Name: Kevin

ID: ____________

Physics 203

Quiz 1

22 August 2022

You will have 80 minutes to complete this quiz. Collaboration is not allowed for the first 60 minutes of the quiz. Collaboration will be allowed with an assigned partner for the final 20 minutes of the quiz. Allowed on your desk are: ten 8.5 × 11 inch doubled sided sheets of notes that are bound together, non-communicating graphing scientific calculator, 1 page of scratch paper, writing utensils, and the quiz.

Solutions

For questions 1 through 3, **fill in the square** next to all correct answers. A given problem may have more than one correct answer. Each correctly bubbled answer will receive two points. There are 7 correct answers in this section and only the first 7 filled-in answers will be graded. There is no partial credit.

1. White light enters a black box (a box that you cannot see what's inside) which contains a single, unknown optical device. The light that comes out from a small hole somewhere on the opposite side of the box is green. What optical device could be inside the black box?

- ☐ Reflection Grating
- ☑ Glass Prism
- ☐ Flat Mirror
- ☑ Diffraction Grating
- ☐ Concave Lens

2. A film camera uses a lens to focus the light reflecting from a 3-meter-tall statue standing several meters away down onto a piece of film that is only a few centimeters wide inside the camera. If the lens has a focal distance f, which of the following would be true to obtain a real image that is in focus on the film?

- ☑ The image should be between f and $2f$ from the lens
- ☐ The camera uses a diverging lens
- ☐ The image is beyond $2f$ from the lens
- ☑ The camera uses a converging lens

3. On a very early morning, two people each throw a pebble into a pond that is completely still, causing waves to travel radially outward from the two points where the pebbles hit the water. Which of the following factors would affect the locations where the two waves would constructively interfere with each other when they meet?

- ☑ The distance between two adjacent crests on the water
- ☐ The maximum height of the waves
- ☑ The difference in the path lengths from each wave source (the spot where the pebble hits the water) to the point where the two waves meet
- ☑ The difference in the times that the first pebble and the second pebble hit the water

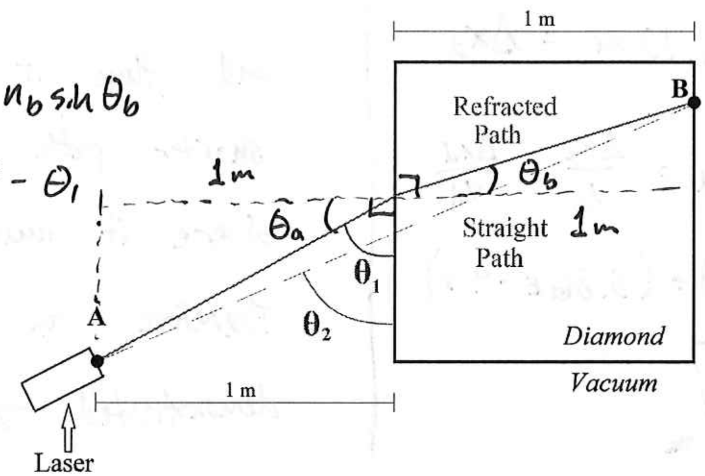
4 ~~8~~. (10 points) This problem will explore the idea that light travels the path of least time.

(a) We have a laser which produces single-frequency light that has a wavelength in vacuum of $\lambda = 590\,\text{nm}$ (yellow). Let's consider what happens as this light travels through diamond, which has an index of refraction of $n = 2.42$ for this light. What is the effective speed of the light as it travels through the diamond?

(b) If we place a laser of this frequency at point A on the diagram below, which is 1 m away from a large block of diamond (a cube which has side lengths of 1 m), and aim it so that it hits the block at an angle of $\theta_1 = 56°$ off the diamond surface, how long does it take the light to travel to point B on the other side of the block?

$n_a \sin\theta_a = n_b \sin\theta_b$

$\theta_a = 90° - \theta_1$



(c) Now, let's consider what would happen if light didn't refract: imagine the light travelled straight from point A to point B, which would hit the diamond surface at an angle of $\theta_2 = 65.5$, but the effective speed of the light in the diamond would still be the same as what you calculated in part (a). How long would it take this light to travel in a straight line from point A to point B?

(d) Explain how it could be possible that light can travel from point A to point B in less time by following the refractive path than if it had simply travelled in a straight line from point A to point B.

(a) $n = \frac{c}{v}$ ← Definition of n

$v = \frac{c}{n} = \frac{3E8\ m/s}{2.42}$ ← Solve for v

$v = 1.24\ E8\ m/s$

(b) $n_a \sin\theta_a = n_b \sin\theta_b$ ← Snell's Law

$n_a = 1$; $\theta_a = 90° - 56°$; $n_b = 2.42$

$\theta_b = \sin^{-1}\left(\frac{n_a}{n_b}\sin\theta_a\right) = 13.4°$ ← Solve for θ_b

(b) (continued) $v = \frac{\Delta x}{\Delta t}$; $\frac{c}{n} = v$

In vacuum: $\cos\theta_a = \frac{1m}{\Delta x_a}$ $\left(\frac{adj}{hyp}\right)$

In diamond: $\cos\theta_b = \frac{1m}{\Delta x_b}$ $\left(\frac{adj}{hyp}\right)$

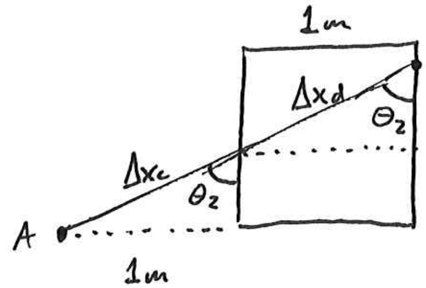
$\Delta x_a = \frac{1m}{\cos(34°)} = 1.21\,m$

$\Delta x_b = \frac{1m}{\cos(13.4°)} = 1.03\,m$

$\Delta t_{total} = \Delta t_a + \Delta t_b = \frac{\Delta x_a}{v_a} + \frac{\Delta x_b}{v_b}$

$\Delta t_{total} = (4.03E-9\ s) + (8.29E-9\ s) = 12.3\ ns$

(c)



In vacuum: $\sin \theta_2 = \frac{1\,m}{\Delta x_c}$ $\left(\frac{opp.}{hyp.}\right)$

In diamond: $\sin \theta_2 = \frac{1\,m}{\Delta x_d}$ $\left(\frac{opp.}{hyp.}\right)$

$$\Delta x_c = \frac{1\,m}{\sin(65.5°)} = 1.10\,m = \Delta x_d$$

$$\Delta t_{total} = \Delta t_c + \Delta t_d = \frac{\Delta x_c}{v_c} + \frac{\Delta x_d}{v_d}$$

$$\Delta t_{total} = (3.66E-9\,s) + (8.86E-9\,s)$$

$$\boxed{\Delta t_{total} = 12.5\,ns}$$

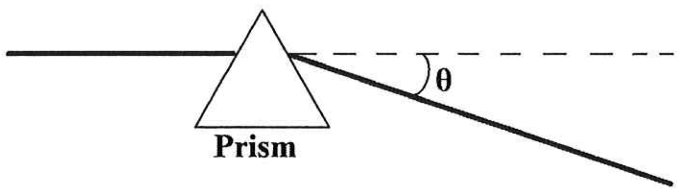
(d) Light moves much faster in vacuum than in diamond (more than double the speed). By taking the refracted path, light travels the greater distance in vacuum toward point B, where it can travel much faster, and thus it can take a much shorter path through the diamond where it must travel slowly. Therefore, as I have mathematically demonstrated, the refractive path dictated by Snell's law is the path which takes less time than a direct path from A to B.

5. (8 points) In this problem, we will explore more about prisms and diffraction gratings.

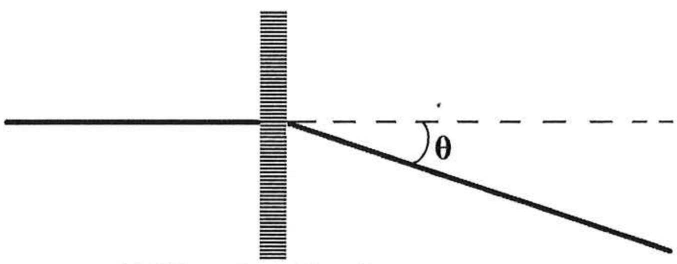
(a) We have seen that both diffraction gratings and prisms can split a beam of white light into its component wavelengths (i.e. white light going in becomes a rainbow of colors going out). Explain one key difference between the **patterns** that white light forms after it passes through a diffraction grating versus after it passes through a prism.

(b) Now consider *how* prisms and diffraction gratings affect light. Explain one key difference between the **physical mechanisms** of how a diffraction grating is able to split white light into its different wavelengths versus how a prism is able to do the same.

(c) We set up a prism so that it deflects blue light (which has a frequency of $f = 6.5 * 10^{14}$ Hz) by an angle of $\theta = 20°$, as shown below. For a diffraction grating to deflect the same frequency of light by the same angle at its first non-central maximum, what must the slit spacing of the diffraction grating be?



Prism



Diffraction Grating

(a) Prisms deflected red light less than blue light, whereas diffraction gratings "deflect" blue light less than red light, so the colors in the pattern will be reversed. Also, the diffraction grating can have multiple rainbow patterns, whereas the prism will only show one rainbow pattern.

(b) Prisms split wavelengths by causing light at different wavelengths to travel at different speeds, and thus be deflected by different angles. Diffraction gratings instead create a pattern through wave intereference, so the different wavelengths constructively interfere at different spatial angles.

(c) $v = f\lambda$ ← velocity of a wave

$\lambda = \frac{v}{f} = 462\ \text{nm}$ ← solve for λ

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$d \sin\theta = m\lambda$ ← Multi-slit bright spots

$\theta = 20°$, $m = 1$ ← Given info

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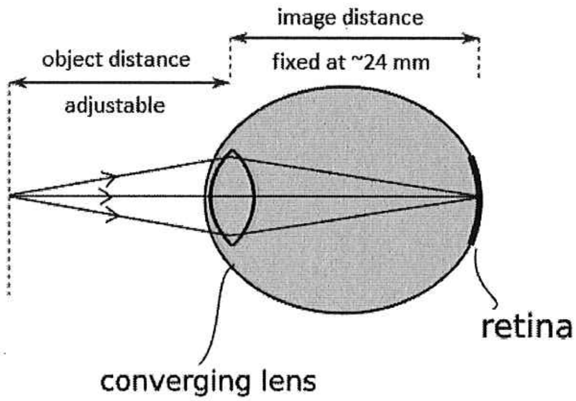
$\boxed{d = \frac{\lambda}{\sin\theta} = 1.35\ \mu\text{m}}$ ← solve for d, slit width

6. (6 points) The eye is a biological wonder, able to take light reflecting off of objects and turn that into an image that your brain can interpret. A key component in this process is that your eye has its own adjustable converging lens, which projects light onto your retina in the back of the eye (see the figure below). The size of your eye does not change as you view objects, which means the distance between the eye's lens and the retina (the *image distance*) is fixed at about 24 mm. However, the eye's muscles can change the shape of the lens itself to adjust the focal length of the lens!



(a) As you're looking at this very quiz, it is probably about 35 cm away from the lens of your eye. Ignoring additional lenses (such as if you are wearing glasses), if the words are appearing in focus, what would the focal length of your eye be right now?

(b) If the average height of the letters you are reading at this moment is about 2 mm, what is the height of the image that these letters form on your retina?

(c) You walk outside on a clear night and look up at the moon (which is very, very far away). What is the focal length of your eye's lens now?

(a) $\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$ ← thin lens equation

$d_i = 2.4\ cm$, $d_o = 35\ cm$

$f = \left[\frac{1}{35\ cm} + \frac{1}{2.4\ cm}\right]^{-1}$ ← solve for f

$\boxed{f = 2.25\ cm}$

(b) $M = -\frac{d_i}{d_o} = \frac{h_i}{h_o}$ ← definition of magnification

$h_i = -h_o \cdot \frac{d_i}{d_o}$ ← solve for h_i

$h_i = -(0.2\ cm)\left(\frac{2.4\ cm}{35\ cm}\right)$

$\boxed{h_i = -0.014\ cm}$

(c) Parallel rays converge at the focus. $\Rightarrow \boxed{f = d_i = 24\ mm}$

or $\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i}$

$d_o \simeq \infty \Rightarrow \frac{1}{d_o} \simeq 0$

$\frac{1}{f} = \frac{1}{d_i} \Rightarrow \boxed{f = d_i = 24\ mm}$