pulley with one object hanging and the other on a horizontal surface. (2 minutes + 2 minutes + 2 minutes + 1 minute)

Learning Objectives: [1, 4, 5, 7]

Problem Statement: A box is on a horizontal frictionless table and is attached via a string and pulley, both of negligible mass, to a hanging mass off the end of the table.



(a) The magnitude tension in the horizontal section of the rope is ____________ the magnitude of tension in the vertical section of the rope.

(1) greater than
(2) less than
(3) equal to

"IDEAL PULLEY & MASSLESS ROPE"

(b) Draw a FBD for both the hanging mass and the box

FBD m_1 FBD m_2

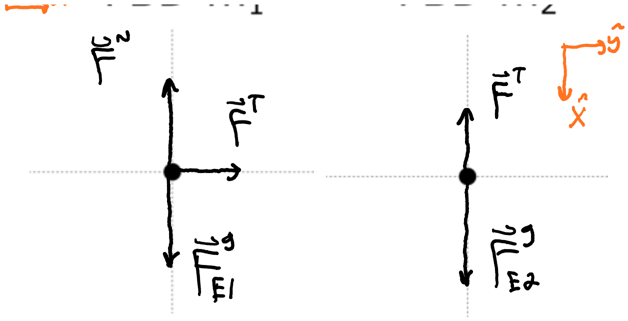
$\hat{y}$ $\hat{x}$ $\vec{F}^N$ $\vec{F}^T$ $\hat{y}$

(c) The magnitude of tension in the cable is _________

(1) greater than $m_2 g$
(2) less than $m_2 g$
(3) equal to $m_2 g$

(d) Rank the magnitude of accelerations for mA and mB

(1) $|\vec{a}_1| > |\vec{a}_2|$
(2) $|\vec{a}_1| < |\vec{a}_2|$
(3) $|\vec{a}_1| = |\vec{a}_2|$



(1) greater than $m_2 g$
(2) less than $m_2 g$
(3) equal to $m_2 g$

(1) $|\mathbf{a}_1| > |\mathbf{a}_2|$
(2) $|\vec{a}_1| < |\vec{a}_2|$
(3) $|\vec{a}_1| = |\vec{a}_2|$

$$a_{1x} = a_{2x} = a_x$$

$$\Sigma F_x = m_1 a_{1x} \qquad \Sigma F_x = m_2 a_{2x}$$

$$F^T = m_1 a_x \qquad -F^T + m_2 g = m_2 a_x$$

$$-m_1 a_x + m_2 g = m_2 a_x$$

$$m_2 g = (m_1 + m_2) a_x$$

$$a_x = \frac{m_2 g}{(m_1 + m_2)}$$

SPECIAL CASES

AS $m_1 \to \infty \quad a_x \to 0$ ✓

AS $m_1 \to 0 \quad a_x \to g$ ✓

AS $m_2 \to \infty \quad a_x \to g$ ✓

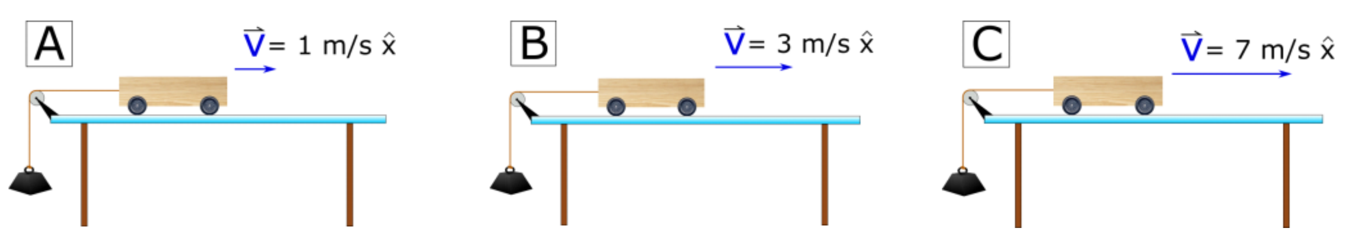
AS $m_2 \to 0 \quad a_x \to 0$ ✓

N3.2-7:

Description: Conceptual question regarding the functional dependence of tension. (2 minutes)

Learning Objectives: [7]

Problem Statement: Rank the following situations based on the magnitude of tension in the string. Assume the hanging mass in each case is the same, friction is negligible, and the pulley and rope have negligible mass.



$$|\vec{F}^{T_A}| = |\vec{F}^{T_B}| = |\vec{F}^{T_C}|$$

$\vec{F}^T$ IS NOT A FUNCTION OF $\vec{V}$

N3.2-8:

Description: Determine the constraints of a coupled system consisting of two objects and a two pulley device. (4 minutes)

Learning Objectives: [1]

Problem Statement: Which of the following are constraints for mass 1 and mass 1?.

(1) $|\Delta\vec{r}_1| = |\Delta\vec{r}_2|$

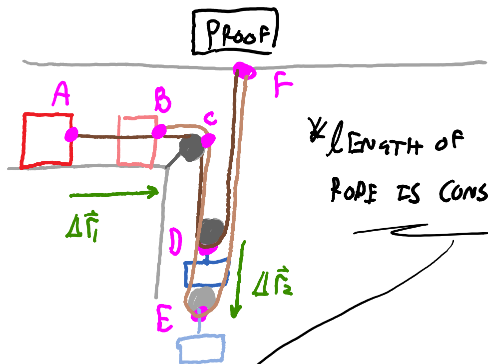
(2) $|\Delta\vec{r}_1| = 2|\Delta\vec{r}_2|$

(3) $|\vec{v}_1| = |\vec{v}_2|$

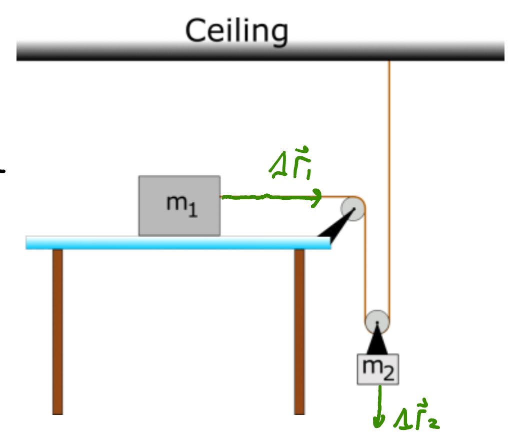
(4) $|\vec{v}_1| = 2|\vec{v}_2|$

(5) $|\vec{a}_1| = |\vec{a}_2|$

(6) $|\vec{a}_1| = 2|\vec{a}_2|$



* LENGTH OF ROPE IS CONSTANT



$$\ell_{BEFORE} = \ell_{AFTER}$$

$$\ell_{AB} + \ell_{BC} + \ell_{CD} + \ell_{DF} = \ell_{BC} + \ell_{CD} + \ell_{DE} + \ell_{ED} + \ell_{DF}$$

$$\ell_{AB} = \ell_{DE} + \ell_{ED}$$

$$|\Delta\vec{r}_1| = |\Delta\vec{r}_2| + |\Delta\vec{r}_2|$$

$$|\Delta\vec{r}_1| = 2|\Delta\vec{r}_2|$$

Act III: Mechanical advantage

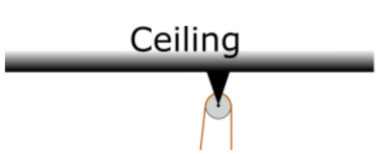
N3.2-9:

Description: Find the mechanical advantage of a pulley device. (4 minutes)

Learning Objectives: [7,8]

Problem Statement: The figure below shows the use of a pulley to obtain mechanical advantage when lifting an object. Mechanical advantage is defined as the magnitude of force required to do a task without the use of a device divided by the magnitude of force required with the device. What is the mechanical advantage of the pulley system shown in the figure?

(1) 1
(2) 2
(3) 3
(4) 4



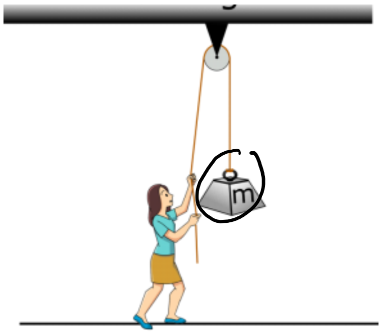
W/OUT DEVICE

NEED Mg TO LIFT

FBD W/ DEVICE

$\uparrow x$ $\quad$ $\uparrow F^T$

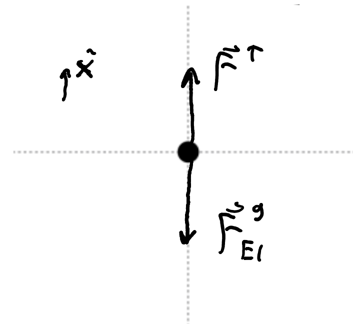
(2) 2
(3) 3
(4) 4



NEED mg TO LIFT

$$M.A. = \frac{mg}{mg} = 1$$

$\boxed{\text{NO M.A.}}$



$\sum F_x = ma_x^0$

$F^T - mg = 0$

$F^T = mg$

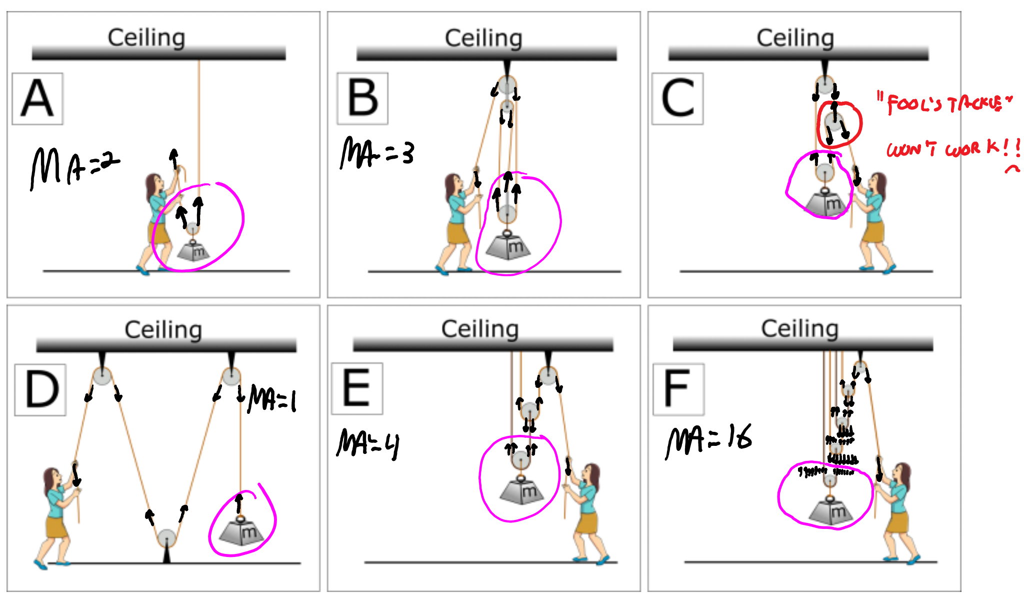
N3.2-10:

Description: Identify which pulley device provides a mechanical advantage larger than 1. (4 minutes)

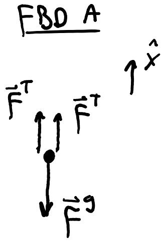
Learning Objectives: [7, 8]

Problem Statement: Below are 4 different devices with pulleys. Which of the devices below have a mechanical advantage greater than 1?

(1) A
(2) B
(3) C
(4) D
(5) E
(6) F



FBD A



$\sum F_x = ma_x^0$

$2F^T - mg = 0$

$F^T = \frac{mg}{2}$

$$MA = \frac{F_{w/out}}{F_w} = \frac{mg}{mg/2} = 2$$

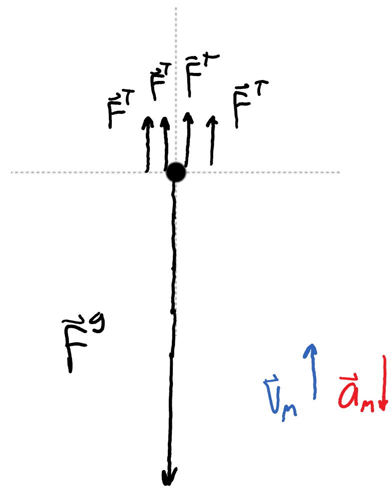
N3.2-11:

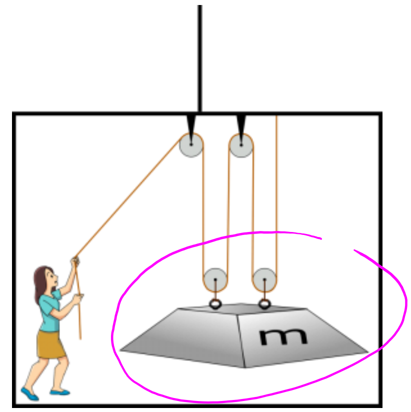
Description: Identify which pulley device provides a mechanical advantage larger than 1. (4 minutes)

Learning Objectives: [7, 8]

Problem Statement: A box of mass m is suspended above the floor of an elevator with the aid of some ideal pulleys. The elevator is moving upwards and slowing down. Which of the following are true regarding the tension in the cable?

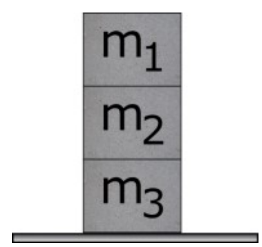
(1) greater than mg
(2) equal to mg
(3) greater than mg/2 and less than mg
(4) equal to mg/2
(5) greater than mg/4 and less than mg/2
(6) equal to mg/4
(7) less than mg/4





Conceptual questions for discussion

1.



The image to the right shows 3 boxes with equal mass. How does the magnitude of the normal force from the top box on the middle box compare to the magnitude of the normal force from the middle box on the bottom box?

i. The magnitude of the normal force from the top box on the middle box is less than the magnitude of the normal force from the middle box on the bottom box.
ii. Both magnitudes of the normal forces are equal because all boxes have the same mass.
iii. Nothing can be said about the relative magnitudes of normal forces without knowing values for each mass.

2. Three physics students are discussing the physics of tug-of-war. Which student do you agree most with?
   i. During a tug-of-war game, both sides are pulling on the same rope, thus the tension that each side experiences is the same magnitude but in the opposite direction. Since each side experiences the same tension, no one will ever win.
   ii. I agree that each side experiences the same magnitude tension but in the opposite direction. However, this is not the only horizontal force acting on each team, there is also friction. Thus the team that wins is the team that has the largest friction force acting on them.
   iii. I don't agree with either of you. The team that wins is the team with the strongest people. The stronger team is able to apply more force on the rope than the weaker team which. This imbalance of force in the rope causes the stronger team to pull the weaker team closer.

3. Is the number of pulleys always equal to the mechanical advantage of a device consisting of pulleys? If not, provide an example of a device that could produce a mechanical advantage greater than the number of pulleys.

Hints

N3.2-1: No hints.

N3.2-2: No hints.

N3.2-3: No hints.

N3.2-4: No hints.

N3.2-5: Do the objects have the same acceleration or different accelerations? How does acceleration relate to net force?

N3.2-6: With no friction, mass 2 will begin to accelerate downwards; draw your FBDs to scale.

N3.2-7: No hints.

N3.2-8: No hints.

N3.2-9: Look at the key equations for definition of mechanical advantage.

N3.2-10: No hints.