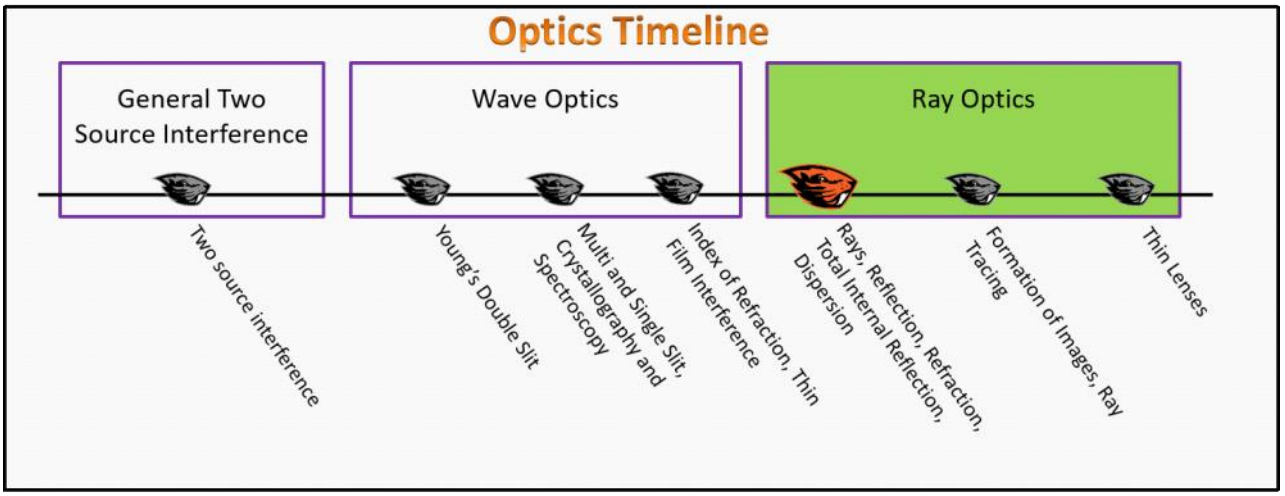
(RO.L1.2-sols) Foundation Stage Solutions

Monday, January 22, 2018 5:44 PM

Ray Optics
Foundation Stage (RO.L1.2)

Lecture 1
Rays, Reflection, Refraction, Total Internal Reflection, Dispersion



Textbook Chapters (* Calculus version)

- **BoxSand** :: KC videos (Snell's Law of Refraction)
- **Knight** (College Physics : A strategic approach 3rd) :: 18.1 ; 18.2 ; 18.3
- ***Knight** (Physics for Scientists and Engineers 4th) :: 34.1 ; 34.2 ; 34.3
- **Giancoli** (Physics Principles with Applications 7th) :: 21-1 ; 21-2 ; 21-5 ; 21-6

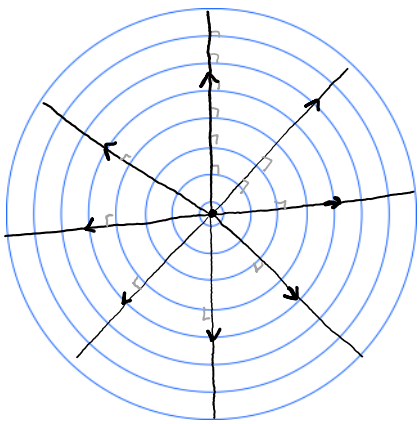
Warm up

RO.2.L1-1:

Description: Conceptual question waves and rays.

Learning Objectives: [?] - Can you identify the objectives from the previous lecture, and this lecture, that this question is relevant to?

Problem Statement: A pebble is dropped into the middle of a very calm lake. At some time later, a snapshot from above was taken as seen below. Each line represents a crest of a ripple. The crests are moving radially outwards. Sketch a few rays overtop of the image that would also describe this system.



Selected Learning Objectives

1. Coming soon to a lecture template near you.

Key Terms

- Ray model of light
- Incident angle
- Reflection
- Reflected angle
- Law of reflection
- Refraction
- Refracted angle
- Snell's law
- Total internal reflection
- Critical angle
- Dispersion

Key Equations

Key Concepts

- Coming soon to a lecture template near you.

Questions

Act I: Rays Model of Light

RO.2.L1-2:

Description: Conceptual question about rays and waves. (3 minutes)

Learning Objectives: [?]

Problem Statement: Which of the following statements are true?

F (1) Sound is a wave, while light is really a ray.
F (2) Some objects emit rays of light, while other sources emit waves.
T (3) Light can be modeled as a ray or a wave at the same time.
F (4) Waves can bend but rays only ever travel in a straight line.

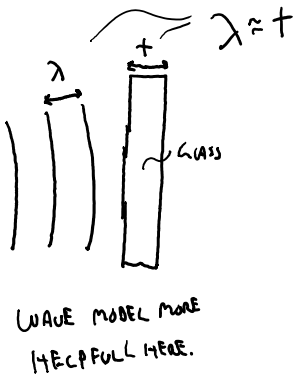
RO.2.L1-3:

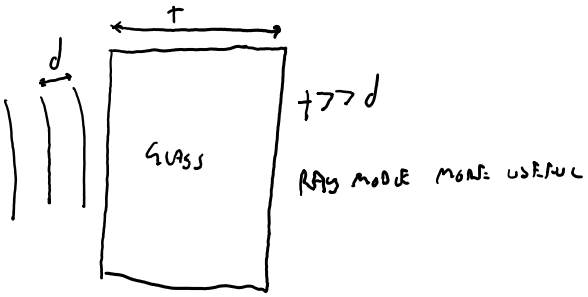
Description: Conceptual question about rays and waves. (3 minutes)

Learning Objectives: [?]

Problem Statement: When would you use the ray model vs the wave model of light to analyze a system?

? (1) Use the wave model when your surfing and the ray model when buying sunglasses.
F (2) Use the wave model for sound and the ray model for light.
T (3) Use the wave model when light interacts with objects that have a size on the order of the light's wavelength.
T (4) Use the ray model when light interacts with objects that have a size much larger than the light's wavelength.
F (5) Use the wave model when light is interacting with anything and the ray model when the light is not interacting with anyt hing.





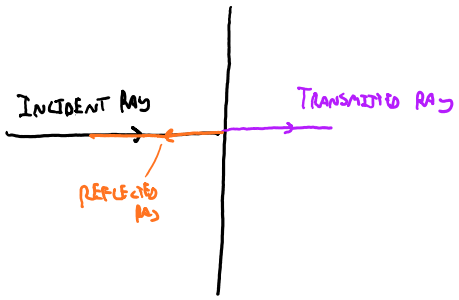
RO.2.L1-4:

Description: Conceptual question about waves when they reach a boundary. (2 minutes)

Learning Objectives: [?]

Problem Statement: Which of the following happens when light crosses a boundary of different index of refractions?

(1) Reflection.
(2) Transmission.
(3) Absorption.
(4) Scattering.
(5) Brody wags his tail.



* COLORS HERE DON'T MEAN ANYTHING *

Act II: Reflection

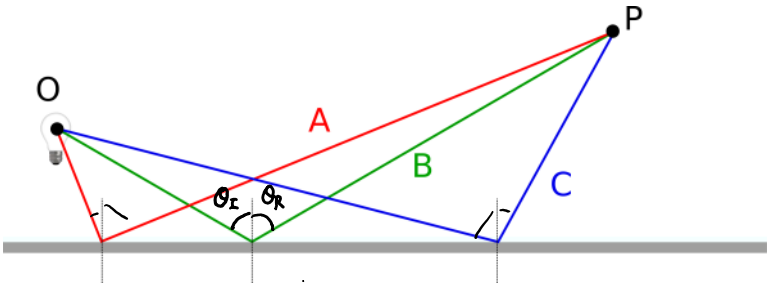
RO.2.L1-5:

Description: Conceptual question reflection. (3 minutes)

Learning Objectives: [?]

Problem Statement: Rays of light travel from an object O to an observer at P via a reflecting surface. Which of the three paths provides the shortest path from **O** to **P**?

(1) Path A.
(2) Path B.
(3) Path C.
(4) All three are the same.
(5) The answer depends on the roughness of the surface.



LAW OF REFLECTION

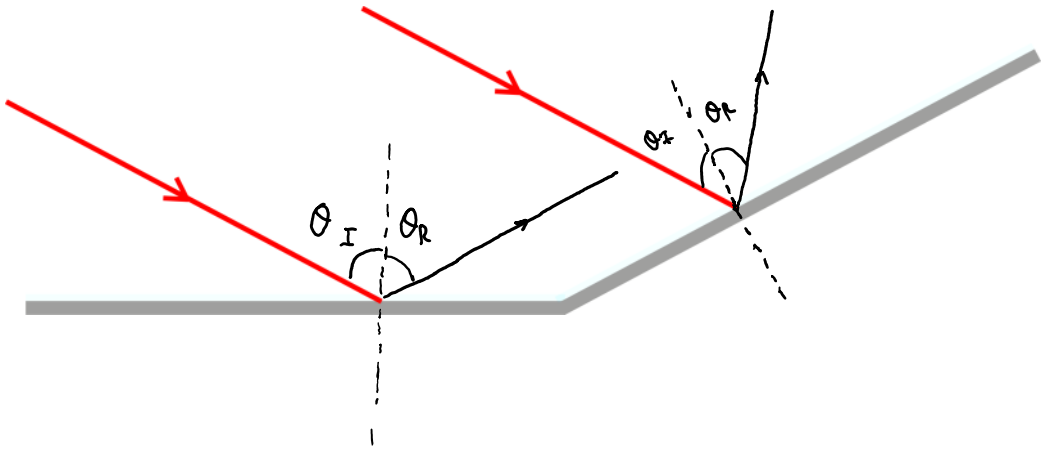
$\theta_{INCIDENT} = \theta_{REFLECTED}$

RO.2.L1-6:

Description: Sketch reflected ray given reflecting surface and incident ray. (3 minutes)

Learning Objectives: [?]

Problem Statement: Two reflecting surfaces are used to make the shape shown below. The sunlight from very far away is represented by the two rays shown in the image. Sketch the path of each ray after reflecting off each surface.



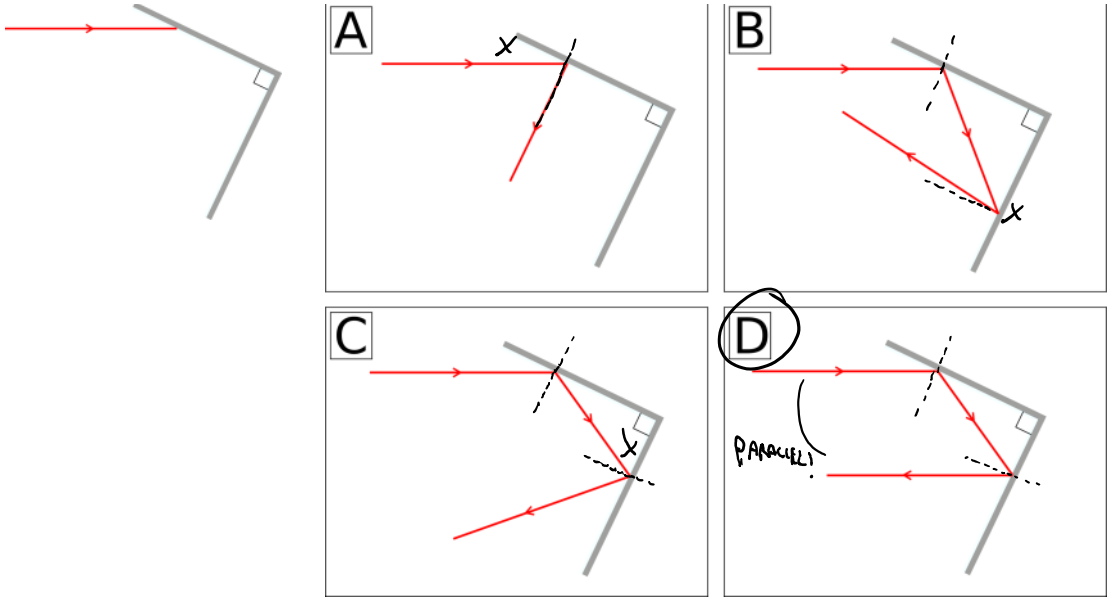
RO.2.L1-7:

Description: Multiple choice sketch reflected ray given incident ray and reflecting surface. (4 minutes)

Learning Objectives: [?]

Problem Statement: Light enters horizontally into the combination of two perpendicular mirrors as shown below. Which of the following images best represent the path the given light ray takes after reflecting off the surfaces?

(1) A
(2) B
(3) C
(4) D



Act III: Refraction

RO.2.L1-8:

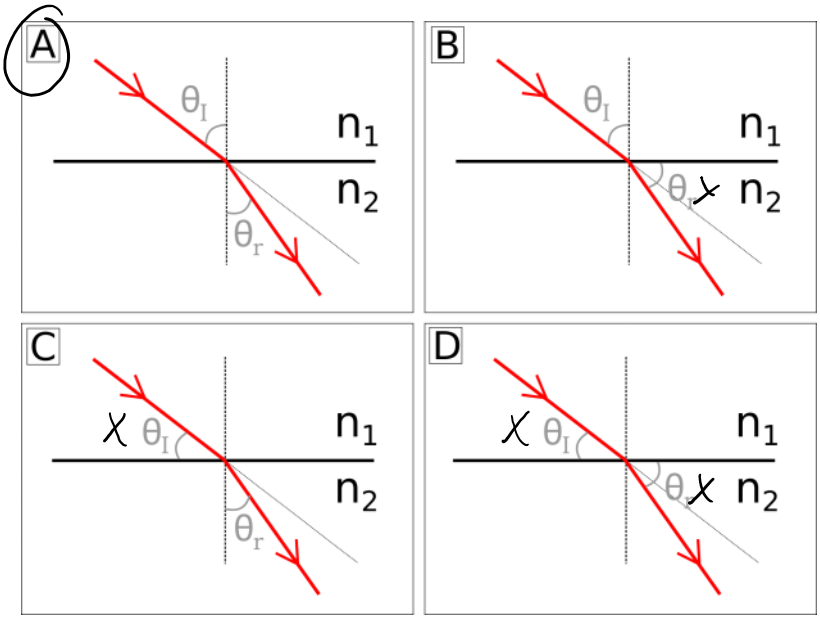
Description: Match physical representation with mathematical model for refraction, Snell's law. (3 minutes)

Learning Objectives: [?]

Problem Statement: The angle of reflection is equal to the angle of incidence: $\theta_I = \theta_R$. The refracted angle is related to the angle of incidence by the mathematical model: $n_1 \sin(\theta_I) = n_2 \sin(\theta_r)$. Which of the following physical representations correctly match the given mathematical model for refraction?

(1) A
(2) B
(3) C
(4) D

θ_I & θ_r RELATIVE TO NORMAL



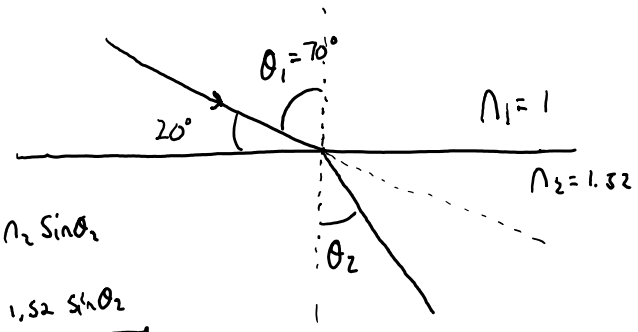
RO.2.L1-9:

Description: Calculate refracted angle given initial angle of ray with respect to boundary, and index of refractions. (5 minutes)

Learning Objectives: [?]

Problem Statement: Light traveling in air strikes a horizontal piece of glass. The light makes an angle of 20 degrees with respect to the horizontal surface. The index of refraction for air and glass are $\mathbf{n_{air}} = 1$ and $\mathbf{n_{glass}} = 1.52$. Calculate the refracted angle?

(1) 38.2°
(2) 13.0°
(3) 31.3°
(4) 90°



$n_1 \sin\theta_1 = n_2 \sin\theta_2$

$1 \sin(70°) = 1.52 \sin\theta_2$

$\theta_2 \approx 38.2°$

$n_1 \sin\theta_1 = n_2 \sin\theta_2$

$\frac{\sin\theta_1}{\sin\theta_2} = \frac{n_2}{n_1}$

IF $n_1 < n_2$ THEN $\theta_2 < \theta_1$

LOW → HIGH BEND TOWARDS NORMAL

IF $n_1 > n_2$ THEN $\theta_2 > \theta_1$

HIGH → LOW BEND AWAY FROM NORMAL

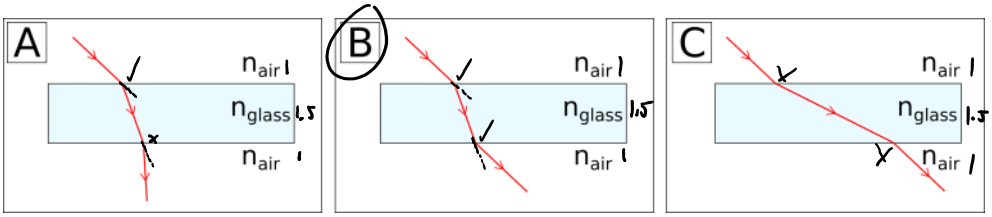
RO.2.L1-10:

Description: Conceptual question about physical representation and Snell's law. (4 minutes)

Learning Objectives: [?]

Problem Statement: Which of the ray diagrams shown below are possible?

(1) A
(2) B
(3) C



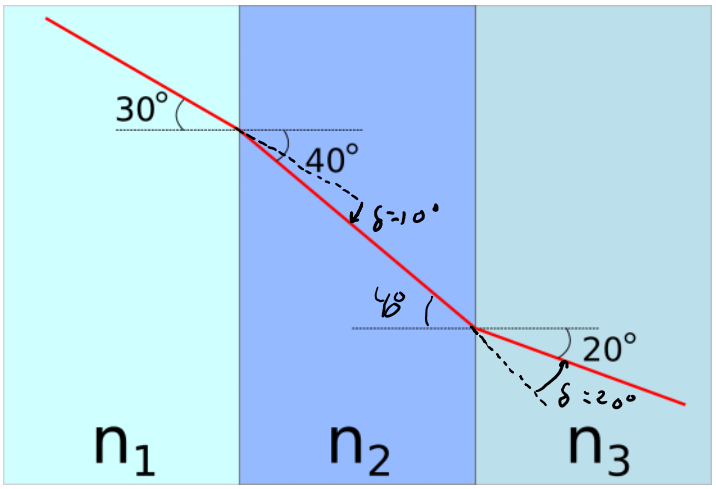
RO.2.L1-11:

Description: Rank index of refractions given incident and refracted angles. (5 minutes)

Learning Objectives: [?]

Problem Statement: Rank the following mediums based on index of refraction.

(1) $n_1 > n_2 > n_3$
(2) $n_3 > n_1 > n_2$
(3) $n_2 > n_3 > n_1$
(4) $n_3 > n_2 > n_1$



$n_1 \rightarrow n_2$

BENDS AWAY FROM NORMAL

SO $n_1 > n_2$ //

$n_2 \rightarrow n_3$

BENDS TOWARDS NORMAL

SO $n_2 < n_3$ //

$n_1 \overset{?}{\ldots} n_3$

FROM 1→2 $\delta = 10°$ ... BENDS LESS THAN FROM 2→3 w/ $\delta = 20°$

SO $n_1 < n_3$ //

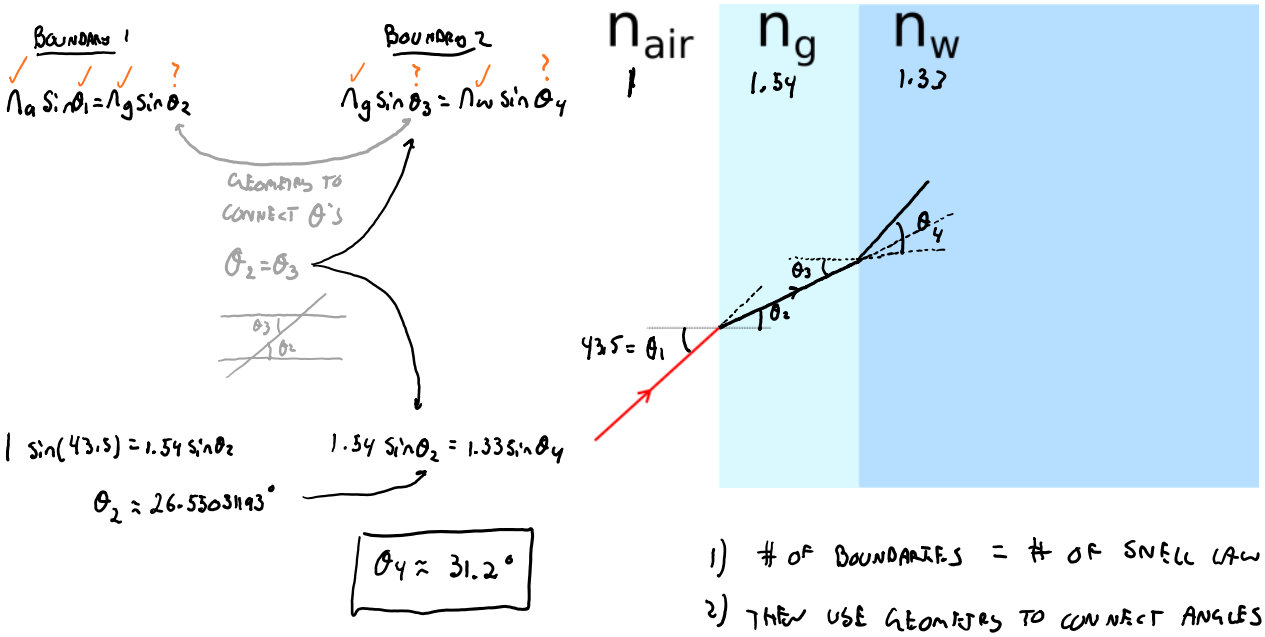
$n_2 < n_1 < n_3$

RO.2.L1-12:

Description: Calculate refracted angle given two boundaries, index of refractions, and initial incident angle. (8 minutes)

Learning Objectives: [?]

Problem Statement: An aquarium filled with water has flat glass sides and an index of refraction of 1.54. A beam of light from the outside air strikes the glass at a 43.5 degree angle to the perpendicular as shown below. What is the angle of refraction when the light ray enters the water? $n_{water} = 1.33$



1) # OF BOUNDARIES = # OF SNELL LAW EQNS
2) THEN USE GEOMETRY TO CONNECT ANGLES FROM ONE BOUNDARY TO ANOTHER
3) MATHY MATH

RO.2.L1-13:

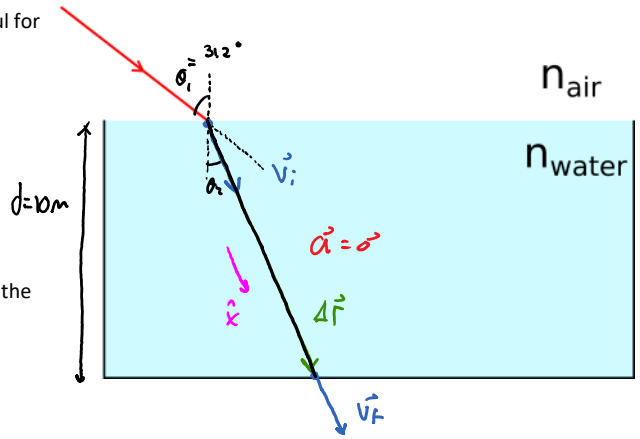
Description: Calculate time it takes light to travel given distance, index of refractions, wavelength, and initial incident angle. (10 minutes)

Learning Objectives: [?]

Problem Statement: Light of wavelength 569 nm in a vacuum strikes the surface of water at an angle of 31.2 degrees with respect to the normal of the surface. The water has an index of refraction of 1.33. The depth of the liquid is 10 meters. We wish to determine the time it takes for the light to travel from the surface of the liquid to the bottom.

(a) Which of the following physics analysis tools might be useful for this system?

(1) Kinematics
(2) Force analysis
(3) Momentum analysis
(4) Energy analysis
(5) Snell's law
(6) Geometry



(b) How long does it take for light to travel from the surface to the bottom?

SNELL'S LAW

$n_a \sin\theta_1 = n_w \sin\theta_2$

$1 \sin(31.2) = 1.33 \sin\theta_2$

KINEMATICS

K	UK
$\Delta x = 10.3574\ m$	

INDEX

$n = \frac{c}{v}$

GEOMETRY

λ

$n_a \sin\theta_1 = n_w \sin\theta_2$

$1 \sin(31.2) = 1.33 \sin\theta_2$

$\theta_2 = 22.923017777^\circ$

	K	UK
$\Delta x = 10.8574\ m$	$v_{i,x} = 2.25564 \times 10^8\ m/s$	Δt
	$v_{f,x} = v_{i,x}$	
	$a_x = 0$	

$\Delta x = v_{i,x}\Delta t + \frac{1}{2}a_x(\Delta t)^2$

$10.8574\ m = (2.25564 \times 10^8\ m/s)\Delta t$

$\Delta t = 4.81 \times 10^{-8}\ m/s$

$\approx 48.1\ ns/s$

INDEX

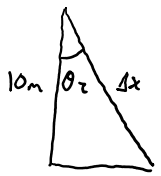
$n = \frac{c}{v}$

$v = \frac{c}{n}$

$v_w = \frac{c}{n_w}$

$v_w = 2.25564 \times 10^8\ m/s$

GEOMETRY



$\cos\theta_2 = \frac{10}{\Delta x}$

$\Delta x = \frac{10}{\cos\theta_2} \approx 10.8574213\ m$

Act IV: Application of Refraction | Total Internal Reflection

RO.2.L1-14:

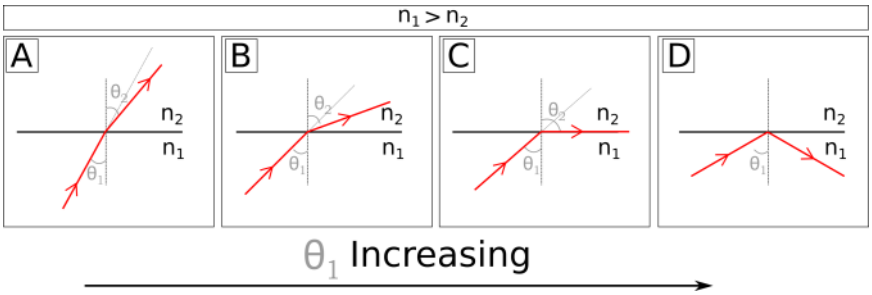
Description: Conceptual question with physical representation Snell's law and total internal reflection. (3 minutes + 1 minute)

Learning Objectives: [?]

Problem Statement: A beam of light is shown onto a horizontal flat surface. The light is originally in a material with an index $\mathbf{n_1}$ and the flat surface has an index $\mathbf{n_2}$. The image below shows this system when the beam is shown at a few different angles.

(a) Use Snell's law to find $\sin(\boldsymbol{\theta_1})$ for the third image **C**.

(1) $\sin(\boldsymbol{\theta_1}) = \mathbf{n_2} / \mathbf{n_1}$
(2) $\sin(\boldsymbol{\theta_1}) = (\mathbf{n_2} / \mathbf{n_1}) \sin(45°)$
(3) $\sin(\boldsymbol{\theta_1}) = \mathbf{n_1} / \mathbf{n_2}$
(4) $\sin(\boldsymbol{\theta_1}) = (\mathbf{n_1} / \mathbf{n_2}) \sin(45°)$



$n_1 \sin\theta_1 = n_2 \sin\theta_2$

$n_1 \sin\theta_1 = n_2 \sin(90^\circ)$

$n_1 \sin\theta_1 = n_2$

$\sin\theta_1 = \frac{n_2}{n_1}$

(b) For this specific system in the third image **C**, $\boldsymbol{\theta_1}$ is often referred to as the

(1) important angle.
(2) critical angle.
(3) angley angle.

$\sin\theta_1 = \frac{n_2}{n_1}$

$\sin\theta_c = \frac{n_2}{n_1}$

$\theta_1 = \theta_c$

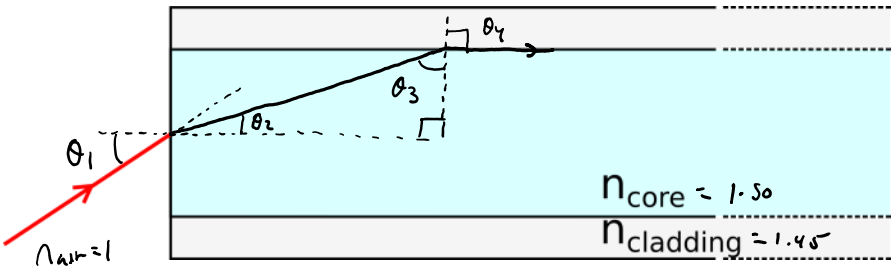
RO.2.L1-15:

Description: Calculate initial incident angle given index of refractions, and critical angle condition. (5 minutes)

Learning Objectives: [?]

Problem Statement: A horizontal section of fiber optic cable consists of an inner glass core with index of refraction of 1.50 and an outer cladding of index 1.45. What is the maximum incident angle such that the light never escapes the inner core?

(1) 14.8°
(2) 75.2°
(3) 22.6°
(4) 42.0°



Boundary 1
$n_a \sin\theta_1 = n_{core} \sin\theta_2$

Boundary 2
$n_{core} \sin\theta_3 = n_{cladding} \sin 90°$

$\sin\theta_3 = \frac{1.45}{1.50}$

$\theta_3 = 75.16488842°$

Geometry $\theta_3 \rightarrow \theta_2$?

$\theta_2 + \theta_3 + 90 = 180$

$\theta_2 = 90 - \theta_3$

$\theta_2 = 14.835115 8°$

$1 \sin\theta_1 = 1.50 \sin(14.83\ldots)$

$\theta_1 = 22.6°$

Act V: Dispersion

RO.2.L1-16:

Description: Conceptual question about definition of dispersion. (3 minutes)

Learning Objectives: [?]

Problem Statement: Which of the following statements are true regarding dispersion?

(1) Dispersion is the phenomena that different frequencies of light refract at different angles.
(2) Dispersion is the phenomena that the index of refraction depends on the frequency of the light.
(3) Dispersion is the phenomena that creates a rainbow when sunlight shines through a prism.

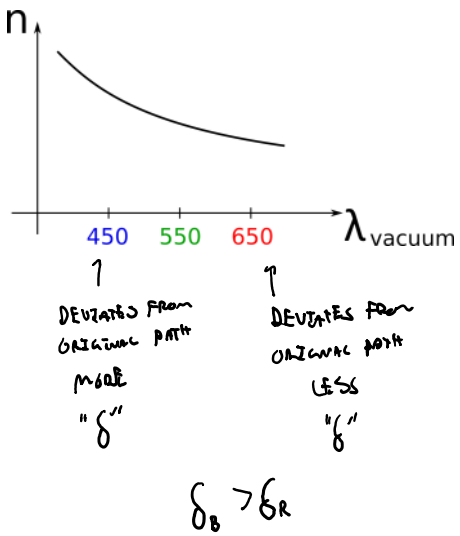
RO.2.L1-17:

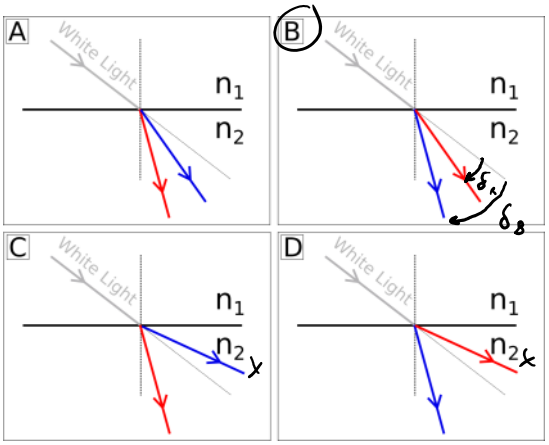
Description: Conceptual question dispersion and separation of wavelengths. (3 minutes)

Learning Objectives: [?]

Problem Statement: The typical frequency dependence of the index of refraction is shown below. Which of the following physical representations correctly represents the refracted blue and red light rays when white light is incident on a piece of glass at some angle relative to the normal?

(1) A
(2) B
(3) C
(4) D





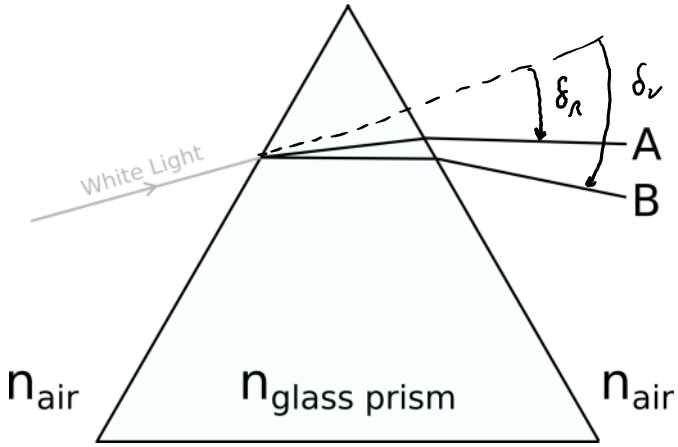
RO.2.L1-18:

Description: Conceptual question dispersion and separation of wavelengths. (3 minutes)

Learning Objectives: [?]

Problem Statement: White light enters a glass prism. When the light leaves the prism, the colors have been separated. Match each ray after leaving the prism, red or violet.

(1) **A** - Violet ; **B** - Red
(2) **A** - Red ; **B** - Violet

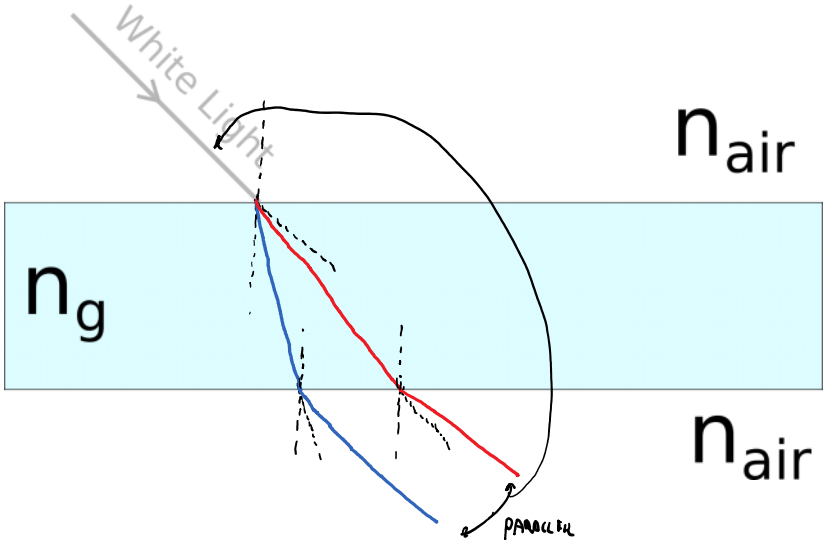


RO.2.L1-19:

Description: Conceptual question dispersion and separation of wavelengths. (3 minutes)

Learning Objectives: [?]

Problem Statement: A rectangular slab of glass surrounded by air on all sides has white light shone onto it at an angle as shown in the image below. Sketch the red and blue rays when the light enters and leaves the glass.



Conceptual questions for discussion

1. Coming soon to a lecture template near you.

Hints

RO.2.L1-1: No hints.

RO.2.L1-2: No hints.

RO.2.L1-3: No hints.

RO.2.L1-4: No hints.

RO.2.L1-5: No hints.

RO.2.L1-6: No hints.

RO.2.L1-7: No hints.

RO.2.L1-8: No hints.

RO.2.L1-9: No hints.

RO.2.L1-10: No hints.

RO.2.L1-11: No hints.

RO.2.L1-12: No hints.

RO.2.L1-13: No hints.

RO.2.L1-14: No hints.

RO.2.L1-15: No hints.

RO.2.L1-16: No hints.

RO.2.L1-17: No hints.

RO.2.L1-18: No hints.

RO.2.L1-19: No hints.