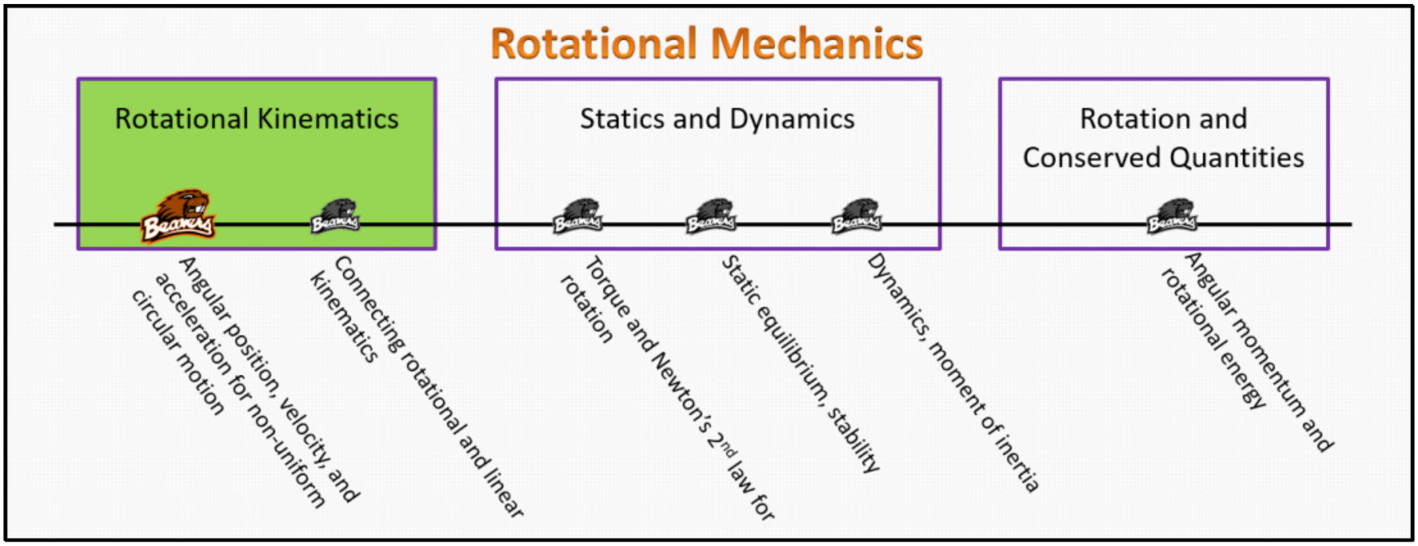
Solutions

Wednesday, January 9, 2019 7:12 AM

# Rotational Kinematics
## Foundation Stage (RK.2.L1)

**lecture 1**

**Angular position, velocity, and acceleration for non-uniform circular motion**




**Textbook Chapters** (* Calculus version)

- **BoxSand** :: KC videos ( rotational kinematics )
- **Knight** (College Physics : A strategic approach 3rd) :: 3.8 ; 7.1 ; 7.2
- ***Knight** (Physics for Scientists and Engineers 4th) :: 4.4 ; 4.5 ; 4.6 ; 12.1
- **Giancoli** (Physics Principles with Applications 7th) :: 5-1 ; 5-2 ; 5-3 ; 5-4 ; 8-1

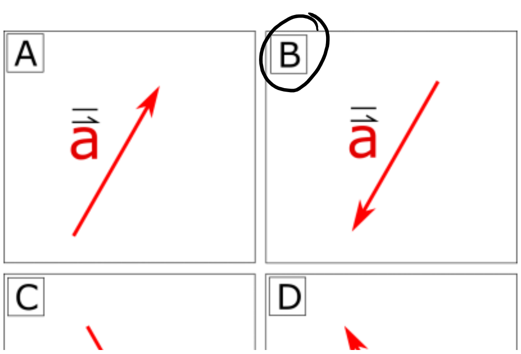
### Warm up

**RK.2.L1-1:**

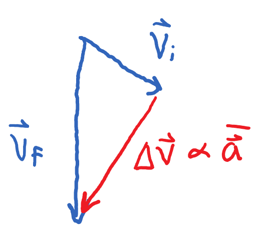
**Description:** Given initial and final velocity vectors, determine direction of average acceleration.

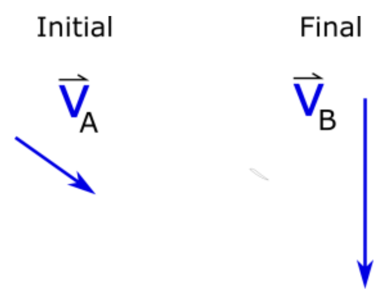
**Learning Objectives:** [?] - Can you identify the objectives from the previous lecture, and this lecture, that this question is relevant to?

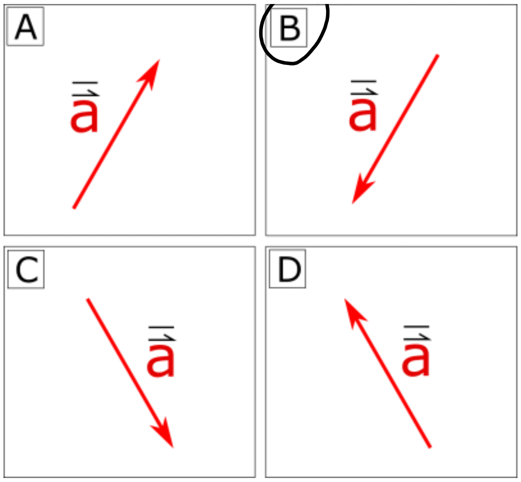
**Problem Statement:** A GPS is used to display the instantaneous velocity vector of Matt the moth at snapshot **A** and **B** as seen below. Use a vector operation diagram to determine the direction of the average acceleration for Matt between snapshots **A** and **B**.



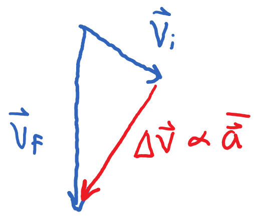

**Vector Operation** ($\Delta\vec{v}$)



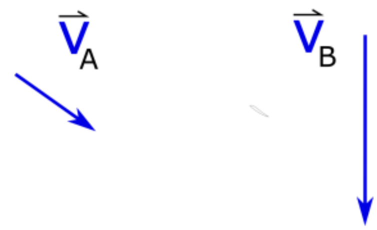






**Vector Operation** ($\Delta\vec{v}$)




Initial | Final




$$\vec{a} = \frac{\Delta\vec{v}}{\Delta t}$$

**Selected Learning Objectives**

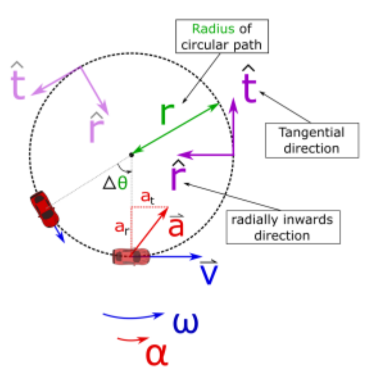
1. Coming soon to a lecture template near you.

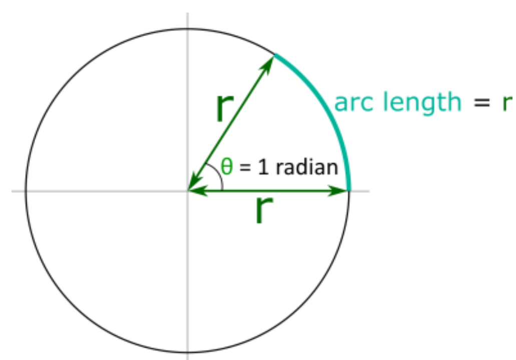
**Key Terms**

- Angular position
- Change in angular position (i.e. angular displacement)
- Angular velocity
- Angular acceleration
- Tangential velocity
- Tangential acceleration
- Radial velocity
- Radial acceleration
- Frequency
- Period

**Key Equations**

Polar coordinates and non-UCM




$\vec{v} = \langle v_r, v_t \rangle \Longrightarrow \vec{v} = \langle 0, v_t \rangle$

$\vec{a} = \langle a_r, a_t \rangle \Longrightarrow \vec{a} = \langle a_r, a_t \rangle$

1 Revolution = 360°

1 Revolution = 2π Radians

360° = 2π Radians

$360° = 2\pi$ Radians

Change in angular position (angular displacement)

Final angular position

Initial angular position

$$\Delta\theta = \theta_f - \theta_i$$

In words: The change in angular position is equal to the final angular position minus the initial angular position.

Average angular velocity

Change in angular position (angular displacement)

$$\bar{\omega} = \frac{\Delta\theta}{\Delta t}$$

Change in time

In words: The average angular velocity is equal to the change in angular position divided by the change in time.

Average angular acceleration

Change in angular velocity

$$\bar{\alpha} = \frac{\Delta\omega}{\Delta t}$$

Change in time

In words: The average angular acceleration is equal to the change in angular velocity divided by the change in time.

Frequency

$$f = \frac{1}{T}$$

Period

*In words:* Frequency is equal to one divied by the period.

Angular velocity

frequency

$$\omega = 2\pi f$$

In words: The angular velocity is equal to 2 pi times the frequency.

## Key Concepts

- When an object is traveling in a circle, the position, velocity, and acceleration (in Cartesian coordinates) are all functions of time. To simplify this analysis, we use polar coordinates which uses an angular position to determine where the object is along the circle. The position, velocity, and acceleration in polar coordinates are vectors, however, the vector nature of these quantities in polar coordinates is beyond the scope of this class. Thus we use angular position (**θ**), angular velocity (**ω**), and angular acceleration (**α**) which are all components of their respective position, velocity, and acceleration vectors in polar coordinates. Even though angular position, angular velocity, and angular acceleration are components (i.e. scalars), they can still be positive or negative. The most widely used convention to determine if these angular components are positive or negative is the following: if the object is traveling counter-clockwise (CCW) then the angular displacement and angular velocity are positive, if traveling clockwise (CW) then negative. The sign of angular acceleration can then be determined based off of the definition of average angular acceleration.
- It's important to emphasize that angular position, angular velocity, and angular acceleration are not vectors; they are components of vectors in polar coordinates. Their sign (+ or -) is determined by the CCW(+) and CW(-) convention. However, we would still like a nice way to communicate which way an object is traveling around a circle with a physical representation, thus we use curved arrows to show CW or CCW direction. Be careful, these angular quantities are not vectors and the curved arrows are not vectors (vectors must be straight arrows).
- Much like assigning an **x** = 0 or **y** = 0 location for linear motion, you can chose where to define your **θ** = 0.
- In polar coordinates, the radial direction points radially inwards or outwards depending on your choice. Choosing radially inwards is often wise because it makes the radial component of acceleration a positive value.
- In circular motion, the velocity vector is always tangent to the circular path. In polar coordinates, the tangential direction is tangent to the circular path; you can choose which way so long as it's tangent.
- Recall in Uniform Circular Motion (UCM), the acceleration is always pointing towards the center, thus there is no tangential component of acceleration. The velocity is always tangent to the circular path in UCM, thus no radial component.
- If an object is traveling in a circle and speeding up or slowing down (i.e. non-UCM), then the acceleration has both a radial and tangential component. The velocity is still always tangent to the circular path, thus no radial component.
- Frequency is the number of repeated events per one unit of time (e.g. 1000 revolutions per minute, 5 cycles per second, 70 beats per second, 1020 hand claps per minute, etc..)
- Period is the amount of time per one repeated event (e.g. 10 seconds per 1 revolution, 24 hours per 1 revolution, 5 seconds per 1 clap, 42 seconds per one click noise).

## Questions

### Act I: Linear acceleration vector

**RK.2.L1-2:**

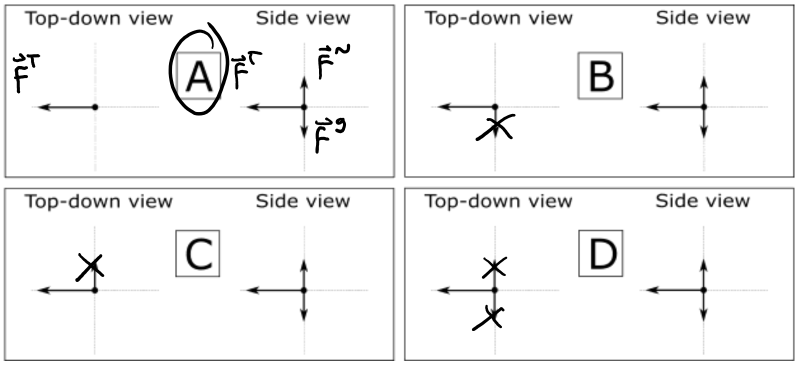
**Description:** Choose FBDs for objects traveling in a circle. (2 minutes + 3 minutes)

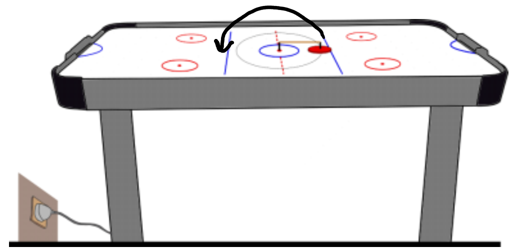
**Learning Objectives: [1, 12, 13]**

**Problem Statement:** An air hockey puck attached to a string is traveling counter-clockwise in a circle at a constant speed (i.e. UCM).

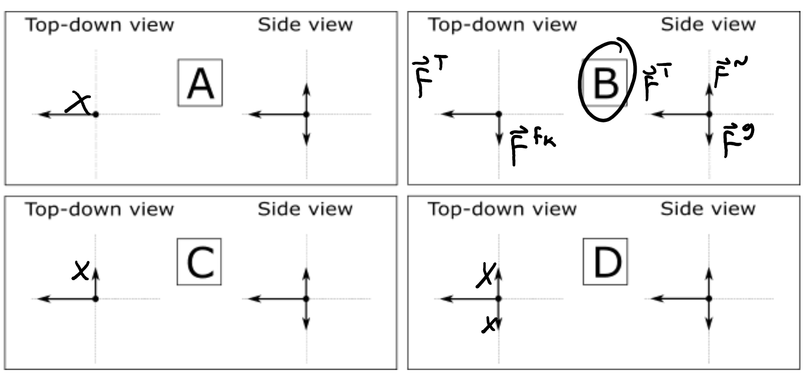
CONSTANT SPEED & RADIUS

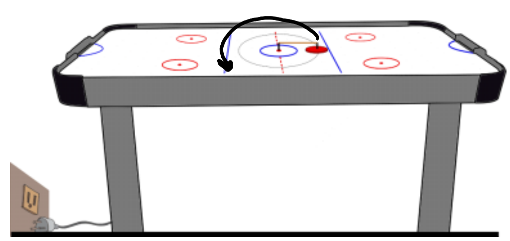
**(a)** Choose the pair of FBDs that correctly describe this scenario.



**(b)** The air is suddenly turned off causing the puck to slow down as it is traveling counter-clockwise around the circle. Choose the pair of FBDs that correctly describe this scenario.



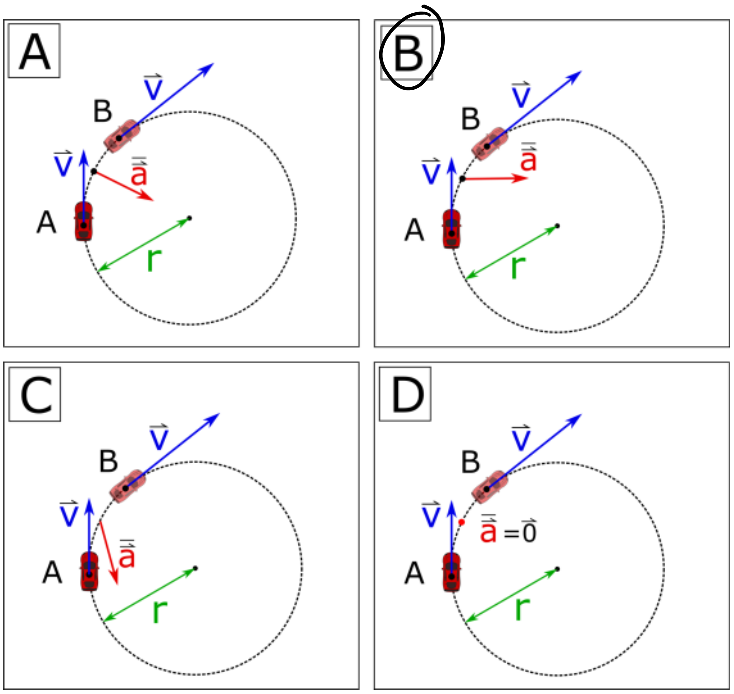



**RK.2.L1-3:**

**Description:** Given initial and final velocity vectors, find direction of average acceleration. (3 minutes)

**Learning Objectives:** [1, 12, 13]

**Problem Statement:** Consider a car speeding up as it travels in a circle. Two snapshots are taken at **A** and **B** with the car's velocity vector shown at each time. What is the direction of the change in velocity and thus the average acceleration of the car between **A** and **B**?




$\vec{a} = \frac{\Delta \vec{v}}{\Delta t}$

VEC. OP.

$\Delta \vec{v} \propto \vec{a}$

$\vec{v}_i$

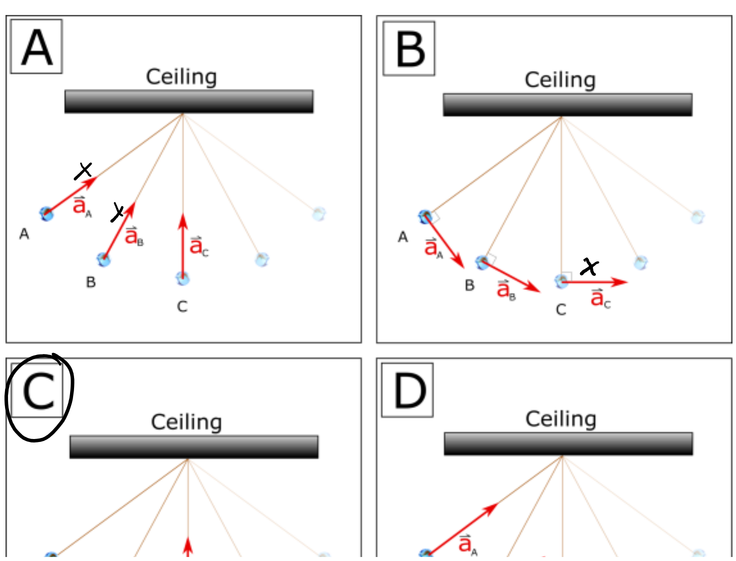
$\vec{v}_f$

**RK.2.L1-4:**

**Description:** Choose the correct acceleration vectors for a pendulum. (4 minutes)

**Learning Objectives:** [1, 12, 13]

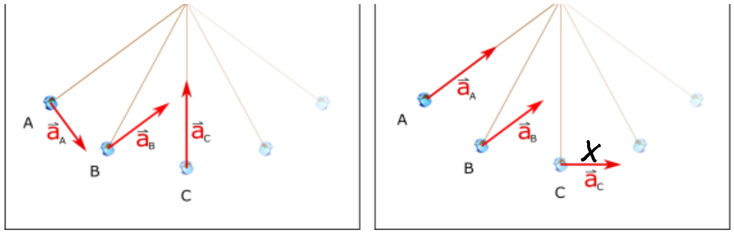
**Problem Statement:** A pendulum is released from rest at snapshot **A**. The pendulum speeds up at a decreasing rate reaching its maximum speed at location **C**. Which of the following set of acceleration vectors correctly describe this scenario.




*NOTE....

...THE FASTER AN OBJECT SPEEDS UP AROUND A CIRCLE, THE MORE

SPEEDS UP AROUND A CIRCLE, THE MORE TANGENT $\vec{a}$ IS TO THE CIRCULAR PATH.




**RK.2.L1-5:**

**Description:** Sketch the acceleration vector of a car going in a circle if given the acceleration of a different car going around the same circle. (4 minutes)

**Learning Objectives:** [1, 12, 13]

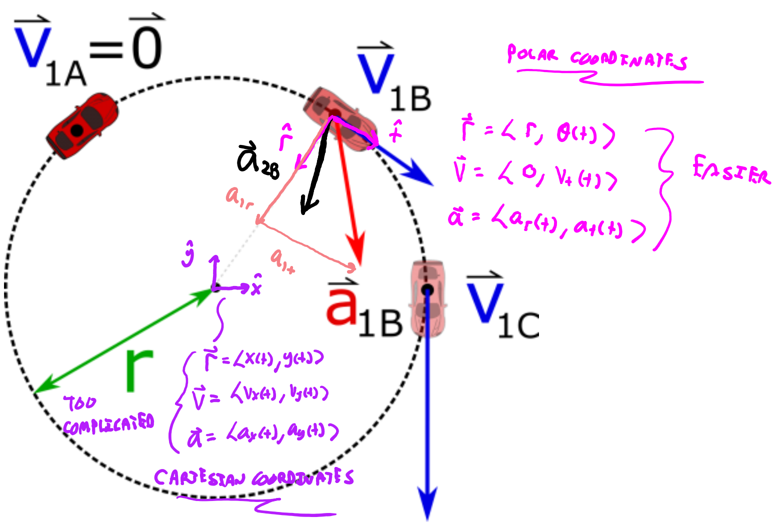
**Problem Statement:** Two cars starts from rest at location **A** and uniformly speed up around the same circle. Car 1 takes 5 seconds to reach location **C** while car 2 takes 15 seconds to reach **C**. The image below shows the acceleration vector of car 1 at location **B**. Sketch the acceleration vector of car 2 at location **B**.

SAME DIST

DIFF. TIMES $\Delta t_1 < \Delta t_2$

SO 1 SPEEDS UP FASTER

THUS $\vec{a}_{1B}$ IS MORE TANGENT




## Act II: Average quantities

**RK.2.L1-6:**

**Description:** Match the form like mathematical representations for average quantities. (2 minutes)

**Learning Objectives:** [1, 12, 13]

**Problem Statement:** Below there are 8 mathematical representations, 4 are linear forms and 4 are rotational forms. Match each rotational mathematical representation with the linear analog.

**(a)** $\theta$

**(b)** $\Delta\theta = \theta_f - \theta_i$

**(c)** $\omega = \frac{\Delta\theta}{\Delta t}$

**(d)** $\alpha = \frac{\Delta\omega}{\Delta t}$

$\theta \longleftrightarrow x$

$\Delta\theta \longleftrightarrow \Delta x$

$\omega \longleftrightarrow v_x$

$\alpha \longleftrightarrow a_x$

$v_x = \frac{\Delta x}{\Delta t}$

$x$

$a_x = \frac{\Delta v_x}{\Delta t}$

$\Delta x = x_f - x_i$

**RK.2.L1-7:**

**Description:** Convert degrees and revolutions to radians and radians to degrees. (2 minutes + 2 minutes + 2 minutes)

**Learning Objectives:** [1, 12, 13]

**Problem Statement:** What is a radian?

**(a)** How many degrees is 1 radian?

$$1 \text{ RAD} \times \frac{360°}{2\pi \text{ RAD}} \approx \boxed{57.3°}$$

**(b)** How many radians is 45 degrees?

$$45° \times \frac{2\pi \text{ RAD}}{360°} \approx \boxed{0.785 \text{ RAD.}}$$

**(c)** How many revolutions is 2 radians?

$$2 \text{ RAD} \times \frac{1 \text{ REV}}{2\pi \text{ RAD}} = \boxed{0.318 \text{ REV.}}$$

**RK.2.L1-8:**

**Description:** Determine the sign of angular kinematic quantities. (3 minutes + 3 minutes + 3 minutes)

**Learning Objectives:** [1, 12, 13]

**Problem Statement:** Fill in each table with the sign (+, -, or 0) for each quantities given the situations described below.

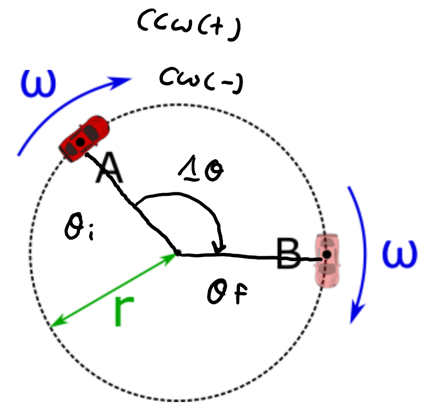
**(a)** A car is traveling clockwise around a circle at a constant speed.

**Problem Statement:** Fill in each table with the sign (+, -, or 0) for each quantities given the situations described below.

**(a)** A car is traveling clockwise around a circle at a constant speed.

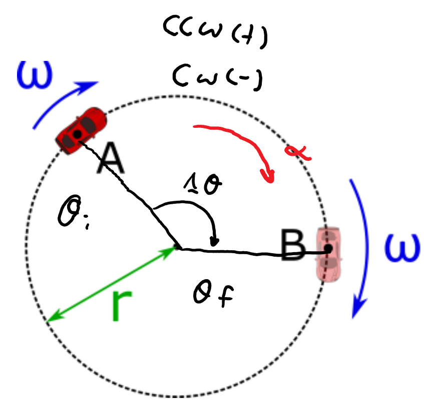
$\Delta\omega = 0$

| $\Delta\theta$ | $\omega$ | $\alpha$ |
|---|---|---|
| – | – | 0 |



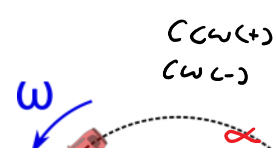

**(b)** A car is traveling clockwise around a circle and speeding up.

| $\Delta\theta$ | $\omega$ | $\alpha$ |
|---|---|---|
| – | – | – |

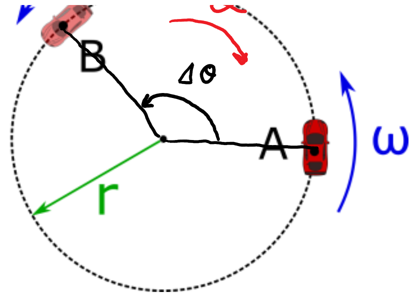



**(c)** A car is traveling counter-clockwise around a circle and slowing down.

| $\Delta\theta$ | $\omega$ | $\alpha$ |
|---|---|---|
| | | |



| | | |
|---|---|---|
| + | + | − |




**RK.2.L1-9:**

**Description:** Compare angular velocity of two objects going around the same circle. (3 minutes)

**Learning Objectives:** [1, 12, 13]

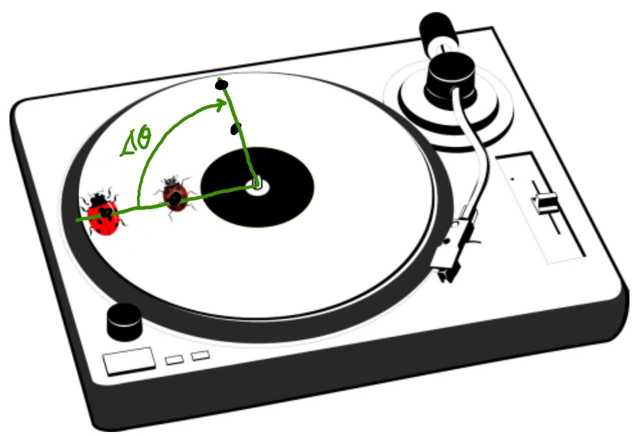
**Problem Statement:** A ladybug sits at the outer edge of a record player which is spinning clockwise, and a gentleman bug sits halfway between her and the axis of rotation. The record player makes 33 1/3 revolutions every 1 minute. The gentleman bug's angular speed is

(1) half the ladybug's.

(2) the same as the ladybug's.

(3) twice the ladybug's.

(4) impossible to determine without radius and angular speed.




$$|\omega| = \left|\frac{\Delta\theta}{\Delta t}\right|$$

SAME $\Delta\theta$
SAME $\Delta t$

**RK.2.L1-10:**

**Description:** Convert frequency to period and angular velocity. (3 minutes)

**Learning Objectives:** [1, 12, 13]

**Problem Statement:** A standard record player record player makes 33 1/3 revolutions every 1 minute.

**(a)** How much time in seconds does it take for the record player to make one full revolution (i.e. what is the period)? → $T = 1.8$ SEC.

$$\frac{1 \text{ MIN}}{33\frac{1}{3} \text{ REV}} = \boxed{0.03 \frac{\text{MIN}}{\text{REV}}} \times \frac{60 \text{SEC}}{1 \text{MIN}} = \boxed{1.8 \frac{\text{SEC}}{\text{REV}}}$$

**(b)** What is the frequency of the record player in SI units?

$$f = \frac{1}{T} = \frac{1}{1.8 \text{ SEC}} = \boxed{0.556 \frac{1}{\text{SEC}}} \equiv \boxed{0.556 \text{ Hz}}$$

**(c)** What is the angular velocity of the record player in SI units?

$$\omega = 2\pi f = \boxed{\pm 3.49 \frac{\text{RAD}}{\text{SEC}}}$$

SPINNING CW or CCW?

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## Conceptual questions for discussion

1. Do you agree with the following statement: the frequency of a cd disk at the outer edge is larger than the frequency at the inner edge?
2. Imagine driving along a straight level horizontal road with the cruise control on (i.e. at a constant speed). Is it possible to accelerate the car without stepping on the gas or brake pedal?
3. Sketch the acceleration vector at the three other snapshots of the pendulum as it swings backup to its maximum height in problem RK.2.L1-4.
4. Consider a pendulum like in problem RK.2.L1-4. Is it possible to break the string without pulling vertically down on the pendulum bob? If so, explain the physics behind the mechanism for the string breaking. If not, explain why you must only pull downwards to break the string.
5. *Beyond the scope of this class but fun: The vector nature of these rotational quantities can be found by using your right hand; if you align your fingers so that they can curl in the CW or CCW direction then your thumb points in the direction of that rotational vector quantity. What is the direction of the angular velocity vector for the car at location **B** in problem RK.2.L1-8 part **c**? Also, what is the direction of the angular acceleration vector?
   i. Straight into the page
   ii. Straight out of the page
   iii. Horizontally to the left
   iv. Horizontally to the right
   v. Vertically up on the paper
   vi. Vertically down on the paper

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## Hints

**RK.2.L1-1**: Recall that average acceleration is the change in velocity divided by change in time. Also, a change in a vector quantity can be found by placing the initial and final vectors tail to tail, with the change pointing from the initial to the final.

**RK.2.L1-2**: Recall that the net external force causes an acceleration of the center of mass of the object, thus first determine which direction the acceleration of the puck is.

**RK.2.L1-3**: Use a vector operation diagram to help determine the direction of the average acceleration.

**RK.2.L1-4**: If an object is speeding up and moving in a circle, there must be a component of acceleration in the tangential direction.

Discuss with your neighbors what "speeding up at a decreasing rate" means with regards to tangential acceleration.

**RK.2.L1-5**: No hints.

**RK.2.L1-6**: No hints.

**RK.2.L1-7**: No hints.

**RK.2.L1-8**: No hints.

**RK.2.L1-9**: No hints.

**RK.2.L1-10**: No hints.