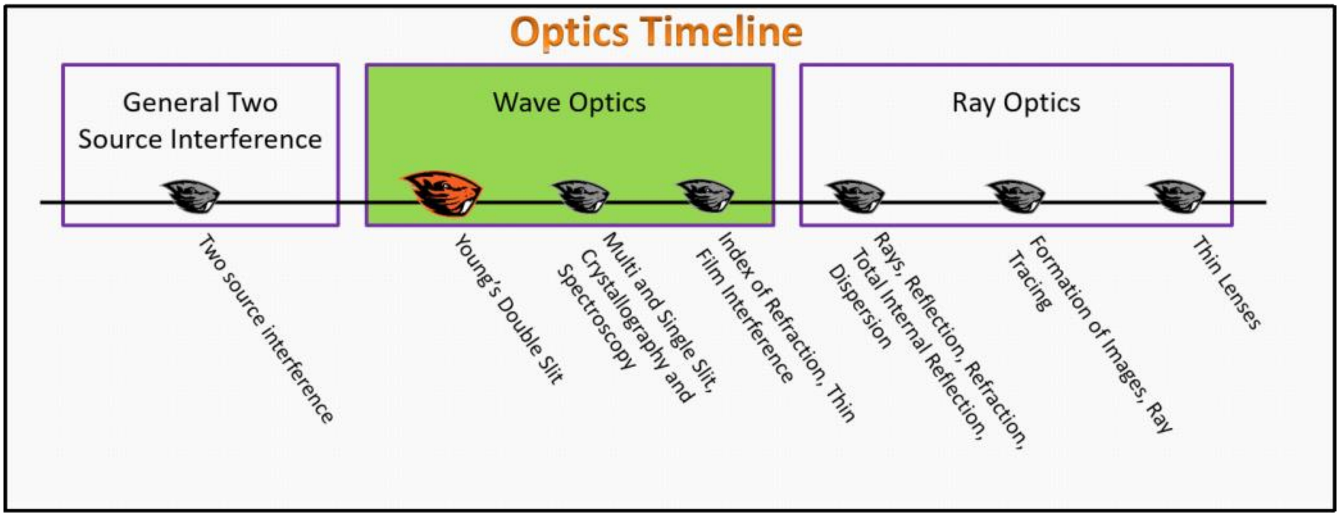
(WO.L1.2-sols) Foundation Stage Solutions

Monday, January 22, 2018 5:44 PM

Wave Optics
Foundation Stage (WO.L1.2)

Lecture 1
Young's Double Slit



Textbook Chapters (* Calculus version)

- **BoxSand** :: KC videos ([Optics]())
- **Knight** (College Physics : A strategic approach 3rd) ::
- ***Knight** (Physics for Scientists and Engineers 4th) ::
- **Giancoli** (Physics Principles with Applications 7th) ::

Warm up

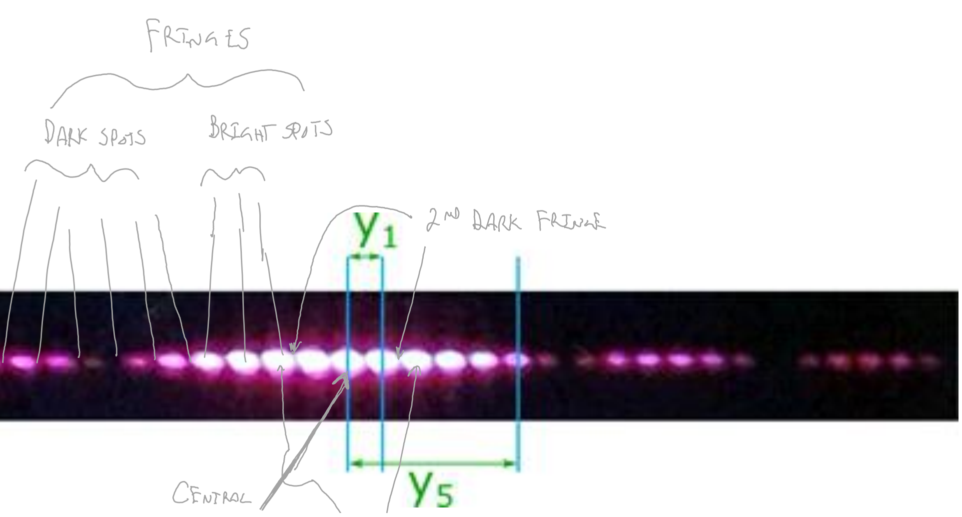
WO.L1.2-01:

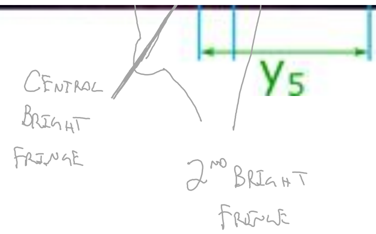
Description:

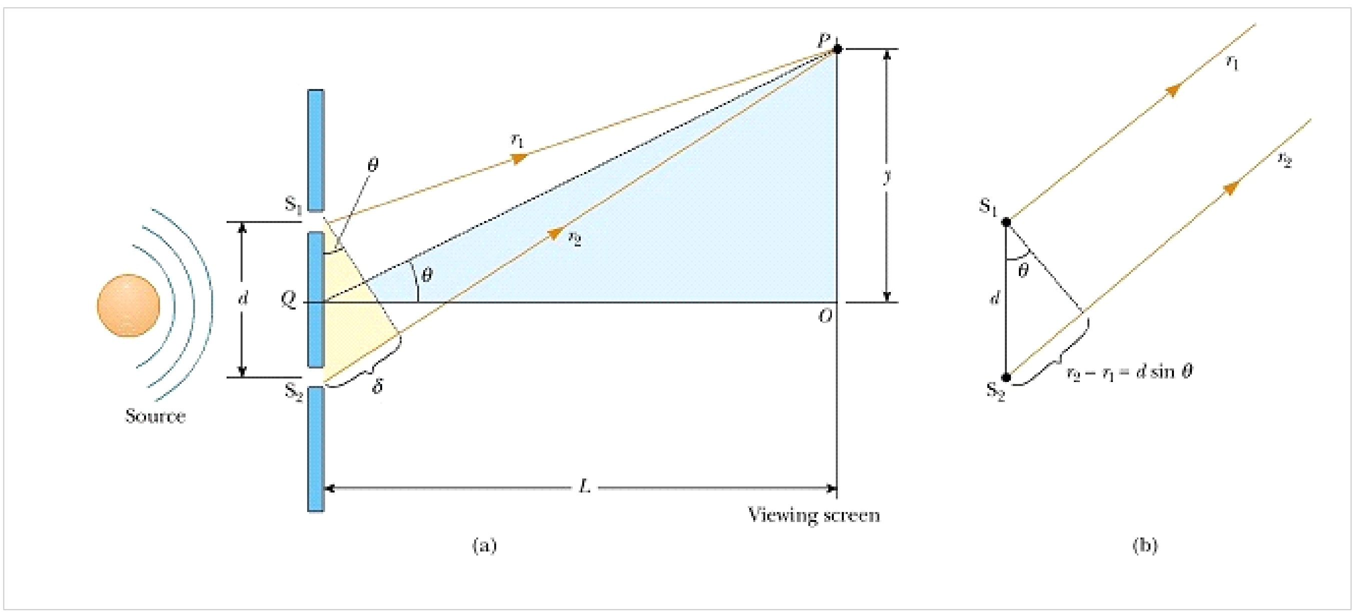
Learning Objectives: [?] - Can you identify the objectives from the previous lecture, and this lecture, that this question is relevant to?

Problem Statement: With your neighbors discuss the following terms with regards to the image below.

Fringe
Central bright fringe
2th bright fringe
2rd dark fringe







Selected Learning Objectives

1. Coming soon to a lecture template near you.

Key Terms

- Path length difference
- Small Angle Approximation

- Slit Separation
- Viewing Screen
- Coherent
- Constructive
- Destructive

Key Equations

$m\lambda = d \sin\theta$	$\tan\theta = \dfrac{y_m}{L}$

Questions

Act I: Applications of the Mathematical Model

WO.L1.2-02:

Problem Statement: Young's double slit experiment is just one example of two source interference. Which of the following is/are necessary to see a spatial interference pattern on a screen opposite two slits?

(1) A single frequency source.
(2) A coherent source of light.
(3) Experimental apparatus needs to be in air.
(4) The source needs to be a light source.

WO.L1.2-03:

Problem Statement: For the double slit experiment, we use a mathematical model $m\lambda = d\sin(\theta)$.

Where in this model is the path length difference?

(1) m
(2) λ
(3) d
(4) $d\sin(\theta)$
(5) $\sin(\theta)$
(6) λd

WO.L1.2-04:

Problem Statement: On observing a double slit experiment, you measure the distance from the slits to the screen to be 1.0 m. You also measure the distance between the central bright spot and the 3rd maximum to be 55 cm. You read on the device that the slits are a distance 4.36 μm apart.

(a) At what angle theta from the central maximum does the 3rd maximum appear?

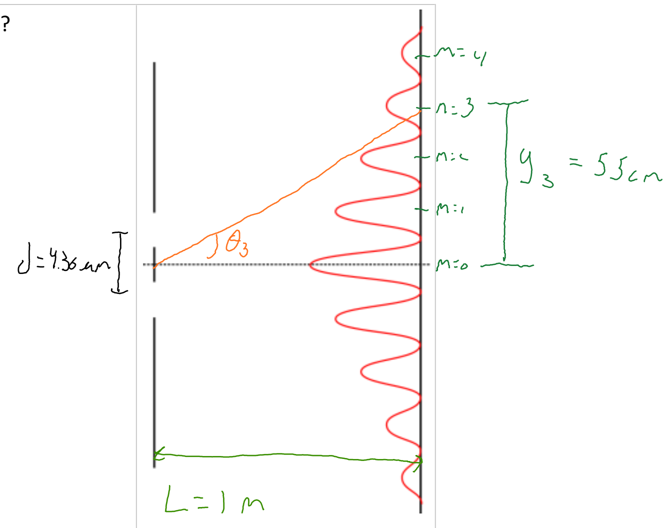
$d \sin\theta_m = m\lambda$ $\qquad$ $y_m = L \tan\theta_m$

✓ ? ? $\qquad$ ✓ ✓ ?

$d \sin\theta_3 = 3\lambda$ $\qquad$ $y_3 = L \tan\theta_3$

$\theta_3 = \tan^{-1}\left(\frac{y_3}{L}\right)$

$\boxed{\theta_3 \simeq 28.8^\circ}$



(b) Find the wavelength of the laser, ***λ*.**

$d \sin\theta_3 = 3\lambda$

$(4.36 \times 10^{-6}) \sin(28.8) = 3\lambda$

$\boxed{\lambda \approx 700 \text{ nm}}$

WO.L1.2-05:

Problem Statement: On observing a double slit experiment, you measure the distance from the slits to the screen to be 1.0 m. You read on the device that the slits are a distance 11.2 µm apart. You also read on the laser that the wavelength it emits is 700 nm. We want to find the distance between the 2nd and 3rd dark fringes on the screen.

(a) What is the distance $\mathbf{y_2}$ for the 2nd dark fringe?

DOUBLE SLIT → DARK SPOTS → $PLD = (m + \frac{1}{2})\lambda$

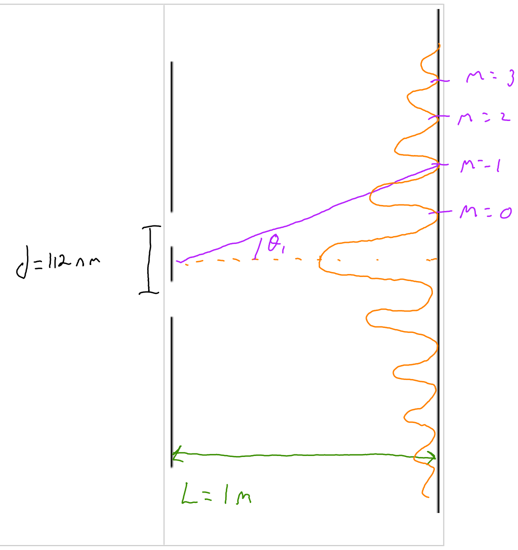
$d\sin\theta_n = (m+\frac{1}{2})\lambda$ $\quad y_n = L\tan\theta_n$

$d\sin\theta_1 = (1+\frac{1}{2})\lambda$ $\quad y_1 = L\tan\theta_1$

$(112\times10^{-6})\sin\theta_1 = \frac{3}{2}(700\times10^{-9})$ $\quad y_1 = (1)\tan(\quad)$

$\theta_1 \simeq 5.37938°$

$\boxed{y_1 \approx 9.42\text{ cm}}$



(b) What is the distance $\mathbf{y_3}$ for the 3rd dark fringe?

$d\sin\theta_2 = (2+\frac{1}{2})\lambda$ $\quad y_2 = L\tan\theta_2$

⋮

$\theta_2 \simeq 8.9893°$

$\boxed{y_2 \simeq 15.8\text{ cm}}$

(c) What is the distance from the 2nd to the 3rd dark fringe on the screen?

$y_2 - y_1 \simeq \boxed{6.38\text{ cm}}$

Act II: Small Angle Approximation

WO.L1.2-06:

Problem Statement: Work together in teams to complete the following table.

θ (deg)	θ (rad)	sin(θ)	tan(θ)	cos(θ)
0.0573	0.001	0.000999999833...	0.001000000003	0.9999995
5.73	0.1	0.0998334....	0.1003346721	0.9950041653
11.5	0.2	0.1986693308	0.2027100355	0.9800665778
17.2	0.3	0.2955202067	0.3093362496	0.9553364891
22.9	0.4	0.3894183423	0.4227932187	0.921060994
28.6	0.5	0.4794255386	0.5463024898	0.8775825619
57.3	1.0	0.8414709848	1.557407725	0.5403023059

(a) Which of the following relationships are true for some portion of the above table?

(1) $\sin\boldsymbol{\theta} = \tan\boldsymbol{\theta}$
(2) $\sin\boldsymbol{\theta} = \cos\boldsymbol{\theta}$
(3) $\cos\boldsymbol{\theta} = \tan\boldsymbol{\theta}$
(4) $\boldsymbol{\theta} = \tan\boldsymbol{\theta}$
(5) $\boldsymbol{\theta} = \sin\boldsymbol{\theta}$
(6) $\boldsymbol{\theta} = \cos\boldsymbol{\theta}$

FOR "SMALL ANGLES"

HOW SMALL? ... YOU DECIDE :)

I'M OK WITH $\theta < 17°$

(b) For what angles are these equations valid?

(c) Are these equations useful when using degrees? Radians?

NOTE ... $5° \neq \sin(5°)$

WO.L1.2-07:

Problem Statement: When observing a double slit interference pattern, you note that it is in a regime where the small angle approximation is a safe approximation to make.

Which of the following relationships can you assume to be true?

(1) $\boldsymbol{\lambda \gg d}$
(2) $\boldsymbol{\lambda \ll d}$
(3) $\boldsymbol{L \gg y_m}$
(4) $\boldsymbol{L \ll y_m}$
(5) $\boldsymbol{m \gg d}$
(6) $\boldsymbol{m \ll d}$

$d\sin\theta_m = m\lambda$ $\qquad y_m = L\tan\theta_m$ $\qquad$ ALL θ'S

$d\theta_m \approx m\lambda$ $\qquad y_m \approx L\theta_m$ $\qquad$ SMALL θ

$\theta_m \approx \frac{m\lambda}{d}$ $\qquad \frac{y_m}{L} \approx \theta_m$

SMALL ... SO $\frac{\lambda}{d} \approx$ SMALL $\qquad$ SMALL

SO..

small .. so

$$\frac{\lambda}{d} \approx \text{small}$$

$$\boxed{\lambda << d}$$

small

so..

$$\boxed{y_m << L}$$

WO.L1.2-08:

Problem Statement: When observing a double slit interference pattern for 500 nm wavelength light incident on slits with a separation of *around* 1 mm, you notice that the distance between the 3rd and 4th order bright spots is 5 mm.

If the screen is 15 meters from the slits, what is the precise slit separation?

CHECK IF SMALL θ

$$\frac{\lambda}{d} \overset{?}{=} \text{small}$$

$$\frac{500 \times 10^{-9}}{1 \times 10^{-3}} = 5 \times 10^{-4} \text{ m}$$

small ∴

$$d\theta_m \approx m\lambda \qquad y_m \approx L\theta_m$$

$$\theta_m = \frac{m\lambda}{d} \qquad \theta_m = \frac{y_m}{L}$$

$$\frac{m\lambda}{d} = \frac{y_m}{L}$$

$$y_m \approx \frac{mL\lambda}{d}$$

$$y_4 - y_3 = \frac{4L\lambda}{d} - \frac{3L\lambda}{d}$$

$$y_4 - y_3 = \frac{L\lambda}{d}$$

$$y_4 - y_3 \approx \boxed{0.600 \text{ mm}}$$

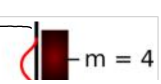
Act III: Proportional Reasoning and Limits of the Model

WO.L1.2-09:

Problem Statement: Suppose the viewing screen in the figure is moved closer to the double slit. What happens to the interference fringes?

(1) They fade out and disappear.

MOVE



(2) They get out of focus.

(3) They get brighter and closer together.

(4) They get brighter and farther apart.

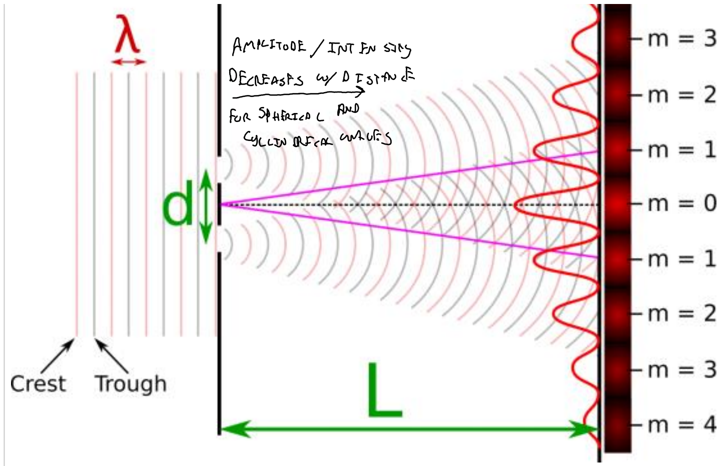
(5) They get brighter but otherwise do not change.

$y_m = L\tan\theta_m$

$\theta_m = \text{CONST}$

$y_m \propto L$

AS $L\downarrow$, $y_m\downarrow$



WO.L1.2-10:

Problem Statement: When observing a double slit experiment, you notice that $\mathbf{y_m}$ for some fringe is getting smaller. Which of the following could be causing this?

(1) The slit width is decreasing.
(2) The slit width is increasing.
(3) The wavelength of light is increasing.
(4) The wavelength of light is decreasing.
(5) The distance between the slits and screen is increasing.

$y_m = L\tan\theta_m$

$L = \text{CONST}$

$y_m \propto \tan\theta_m$

IF $y_m\downarrow$

THEN $\tan\theta_m\downarrow$

AND $\theta_m\downarrow$

$d\sin\theta_m = m\lambda$ (CONST)

$\sin\theta_m \propto \lambda$

IF $\theta_m\downarrow$

THEN $\sin\theta_m\downarrow$

AND $\lambda\downarrow$

OR $d\sin\theta_m = \underset{\sim}{m}\lambda$ (CONST)

$d \propto \frac{1}{\sin\theta_m}$

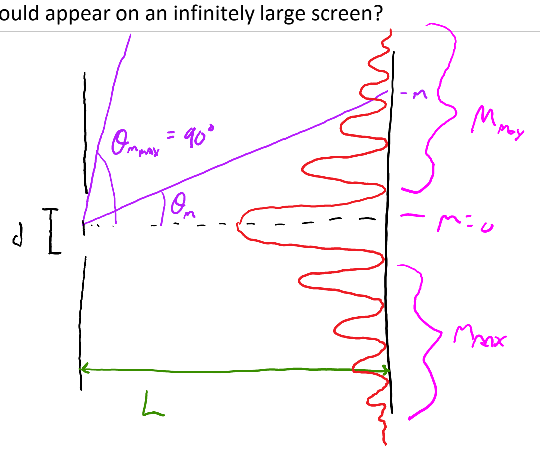
IF $\theta_m\downarrow$

THEN $\sin\theta_m\downarrow$

AND $d\uparrow$

WO.L1.2-11:

Problem Statement: A particular double slit experiment using 600 nm wavelength light and 50 μm slit separation. How many total fringes would appear on an infinitely large screen?



$$d \sin\theta_m = m\lambda$$

$$d \sin(90) = M_{max}\lambda$$

$$M_{max} = \frac{d}{\lambda}$$

$$M_{max} = 83.333$$

BUT M'S ARE INTEGERS

SO $M_{max} = 83$

TOTAL # $= 2M_{max} + 1$

$= \boxed{167 \text{ BRIGHT FRINGES}}$

WO.L1.2-12:

Problem Statement: A particular double slit experiment uses a slit separation **d**. Is there a limit to the wavelength of the light you can use?

(1) Yes, $\lambda < d$
(2) Yes, $\lambda > d$
(3) No, any slit separation will produce an interference pattern.

MAX $\theta = 90°$

$M = 1$ MUST HAVE $\theta < 90°$

$$d \sin\theta_m = m\lambda$$

$$d \sin\theta_m = m\lambda$$

$$\underbrace{\sin\theta_m}_{\text{CAN'T BE} > 1} = \frac{m\lambda}{d}$$

So $\frac{m\lambda}{d} < 1$

$$\boxed{\lambda < d}$$

WO.L1.2-13:

Problem Statement: In a particular double slit experiment you notice that the small angle approximation is a good approximation. If the wavelength decreases by a factor of 2, what happens to $\mathbf{y_2}$?

(1) It quadruples.
(2) It doubles.
(3) It doesn't change.
(4) It decreases by a factor of 2.
(5) It decreases by a factor of 4.

$$d\,\theta_m = m\lambda \qquad y_m = L\theta_m$$

$$\frac{y_m}{L} = \frac{m\lambda}{d}$$

$$y_m = \frac{m\lambda L}{d}$$

IF $L = \text{const}$
$m = \text{const}$
$d = \text{const}$

$$y_m \propto \lambda$$

Conceptual questions for discussion

1. Coming soon to a lecture template near you.

Hints

WO.L1.2-01: No hints.

WO.L1.2-02: No hints.

WO.L1.2-03: No hints.

WO.L1.2-04: No hints.

WO.L1.2-05: No hints.

WO.L1.2-06: No hints.

WO.L1.2-07: No hints.

WO.L1.2-08: No hints.

WO.L1.2-09: No hints.

WO.L1.2-10: No hints.

WO.L1.2-11: No hints.

WO.L1.2-12: No hints.

WO.L1.2-12: No hints.