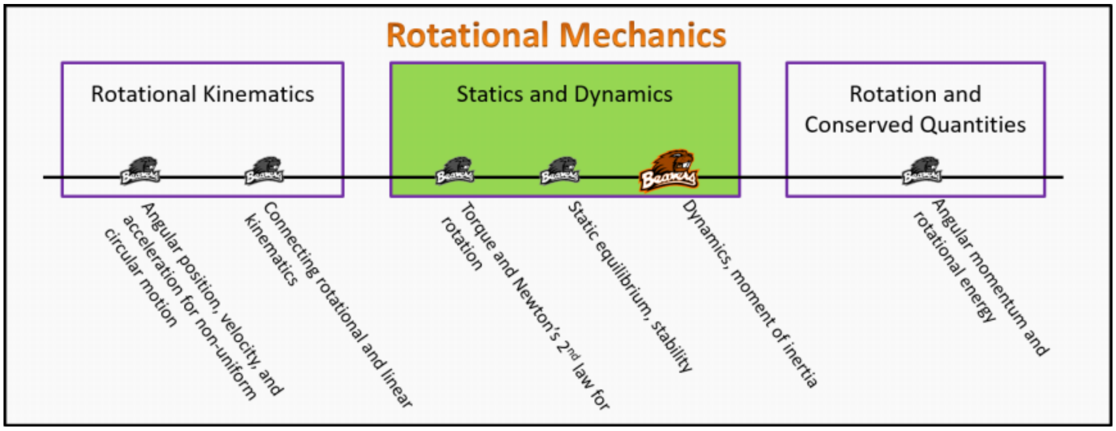
SD.2.L3 - Solutions

Wednesday, January 16, 2019 7:51 PM

Statics and Dynamics
Foundation Stage (SD.2.L3)

lecture 3
Dynamics, momentum of inertia



Textbook Chapters (* Calculus version)

- **BoxSand** :: KC videos (statics & dynamics)
- **Knight** (College Physics : A strategic approach 3rd) :: 7.5 ; 7.6
- ***Knight** (Physics for Scientists and Engineers 4th) :: 12.6 ; 12.7
- **Giancoli** (Physics Principles with Applications 7th) :: 8-5 ; 8-6

Warm up

SD.2.L3-1:

Description: Apply Newton's 2nd law for rotation given torques and moment of inertia to solve for angular acceleration.

Learning Objectives: [?] - Can you identify the objectives from the previous lecture, and this lecture, that this question is relevant to?

Problem Statement: Consider a system with a moment of inertia of 4 kg·m² with one $\boxed{\text{counter-clockwise}}$ torque about reference axis **o** of magnitude $\boxed{12\ \text{N·m}}$, and two clockwise torques about reference axis **o** of magnitudes 8 N·m and 2 N·m. What is the angular acceleration of this system?

$$\Sigma \tau_{EXT,o} = I_o \alpha$$

$$+12\ Nm - 8\ Nm - 2\ Nm = 4\ kg\,m^2\ \alpha$$

$$\boxed{\alpha = 0.5\ \frac{RAD}{s^2}}$$

Selected Learning Objectives

1. Coming soon to a lecture template near you.

Key Terms

- Moment of inertia

Key Equations

Torque
Moment of inertia
Angular acceleration

$$\sum \vec{\tau}_{ext,o} = I_{sys,o}\,\vec{\alpha}_{sys,o}$$

Net (i.e. add up all of the __)
External to system, about reference axis o
System, about reference axis o

In words: The net torque external to the system about reference axis o is equal to the momentum of inertia of the sysem about reference axis o multiplied by the angular acceleration of the system about reference axis o.

Moment of inertia
mass of point particle
perpendicular distance of point particle from reference axis o

$$I_{point\ particle,o} = m\, r_o^2$$

Point particle, about reference axis o

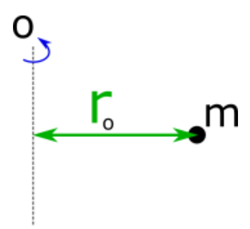
Moment of inertia
mass of point particle
perpendicular distance of point particle from reference axis o

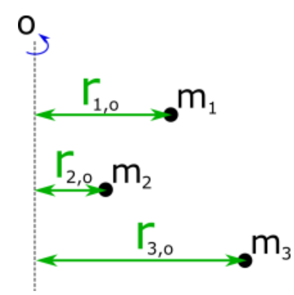
$$I_{sys,o} = \sum m_i\, r_{i,o}^2 = m_1 r_{1o}^2 + m_2 r_{2o}^2 + \dots$$

System, about reference axis o
Net (i.e. add up all of the __)

In words: The moment of inertia for a point particle about axis o is equal to the mass of the point particle times the perpendicular distance of the point particle from o squared.

In words: The moment of inertia for a system of point particles about axis o is equal to the sum of the products between the mass of each point particle and its pendicular distance from o squared.





Key Concepts

- Recall that the net force acting on a system caused the center of mass of the system to accelerate. For a given net force, the magnitude of the acceleration was scaled by the mass of the system. Similarly, Newton's 2nd law for rotation tells us that a net torque acting on a system about a reference axis will cause an angular acceleration. For a given net torque, the magnitude of the angular acceleration is scaled by a new quantity known as the moment of inertia. Thus the moment of inertia is analogous to mass in the sense that both scale the magnitude of acceleration/angular-acceleration given a net force/net torque.
- The moment of inertia depends on the reference axis and on the distribution of the mass about that reference axis. For example, the moment of inertia about a reference axis orientated vertically going through your head to the floor is larger if you stretch your arms out (more mass further away from axis) compared to if you kept your arms at your side.
- For objects that are not point-like particles about a reference axis, you can break the object up into multiple point particles to approximate the moment of inertia of the object.
- A force analysis and a torque analysis can be used together to analysis a scenario, just remember that the net force relates to the acceleration of the center of mass and the net torque relates to the angular acceleration about a reference axis.

Questions

Act I: Identifying equilibrium in systems

SD.2.L3-2:

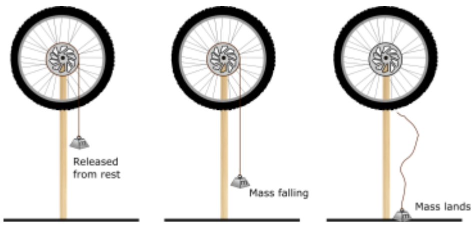
Description: Identify equilibrium status of given systems. (2 minutes + 2 minutes)

Learning Objectives: [1, 12, 13]

Problem Statement: A bicycle wheel is attached to a post as shown in the images below. A rope attached to a mass is wrapped around the inner radius of the wheel and released from rest.

(a) While the weight is falling, the wheel is in

F (1) translational static equilibrium and rotational static equilibrium
F (2) translational dynamic equilibrium only
F (3) translational dynamic and rotational static equilibrium
T (4) translational static equilibrium only
F (5) rotational dynamic equilibrium only



(b) When the rope and the mass are no longer attached to the wheel, the wheel is in

* IGNORE FRICTION + AIR RESISTANCE

T (1) translational static equilibrium and rotational dynamic equilibrium
F (2) translational dynamic equilibrium only
F (3) translational dynamics and rotational static equilibrium
F (4) translational static equilibrium only
F (5) rotational dynamic equilibrium only

SD.2.L3-3:

Description: Conceptual application of torque analysis and force analysis to determine motion of object. (5 minutes + 3 minutes + 2 minutes)

Learning Objectives: [1, 12, 13]

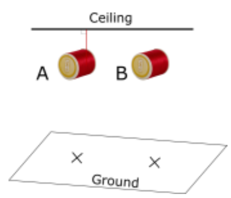
Problem Statement: Two identical spools of thread are released from rest. Spool A has it's thread attached to a ceiling about it, and spool B is not attached to the ceiling.

(a) Which spool reaches the floor first?

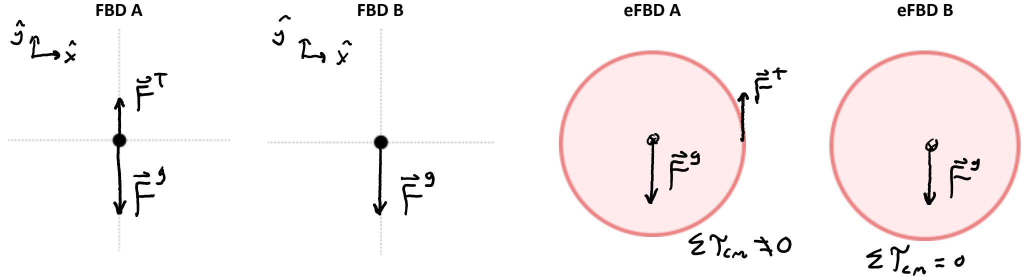
$\Sigma \vec{F} = m\vec{a}$

(1) A
(2) B
(3) Same time

$|\Sigma \vec{F}_A| < |\Sigma \vec{F}_B|$ w/ $m_A = m_B$ $a_A < a_B$



FBD A **FBD B** **eFBD A** **eFBD B**



(b) Where does spool A land on the floor?

(1) To the left of the X
(2) To the right of the X
(3) On the X

B/c $\Sigma F_x = 0 \rightarrow a_x = 0$
w/ $V_{ix} = 0$ NO ΔV_x

(c) Match which state each spool is in.

(1) Translational dynamic equilibrium and rotational dynamics
(2) Translational dynamic equilibrium and rotational dynamic equilibrium
B (3) Translational dynamics and rotational static equilibrium
A (4) Translational dynamics and rotational dynamics

Act II: Moment of inertia

SD.2.L3-4:

Description: Calculate moment of inertia for system of point particles. (4 minutes)

Learning Objectives: [1, 12, 13]

Problem Statement: Consider 3 point particles each of mass **m** arranged in the configuration shown below (equilateral triangle of side lengths **L**). Find the moment of inertia about the axis labeled "**o**".

(1) **m** L^2
(2) 1/2 **m** L^2
(3) 3/4 **m** L^2
(4) 5/4 **m** L^2

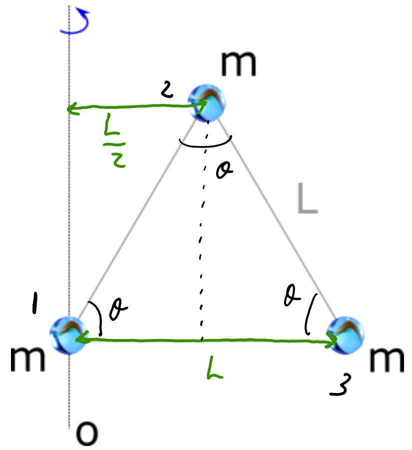
$$I_{sys,o} = \sum_i m_i r_{i,o}^2$$

$$= m_1 \cancel{r_{1o}}^{2} + m_2 r_{2o}^2 + m_3 r_{3o}^2$$

$$= m\left(\frac{L}{2}\right)^2 + m(L)^2$$

$$= \frac{1}{4} mL^2 + mL^2$$

$$\boxed{I_{sys,o} = \frac{5}{4} mL^2}$$



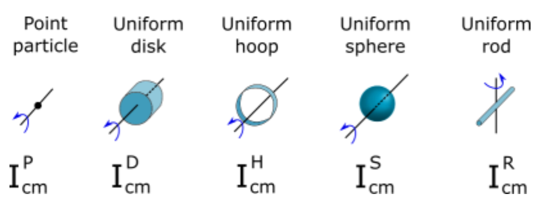
SD.2.L3-5:

Description: Rank moment of inertias. (3 minutes + 3 minute)

Learning Objectives: [1, 12, 13]

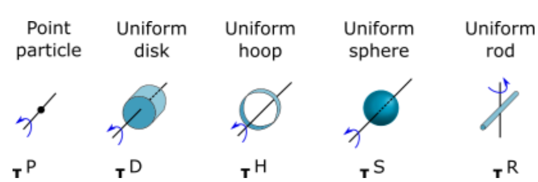
Problem Statement: Consider a point particle, a rod with uniform mass distribution and length **R**, a hoop with uniform mass distribution and radius **R**, a solid disk with uniform mass distribution and radius **R**, and a sphere with uniform mass distribution and radius **R**. All objects have the same mass **m**.

(a) Rank the moment of inertia about the center of mass of each object as shown below.



$$\boxed{I^P_{cm} = 0 < I^R_{cm} < I^S_{cm} < I^D_{cm} < I^H_{cm}}$$

(b) Below shows mathematical representations of moments of inertia. Match each representation with each object about the center of mass as show in the image below.



(1) 0 POINT PARTICLE

(2) 1/12 m R^2 ROD

center of mass as show in the image below.

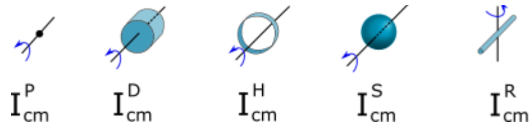
(1) 0 POINT PARTICLE

(2) 1/12 **m R**^{2} ROD

(3) 2/5 **m R**^{2} SPHERE

(4) 1/2 **m R**^{2} DISK

(5) **m R**^{2} HOOP



SD.2.L3-6:

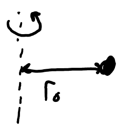
Description: Rank moment of inertias. (4 minutes)

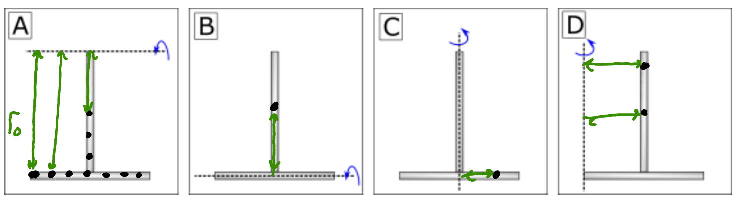
Learning Objectives: [1, 12, 13]

Problem Statement: Four T handles are made from two identical rods of equal mass and length. Rank in order, from largest to smallest, the moment of inertia about the reference axis shown by the dotted lines below.

POINT PARTICLE

$I_o = m r_o^2$





$\boxed{I_A > I_D > I_B > I_C}$

Act III: Dynamics

SD.2.L3-7:

Description: Rotational dynamics problem solving for moment of inertia. (2 minutes + 2 minutes + 6 minutes)

Learning Objectives: [1, 12, 13]

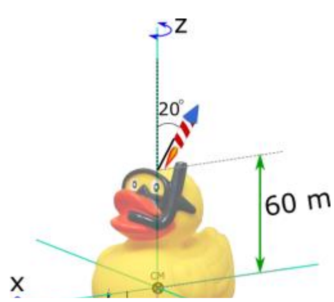
Problem Statement: Mega Puddles, the 417,000-kg giant spaceship designed by U of O architects is falling miserably with two constant 2560 N thrusters stuck on as shown in the image below. While cruising along on his Scooty Puff Jr. Benny determines that the angular acceleration of the spaceship about its center of mass is 0.0477 rad/s^2.

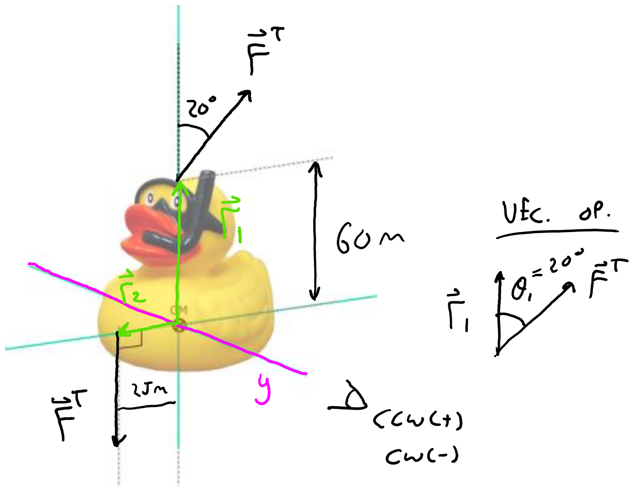
(a) What axis should be used to determine the moment of inertia?

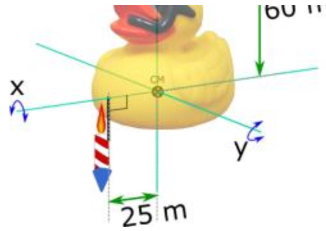
(1) x
(2) y
(3) z
(4) some other axis

(b) Draw an eFBD for the spaceship. The ship is far away from other massive objects.

$\vec{F}^T$







(c) Mega Puddles, the 417,000-kg giant spaceship designed by U of O architects is falling miserably with two constant 2560 N thrusters stuck on as shown in the image below. While cruising along on his Scooty Puff Jr. Benny determines that the angular acceleration of the spaceship about its center of mass is 0.0477 rad/s^2. **What is the moment of inertia about the reference axis you hose in part (a)?**

$$\sum \tau_y = I_y \alpha$$

$$-|\vec{r}_1||\vec{F}^T|\sin\theta_1 + |\vec{r}_2||\vec{F}^T|\cancel{\sin\theta_2}^{\,1} = I_y \alpha$$

$$-(60\text{ m})(2560\text{ N})\sin(20°) + (25\text{ m})(2560\text{ N}) = I_y\,(0.0477\text{ rad/s}^2)$$

$$\boxed{I_y \approx 240000\ \text{kgm}^2}$$

Act IV: Connecting torque analysis to kinematics

SD.2.L3-8:

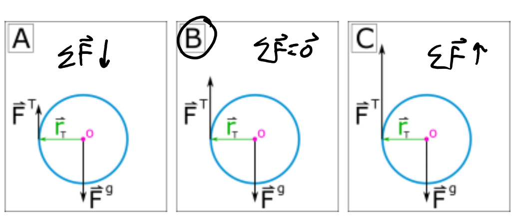
Description: Rotational dynamics and translational static problem solving for linear and rotational quantities. (4 minutes + 5 minutes + …)

Learning Objectives: [1, 12, 13]

Problem Statement: A light string is wrapped around a 0.50-kg hollow 0.080-m-radius hoop. The moment of inertia about the center of mass for a hoop is $\mathbf{I_{cm}} = \mathbf{m\,r^2}$, where **m** is the mass of the hoop and **r** is the radius. The hoop is released from rest above the surface of the earth and the free end of the string is pulled upwards by a hand such that the center of mass of the ring does not move.

$\sum \vec{F} = \vec{0}$

(a) Which eFBD correctly represents this scenario?



(b) Which of the following is the correct expression for the angular acceleration of the hoop?

(1) -g
(2) r g
(3) -g/r
(4) r/g

$$\Sigma \tau_{cm} = I_{cm}\,\alpha$$

FROM FBD

$$|\vec{F}^T| = |\vec{F}^g| = mg$$

$$-|\vec{r}_T|\,|\vec{F}^T|\,\cancel{\sin\theta_T}^{\,1} + \cancel{|\vec{r}_g|}^{\,0}\,|\vec{F}^g|\sin\theta_g = mr^2\alpha$$

$$-r\,|\vec{F}^T| = mr^2\alpha$$

$$-r\,\cancel{m}g = \cancel{m}r^2\alpha$$

$$\boxed{-\frac{g}{r} = \alpha} = 122.5\ \frac{\text{RAD}}{s^2}$$

(c) Find the change in angular position after 0.5 seconds.

$$\Delta\theta = \cancel{\omega_i}^{\,0}\Delta t + \tfrac{1}{2}\alpha\Delta t^2$$

$$\Delta\theta = \tfrac{1}{2}\left(-122.5\ \tfrac{\text{RAD}}{s^2}\right)(0.5)^2$$

$$\boxed{\Delta\theta \simeq 15.31\ \text{RAD}}$$

(d) What is the arc length for a point on the rim during this 0.5 seconds?

$$S = r\,\Delta\theta$$

$$= (0.08\,\text{m})(15.31\ \text{RAD})$$

$$\boxed{S \simeq 1.225\ \text{m}}$$

(e) Which of the following is the correct tangential component of acceleration?

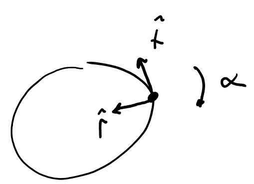
(1) -g
(2) r g
(3) -g/r
(4) r/g

$$a_t = \alpha r$$

$$a_t = \left(-\frac{g}{r}\right)r$$

$$a_t = -g$$

?



COULD BE + IF $\hat{r}$ $\hat{t}$

(f) What is the distance a point on the rim traveled during this 0.5 seconds?

$$\Delta x = \cancel{V_{i,x}}^{\,0}\Delta t + \tfrac{1}{2}a_x\Delta t^2$$

$$\Delta x = \tfrac{1}{2}a_t\,\Delta t^2$$

$$\Delta x = \tfrac{1}{2}(g)(0.5)^2$$

$$\boxed{\Delta x \simeq 1.225\ \text{m}}$$

SAME AS PART d :)

(g) When a marked spot on the hoop goes through 180 degrees, how much string has been unwound?

STRING ATTACHED @ RADIUS

$$S = r\,\Delta\theta$$

$$= (0.08\,\text{m})(\pi\ \text{RAD})$$

$$\boxed{S \approx 0.251\ \text{m}}$$

(h) How fast (in m/s) is a point on the rim going at the 0.5 second mark?

OPTION 1 (LINEAR)

$$V_f = \cancel{V_i}^{\,0} + a_t\Delta t$$

OPTION 2 (ROTATIONAL)

$$\omega_f = \cancel{\omega_i}^{\,0} + \alpha\Delta t$$

OPTION 1 (LINEAR)

$V_{fx} = \cancel{V_{ix}}^{0} + a_x \Delta t$

$V_{fx} = a_t \Delta t$

$V_{fx} = g \Delta t$

$\boxed{V_{fx} = 4.9 \text{ m/s}}$

SAME :)

OPTION 2 (ROTATIONAL)

$\omega_f = \cancel{\omega_i}^{0} + \alpha \Delta t$

$\omega_f = \frac{g}{r} \Delta t$ w/ $V_t = \omega r$

$\omega = \frac{V_t}{r}$

$\frac{V_t}{r} = \frac{g \Delta t}{r}$

$V_t = g \Delta t$

$\boxed{V_t = 4.9 \text{ m/s}}$

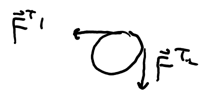
SD.2.L3-9:

Description: Rotational dynamics and translational dynamics problem solving for linear and rotational quantities. (3 minutes + 3 minutes + 4 minutes + 8 minutes + ...)

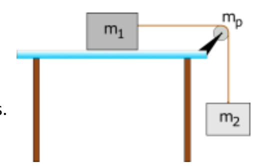
Learning Objectives: [1, 12, 13]

Problem Statement: A block $\mathbf{m_1}$ = 5 kg rests on a horizontal frictionless surface. A rope connecting the block is draped around a uniform solid disk (pulley) of mass $\mathbf{m_p}$ = 2 kg to a hanging mass of $\mathbf{m_2}$ = 10 kg. When released from rest, we wish to determine information about the acceleration of each block, distances they travel after a given amount of time, etc...

(a) You have seen this problem before with two masses and an ideal pulley, only then the pulley had negligible mass. Now we are considering a more real system where the pulley does have mass. We are still assuming negligible friction in the pulley's bearing. Which of the following features of the system have changed?



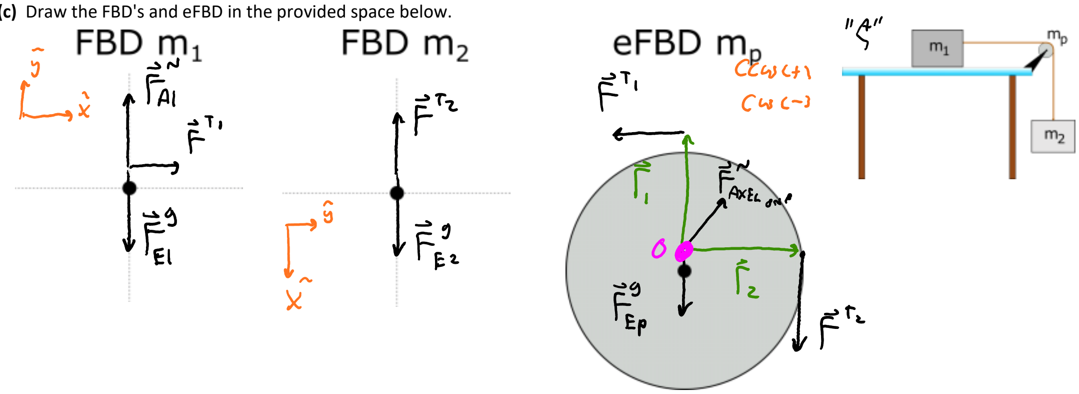
T (1) $|\vec{\mathbf{F}}^{T_1}|$ is no longer equal to $|\vec{\mathbf{F}}^{T_2}|$
F (2) $|\vec{\mathbf{a}}_1|$ is no longer equal to $|\vec{\mathbf{a}}_2|$
T (3) The acceleration of block 1 will be less now that the pulley has mass.
F (4) The final speed of mass 2 will be larger after a given unit of time now that the pulley has mass.



(b) Which of the following are constraints on the system?

T (1) $|\vec{\mathbf{a}}_1| = |\vec{\mathbf{a}}_2|$
T (2) $|\vec{\mathbf{a}}_1| = |\mathbf{a}_t|$
T (3) $|\mathbf{a}_t| = |\boldsymbol{\alpha}|\mathbf{r}$
T (4) $|\vec{\mathbf{v}}_1| = |\vec{\mathbf{v}}_2|$
T (5) $|\vec{\mathbf{v}}_2| = |\mathbf{v}_t|$
T (6) $|\mathbf{v}_t| = |\boldsymbol{\omega}|\mathbf{r}$

(c) Draw the FBD's and eFBD in the provided space below.



(d) A block $\mathbf{m_1}$ = 5 kg rests on a horizontal frictionless surface. A rope connecting the block is draped around a uniform solid disk (pulley) of mass $\mathbf{m_p}$ = 2 kg to a hanging mass of $\mathbf{m_2}$ = 10 kg. When released from rest, **what is the acceleration of the center of mass of block 1?**

$\Sigma F_x = m_1 a_{1x}$ $\Sigma F_x = m_2 a_{2x}$ $\Sigma \tau_o = I_o \alpha$

$$\Sigma F_x = m_1 a_{1x}$$

$$|\vec{F}^{T_1}| = m_1 a_x$$

$$\Sigma F_x = m_2 a_{2x}$$

$$-|\vec{F}^{T_2}| + |\vec{F}^{g}_{E2}| = m_2 a_x$$

$$-|\vec{F}^{T_2}| + m_2 g = m_2 a_x$$

ADD TOGETHER

$$|\vec{F}^{T_1}| - |\vec{F}^{T_2}| = m_1 a_x + m_2 a_x - m_2 g$$

$$\Sigma \tau_o = I_o \alpha$$

$$|\vec{r}_1||\vec{F}^{T_1}| \cancel{\sin\theta_1} - |\vec{r}_2||\vec{F}^{T_2}| \cancel{\sin\theta_2} = -\tfrac{1}{2} m_p R^2 \alpha$$

$$R|\vec{F}^{T_1}| - R|\vec{F}^{T_2}| = -\tfrac{1}{2} m_p R^2 \alpha$$

$$|\vec{F}^{T_1}| - |\vec{F}^{T_2}| = -\tfrac{1}{2} m_p R \alpha$$

w/ $a_x = a_t = \alpha R$

$\alpha = \frac{a_x}{R}$

$$|\vec{F}^{T_1}| - |\vec{F}^{T_2}| = -\tfrac{1}{2} m_p a_x$$

EQUAL

$$m_1 a_x + m_2 a_x - m_2 g = -\tfrac{1}{2} m_p a_x$$

$$m_1 a_x + m_2 a_x + m_p a_x = m_2 g$$

$$\boxed{a_x = \frac{m_2 g}{(m_1 + m_2 + m_p)}} \approx \boxed{5.76 \text{ m/s}^2}$$

(e) What is the angular acceleration of the 10 cm radius pulley?

$$a_t = \alpha r$$

$$a_x = \alpha r$$

$$\alpha = \frac{a_x}{r} = \frac{5.76 \text{ m/s}^2}{0.1 \text{ m}} \approx \boxed{57.6 \tfrac{\text{RAD}}{s^2}}$$

(f) How far does the block 1 travel after the first 2 seconds after being released from rest?

$$\Delta X_1 = \cancel{v_{i1x}}^{0} \Delta t + \tfrac{1}{2} a_{1x} \Delta t^2$$

$$\Delta X = \tfrac{1}{2}(5.76 \text{ m/s}^2)(2)^2$$

$$\boxed{\Delta X \approx 11.5 \text{ m}}$$

(g) How many revolutions does the pulley make in this 2 seconds?

$$S = r\Delta\theta$$

$$S = r\Delta\theta$$

$$\Delta x = r\Delta\theta$$

$$\Delta\theta = \frac{\Delta x}{r} \approx \boxed{115 \text{ RAD}}$$

(h) How far does a point on the edge of the pulley travel in this 2 seconds?

$$\boxed{S = \Delta x = 11.5 \text{ m}}$$

(i) What are the tensions in each section of rope?

$$|\vec{F}^{T_1}| = m_1 a_{1x}$$

$$\boxed{|\vec{F}^{T_1}| = 28.8 \text{ N}}$$

$$|\vec{F}^{T_2}| = m_2 g - m_2 a_{2x}$$

$$\boxed{|\vec{F}^{T_2}| = 40.4 \text{ N}}$$

Conceptual questions for discussion

1. Do you agree with the following statement: Every object has only one moment of inertia. Support your answer with examples.
2. What happens to the moment of inertia about Earth's rotational axis, if anything, when tall building are built near the equator?
3. Is there a location on Earth that you could build a tall building without affecting the moment of inertia of Earth about its rotational axis?
4. Use your knowledge of Newton's laws of motion for rotation to explain why sports car use wheels of lighter mass than normal wheels.
5. Tight rope walkers often use long poles as seen in the cartoon below. Use your knowledge of Newton's laws of motion for rotation to explain why it is advantage to use such poles when walking along a tight rope.

Hints

SD.2.L3-1: No hints.

SD.2.L3-2: No hints.

SD.2.L3-3: Draw FBDs for each spool to determine which one lands first and where A lands relative to the X.

SD.2.L3-4: The "**r**" in the moment of inertia equation is the perpendicular distance from the reference axis.

SD.2.L3-5: No hints.

SD.2.L3-6: Break the T-handle up into multiple pieces of equal mass and look at their "**r**" distance away from the reference axis.

SD.2.L3-7: No hints.

SD.2.L3-8: No hints.

SD.2.L3-9: No hints.